CALCULUL INTEGRALELOR DEFINITE

FOLOSIND METODA INTEGRĂRII PRIN PĂRȚI

Calculați:

1) $\int_0^1 x^2 e^x dx$

2) $\int_e^{e^2} \ln x \, dx$

3) $\int_0^{\pi/3} x \cdot \sin x \, dx$

4) $\int_0^1 x e^{-x} dx$

5) $\int_0^{\pi/2} e^x \sin 2x \, dx$

6) $\int_0^1 (x + e^{-2x}) dx$

7) $\int_0^{\pi/2} x \cdot \cos 3x \, dx$

8) $\int_1^e x^3 \cdot \ln^2 x \, dx$

9) $\int_0^{\pi/2} e^{-x} \cdot \sin \frac{x}{5} dx$

10) $\int_0^{\pi/2} e^{-\frac{x}{4}} \cos(2x) dx$

11)

Calculați $\int_0^1 x^2 \cdot e^x dx$

E_1) prin părți, $\int f \cdot g' = f \cdot g - \int f' g$ sau

$$\int_a^b f \cdot g' = f \cdot g \Big|_a^b - \int_a^b f' \cdot g$$

E_2) $I = \int_0^1 x^2 \cdot e^x dx$

$f(x) = x^2 \Rightarrow f'(x) = 2x$

$g'(x) = e^x \Rightarrow g(x) = e^x$

E_3) $I = x^2 \cdot e^x \Big|_0^1 - \int_0^1 2x \cdot e^x dx$

$I = e - 0 - 2 \underbrace{\int_0^1 x \cdot e^x dx}_{I_1}$

E_4) $I_1 = \int_0^1 x e^x dx$

$f(x) = x \Rightarrow f'(x) = 1$

$g'(x) = e^x \Rightarrow g(x) = e^x$

$I_1 = x e^x \Big|_0^1 - \int_0^1 e^x dx =$

$= 1e^1 - 0 - e^x \Big|_0^1 = e - (e^1 - e^0) = e - e + 1 = 1$

E_5) $I = e - 2(0+1) = e - 2$

Calculați: $\int_e^{e^2} \ln x \, dx$

$E_1)$ $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$

$g'(x) = 1 \Rightarrow g(x) = x$

$E_2 | I = x \cdot \ln x \Big|_e^{e^2} - \int_e^{e^2} \frac{1}{x} \cdot x \, dx$

$= e^2 \ln e^2 - e \ln e - x \Big|_e^{e^2}$

$= e^2 \cdot 2 \ln e - e - (e^2 - e)$

$= 2e^2 - e - e^2 + e = e^2$