DERIVAREA INTEGRALELOR DEFINITE

Calculați:

1) $f'(x)$ știind $f(x)=\int_{\frac{\pi}{6}}^{\arccos x} \ln(2+3\cos^2 t)\,dt$

2) $f'(x)$ știind $f(x)=\int_0^{x^2} t^3\cdot e^{t^2+t}\,dt$

3) $f'(x)$ știind $f(x)=\int_1^{\arcsin x} (\sin t+1)e^{\sin^2 t}\,dt$

4) $\lim\limits_{x\to 0} \dfrac{\int_0^x \ln(1+t^2)\,dt}{\sin^2 x}$

5) $\lim\limits_{x\to 0} \dfrac{\int_x^{2x} \sin^3 t\,dt}{x^3-4x^5}$

6) $\lim\limits_{x\to 0} \dfrac{\int_0^{x^2} \ln(1+t+t^2)\,dt}{\int_0^x \ln(1+t^2)\,dt}$

7) $\lim\limits_{x\to\infty} \dfrac{\int_0^x (\operatorname{arctg} t)^2\,dt}{\sqrt{x^2+4}}$

8) $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\int_0^{\cos x} e^{t^2}\,dt}{\int_0^{\operatorname{ctg} x} \ln(t^2+2)\,dt}$

9) $\lim\limits_{x\to 0} \dfrac{\int_0^{x^2} \sin t^2}{\int_0^x e^{t^2}\,dt}$

10) $\lim\limits_{x\to 1} \dfrac{\int_1^x \frac{\ln t}{t}\,dt}{(x-1)^2}$

11) $\lim\limits_{x\to 0} \dfrac{\int_0^x t\cdot\sqrt{2-t^2}\,dt}{x^2}$

12) Pct de inflexiune pt

$F_n(x)=\int_0^x t^n\cdot e^{-t}\,dt$

Calculați: $f'(x)$ pentru $f(x)=\int_{\pi/6}^{\arccos x} \ln(2+3\cos^2 t)\,dt$

E1) Fie $g:\left[\frac{\pi}{6}, \arccos t\right] \to \mathbb{R}$, $g(t)=\ln(2+3\cos^2 t)$

g continuă pe $\left[\frac{\pi}{6}; \arccos t\right] \Rightarrow g$ admite primitive,

fie $G(t)$ o primitivă a sa

E2) $f'(x)=\left(\int_{\pi/6}^{\arccos x} \ln(2+3\cos^2 t)\,dt\right)'=$

$=\left(G(\arccos x)-G\left(\frac{\pi}{6}\right)\right)'=$

$=G'(\arccos x)\cdot(\arccos x)'-0$

$=g(\arccos x)\cdot\left(-\frac{1}{\sqrt{1-x^2}}\right)=$

$=\ln(2+3\cos^2(\arccos x))\cdot\left(-\frac{1}{\sqrt{1-x^2}}\right)=$

$=\ln(2+3x^2)\cdot\left(-\frac{1}{\sqrt{1-x^2}}\right)$

Fie $f:(0;\infty)\to\mathbb{R}$, $f(x)=\frac{\ln x}{x}$. Calculați

$$\lim_{x\to 1}\frac{\int_1^x f(t)dt}{(x-1)^2}$$

E_1) studiem în ce caz ne situăm

$$l_1=\lim_{x\to 1}\int_1^x f(t)dt=\int_1^1 f(t)dt=0$$

E_2) $l=\lim_{x\to 1}\frac{\int_1^x f(t)dt}{(x-1)^2}\overset{\frac{0}{0}}{\underset{l'H}{=}}\lim_{x\to 1}\frac{\left(\int_1^x f(t)dt\right)'}{\left((x-1)^2\right)'}=$

$$=\lim_{x\to 1}\frac{f(x)}{2(x-1)}=\lim_{x\to 1}\frac{\frac{\ln x}{x}}{2(x-1)}=$$

$$=\lim_{x\to 1}\frac{\ln x}{2x^2-2x}$$

E_3) $l\overset{\frac{0}{0}}{\underset{l'H}{=}}\lim_{x\to 1}\frac{\frac{1}{x}}{4x-2}=\frac{1}{2}$

Obs.: s-a folosit $\left(\int_a^x f(t)dt\right)'=f(x)$

Fie $f(x) = x\sqrt{2-x^2}$, calculați

$$\lim_{x\to 0} \frac{\int_0^x f(t)\,dt}{x^2}$$

E_1) Dacă se poate, calculăm $\int_0^x t\sqrt{2-t^2}\,dt$

însă în cazul nostru nu se poate (complicat)

E_2) $l = \lim_{x\to 0} \int_0^x f(t)\,dt = \int_0^0 f(t)\,dt = 0$

$\Rightarrow$ venim în cazul $\frac{0}{0}$

E_3) $l = \lim_{x\to 0} \frac{\int_0^x f(t)\,dt}{x^2} \overset{\frac{0}{0}}{\underset{l'H}{=}}$

$$= \lim_{x\to 0} \frac{f(x)}{2x} = \lim_{x\to 0} \frac{x\sqrt{2-x^2}}{2x} = \frac{\sqrt{2}}{2}$$

Obs.: s-a folosit $\left(\int_a^x f(t)\,dt\right)' = f(x)$

Fie $F_n : (0;\infty) \to \mathbb{R}$, $F_n(x) = \int_0^x t^n e^{-t}\,dt$

Determinați punctele de inflexiune pentru F_n

E1) Calculez $F_n''(x)$

E2) $F_n'(x) = \left(\int_0^x t^n e^{-t}\,dt\right)' =$

E3) Se poate folosi direct relația

$$\left(\int_a^x f(t)\,dt\right)' = f(x)$$ dar numai

dacă avem integrală de la număr la x

E4) $\Rightarrow F_n'(x) = \left(\int_0^x t^n e^{-t}\,dt\right)' = x^n e^{-x}$

E5) $F_n''(x) = \left(x^n e^{-x}\right)' = n x^{n-1} e^{-x} + x^n e^{-x}(-1) =$

$$= x^{n-1} e^{-x}(n - x)$$

E6) $F_n''(x) = 0 \Rightarrow$ $x^{n-1} e^{-x} = 0 \Rightarrow$ $x = 0 \notin A_{F_n}$ sau $e^{-x} = 0$ fals

sau

$n - x = 0 \Rightarrow x = n$

E7)

x	0		n	
F_n''	$+$	$+$	0	$-$
F_n	$($	$)$		$\cap$

$\Rightarrow x = n$ not inflexiune