GĂSIREA RELAȚIILOR DE RECURENȚĂ PENTRU INTEGRALE DEFINITE

Găsiți o relație de recurență pentru:

1) $I_n = \int_0^1 \frac{x^n}{\sqrt{1+x^2}}\,dx$, $n \geq 1$

2) $I_n = \int_1^e \ln^n x\,dx$, $n \geq 1$

3) $I_n = \int_0^{\pi/2} x^n \cos x\,dx$, $n \geq 1$

4) $I_n = \int_0^{\pi/2} x^n \sin x\,dx$, $n \geq 1$

5) $I_n = \int_0^1 x^n e^x\,dx$, $n \geq 1$

6) $I_n = \int_0^1 \arcsin^n x\,dx$, $n \geq 1$

7) $I_n = \int_0^{\pi/2} e^x \sin x\,dx$

8) $I_n = \int_0^{\pi/4} \operatorname{tg}^n x\,dx$

9) $I_n = \int_0^{\pi/2} \sin^n x\,dx$

10) $I_n = \int_0^{\pi/2} \cos^n x\,dx$

Găsiți o relație de recurență:

$$I_n = \int_0^1 \frac{x^n}{\sqrt{1+x^2}}\,dx$$

$E_1)$ $f(x) = x^{n-1} \Rightarrow f'(x) = (n-1)x^{n-2}$

$g'(x) = \frac{x}{\sqrt{1+x^2}} \Rightarrow g(x) = \sqrt{1+x^2}$

$E_2)$ $I_n = x^{n-1}\sqrt{1+x^2}\,\Big|_0^1 - (n-1)\int_0^1 x^{n-2}\cdot\sqrt{1+x^2}\,dx$

$$I_n = 1 - 0 - (n-1)\int_0^1 \frac{x^{n-2}(1+x^2)}{\sqrt{1+x^2}}\,dx$$

$$I_n = 1 - (n-1)\left(\int_0^1 \frac{x^{n-2}}{\sqrt{1+x^2}}\,dx + \int_0^1 \frac{x^n}{\sqrt{1+x^2}}\,dx\right)$$

$E_3)$ $I_n = 1 - (n-1)(I_{n-2} + I_n)$

$I_n = 1 - (n-1)I_{n-2} - (n-1)I_n$

$\Rightarrow I_n(1+n-1) = 1 - (n-1)I_{n-2} \quad | : n$

$$I_n = \frac{1}{n} - \frac{n-1}{n} I_{n-2}$$

Găsiți o relație de recurență pentru

$$I_n = \int_0^{\pi/2} x^n \cdot \sin x \, dx$$

E1) $f = x^n \Rightarrow f' = n x^{n-1}$

$g' = \sin x \Rightarrow g = -\cos x$

E2) $I_n = -x^n \cos x \Big|_0^{\pi/2} + n \int_0^{\pi/2} x^{n-1} \cos x \, dx$

$I_n = -\left(\left(\frac{\pi}{2}\right)^n \cos\frac{\pi}{2} - 0\cos 0\right) + n \int_0^{\pi/2} x^{n-1} \cos x \, dx$

$I_n = -\left(\frac{\pi}{2}\right)^n + n \underbrace{\int_0^{\pi/2} x^{n-1} \cos x \, dx}_{J_{n-1}}$

E3) pt J_{n-1}, $f = x^{n-1} \Rightarrow f' = (n-1) x^{n-2}$

$g' = \cos x \Rightarrow g = \sin x$

$J_{n-1} = x^{n-1} \sin x \Big|_0^{\pi/2} - (n-1) \underbrace{\int_0^{\pi/2} x^{n-2} \sin x \, dx}_{I_{n-2}}$

$\Rightarrow J_{n-1} = \left(\frac{\pi}{2}\right)^{n-1} \cdot \underbrace{\sin\frac{\pi}{2}}_{1} - 0 - (n-1) I_{n-2}$

$\Rightarrow J_{n-1} = \left(\frac{\pi}{2}\right)^{n-1} - (n-1) I_{n-2}$

E4) $I_n = -\left(\frac{\pi}{2}\right)^n + n\left(\left(\frac{\pi}{2}\right)^{n-1} - (n-1) I_{n-2}\right)$

$$\boxed{I_n = -\left(\frac{\pi}{2}\right)^n + n \cdot \left(\frac{\pi}{2}\right)^{n-1} - n(n-1) I_{n-2}}$$

Găsiți relația de recurență:

$$I_n = \int_0^{\pi/2} \sin^n x \, dx$$

$E_1)$ $f(x) = \sin^{n-1} x \Rightarrow f'(x) = (n-1)\sin^{n-2} x \cdot \cos x$

$g'(x) = \sin x \Rightarrow g(x) = -\cos x$

$E_2)$ $I_n = -\sin^{n-1} x \cdot \cos x \Big|_0^{\pi/2} + (n-1)\int_0^{\pi/2} \sin^{n-2} x \cdot \cos^2 x \, dx$

$$I_n = -\left(\sin^{n-1}\frac{\pi}{2}\underbrace{\cos\frac{\pi}{2}}_{0} - \underbrace{\sin^{n-1} 0}_{0}\cdot\cos 0\right) + (n-1)\int_0^{\pi/2} \sin^{n-2} x \cdot (1-\sin^2 x)\, dx$$

$$I_n = (n-1)\left(\underbrace{\int_0^{\pi/2} \sin^{n-2} x \, dx}_{I_{n-2}} - \underbrace{\int_0^{\pi/2} \sin^n x \, dx}_{I_n}\right)$$

$$I_n = (n-1)\, I_{n-2} - (n-1)\, I_n$$

$E_3)$ $I_n\,(1 + (n-1)) = (n-1)\, I_{n-2}$

$$n\, I_n = (n-1)\, I_{n-2}$$

$$\Rightarrow I_n = \frac{n-1}{n} I_{n-2}$$