5 CALCULUL PRIMITIVELOR DE TIPUL $\int \sqrt{x^2-a^2}\,dx$, $\int \sqrt{x^2+a^2}\,dx$, $\int \sqrt{a^2-x^2}\,dx$

Calculați:

1) $\int \sqrt{x^2-4}\,dx$

2) $\int \sqrt{9-x^2}\,dx$

3) $\int \sqrt{x^2-16}\,dx$

4) $\int \sqrt{x^2-1}\,dx$

5) $\int \sqrt{x^2+3}\,dx$

6) $\int \sqrt{5-x^2}\,dx$

7) $\int \sqrt{x^2+7}\,dx$

8) $\int \sqrt{4-x^2}\,dx$

9) $\int \sqrt{3-x^2}\,dx$

10) $\int \sqrt{x^2-8}\,dx$

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Calculați $\int \sqrt{1-x^2}\,dx$

$E_1)\ I=\int \sqrt{1-x^2}\,dx=\int \frac{\sqrt{1-x^2}\cdot\sqrt{1-x^2}}{1}\,dx=$

$=\int \frac{1-x^2}{\sqrt{1-x^2}}\,dx=\underbrace{\int \frac{1}{\sqrt{1-x^2}}\,dx}_{I_1}-\underbrace{\int \frac{x^2}{\sqrt{1-x^2}}\,dx}_{I_2}$

$E_2)\ I_1=\int \frac{1}{\sqrt{1-x^2}}\,dx=\arcsin\frac{x}{1}+C$

$E_3)\ I_2=\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$f=x \to f'=1$

$g'=\frac{x}{\sqrt{1-x^2}} \to g=-\sqrt{1-x^2}$

$I_2=-x\sqrt{1-x^2}+\underbrace{\int \sqrt{1-x^2}\,dx}_{I}$

$E_4)\ I=I_1-I_2 \Rightarrow I=\arcsin x+x\sqrt{1-x^2}-I$

$\Rightarrow 2I=\arcsin x+x\sqrt{1-x^2}+C$

$I=\frac{1}{2}(\arcsin x+x\sqrt{1-x^2})+C$

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Calculați: $I=\int \sqrt{4-x^2}\,dx$

$E_1)\ I=\int \frac{\sqrt{4-x^2}}{1}dx=\int \frac{4-x^2}{\sqrt{4-x^2}}dx \Rightarrow$

$\Rightarrow I=4\underbrace{\int \frac{1}{\sqrt{4-x^2}}dx}_{I_1}-\underbrace{\int \frac{x^2}{\sqrt{4-x^2}}dx}_{I_2}$

$E_2)\ I_1=\int \frac{1}{\sqrt{4-x^2}}dx=\arcsin\frac{x}{2}+C$

$E_3)\ I_2=\int \frac{x^2}{\sqrt{4-x^2}}dx$

$f(x)=x \Rightarrow f'(x)=1$

$g'(x)=\frac{x}{\sqrt{4-x^2}} \Rightarrow g(x)=\int \frac{x}{\sqrt{4-x^2}}dx=-\sqrt{4-x^2}$

$E_4)\ I_2=x\cdot(-\sqrt{4-x^2})-\int 1\cdot(-\sqrt{4-x^2})dx$

$I_2=-x\sqrt{4-x^2}+\underbrace{\int \sqrt{4-x^2}\,dx}_{I} \Rightarrow$

$\Rightarrow I_2=-x\sqrt{4-x^2}+I$ înlocuim în $E_1)$

$E_5)\ I=4\arcsin\frac{x}{2}+x\sqrt{4-x^2}-I$

$2I=4\arcsin\frac{x}{2}+x\sqrt{4-x^2}+C \Rightarrow I=\frac{1}{2}\left(x\sqrt{4-x^2}+4\arcsin\frac{x}{2}\right)+C$

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Calculați: $I=\int \sqrt{8+4x+x^2}\,dx$

$E_1)\ I=\int \sqrt{x^2+4x+8}\,dx \xrightarrow[a(x+\frac{b}{2a})^2-\frac{\Delta}{4a}]{\text{forma canonică}}$

$\Rightarrow I=\int \sqrt{1(x+2)^2+4}\,dx$

$E_2)$ not $t=x+2 \mid (\,)' \Rightarrow dt=dx$

$I(t)=\int \sqrt{t^2+4}\,dt=\int \frac{t^2+4}{\sqrt{t^2+4}}dt=\underbrace{\int \frac{t^2\,dt}{\sqrt{t^2+4}}}_{I_1}+4\underbrace{\int \frac{dt}{\sqrt{t^2+4}}}_{I_2}$

$E_3)$ Pt $I_1=\int \frac{t^2}{\sqrt{t^2+4}}dt$

$f(t)=t \Rightarrow f'(t)=1$

$g'(t)=\frac{t}{\sqrt{t^2+4}} \Rightarrow g(t)=\sqrt{t^2+4}$

$I_1=t\sqrt{t^2+4}-\int \sqrt{t^2+4} \Rightarrow \boxed{I_1=t\sqrt{t^2+4}-I}$

$E_4)\ I_2=\int \frac{dt}{\sqrt{t^2+4}}=\ln\left(t+\sqrt{t^2+4}\right)+C$

$E_5)\ I=I_1+4I_2 \Rightarrow t\sqrt{t^2+4}-I+4\ln\left(t+\sqrt{t^2+4}\right)=I$

$2I=t\sqrt{t^2+4}+4\cdot\ln\left(t+\sqrt{t^2+4}\right)+C$

$I=\frac{1}{2}\left(t\sqrt{t^2+4}+4\ln\left(t+\sqrt{t^2+4}\right)\right)+C$

$I(x)=\frac{1}{2}\left((x+2)\sqrt{x^2+4x+8}+4\ln\left(x+2+\sqrt{x^2+4x+8}\right)\right)+C$

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