CALCULUL PRIMITIVELOR FOLOSIND A DOUA METODĂ A SCHIMBĂRII DE VARIABILĂ

Calculați (a 2-a metod och. var)

1) $\int \frac{\sqrt{x}}{x+2}\,dx$ și $\int \frac{tg^2x+1}{tgx+1}\,dx$

2) $\int e^{\sqrt{x}}\,dx$ și $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}\,dx$

3) $\int x\sqrt[3]{x-2}\,dx$ și $\int \frac{dx}{x\sqrt{3x+1}}$

4) $\int \frac{dx}{1+e^x}$ și $\int \frac{e^{2x}}{1+e^x}\,dx$

5) $\int \frac{1}{x+\sqrt{x}}\,dx$ și $\int \frac{x+1}{1+\sqrt{x+2}}\,dx$

6) $\int \frac{1}{x\sqrt{1+x}}\,dx$ și $\int \frac{tgx}{1+tgx}\,dx$

7) $\int \sqrt{1+\sqrt{x}}\,dx$ și $\int \frac{1+x}{1+\sqrt{x+2}}\,dx$

8) $\int tg^2x\,dx$

Prof. Ovidiu Bădescu

Calculați: a) $\int \frac{\sqrt{x}}{x+2}\,dx$

b) $\int \frac{tg^2 x+1}{tg x+1}\,dx$

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a) $I=\int \frac{\sqrt{x}}{x+2}\,dx$

$t=\sqrt{x}\,|()^2 \Rightarrow x=t^2\,|()' \Rightarrow dx=2t\,dt$

$I(t)=\int \frac{t}{t^2+2}\cdot 2t\,dt=2\int \frac{t^2}{t^2+2}\,dt \Rightarrow$

$I(t)=2\int \frac{t^2+2-2}{t^2+2}\,dt=2\left(\int 1\,dt-2\int \frac{1}{t^2+2}\,dt\right)=$

$=2\left(t-2\cdot\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{2}}\right)+C$

$$\boxed{I(x)=2\left(\sqrt{x}-\frac{2}{\sqrt{2}}\operatorname{arctg}\frac{\sqrt{x}}{\sqrt{2}}\right)+C}$$

b) $I=\int \frac{tg^2 x+1}{tg x+1}\,dx$

$t=tg\,x \Rightarrow x=\operatorname{arctg} t\,|()' \Rightarrow dx=\frac{1}{1+t^2}\,dt$

$I(t)=\int \frac{t^2+1}{t+1}\cdot\frac{1}{1+t^2}\,dt=\int \frac{1}{t+1}\,dt \Rightarrow$

$I(t)=\ln|t+1|+C$

$$\boxed{I(x)=\ln|tg\,x+1|+C}$$

Prof. Ovidiu Badescu

Calculați, folosind a 2-a metodă de sch. de variabilă: a) $\int x\sqrt{x+1}\,dx$

b) $\int \frac{x}{\sqrt[3]{x-2}}\,dx$

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a) E1) $I=\int x\sqrt{x+1}\,dx$

not $t=\sqrt{x+1}\,|()^2 \Rightarrow t^2=x+1 \Rightarrow x=t^2-1\,|()' \Rightarrow dx=2t\,dt$

E2) $I(t)=\int (t^2-1)\cdot t\cdot 2t\,dt=2\int (t^4-t^2)\,dt \Rightarrow$

$I(t)=2\cdot\left(\frac{t^5}{5}-\frac{t^3}{3}\right)+C$

E3) $I(x)=2\left(\frac{\sqrt{(x+1)^5}}{5}-\frac{\sqrt{(x+1)^3}}{3}\right)+C$

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b) E1) $I=\int \frac{x}{\sqrt[3]{x-2}}\,dx$

not $t=\sqrt[3]{x-2}\,|()^3 \Rightarrow t^3=x-2 \Rightarrow x=t^3+2\,|()' \Rightarrow$

$\Rightarrow dx=3t^2\,dt$

E2) $I(t)=\int \frac{t^3+2}{t}\cdot 3t^2\,dt=\int 3t(t^3+2)\,dt=$

$=3\int (t^4+2t)\,dt=3\left(\frac{t^5}{5}+2\cdot\frac{t^2}{2}\right)+C$

E3) $I(x)=3\left(\frac{\sqrt[3]{(x-2)^5}}{5}+\sqrt[3]{(x-2)^2}\right)+C$

Prof. Ovidiu Bădescu