CALCULUL PRIMITIVELOR FOLOSIND PRIMA METODĂ DE SCHIMBARE DE VARIABILĂ

Calculați (prima met de sch var)

1) $\int (2x+1)^5 dx$ și $\int x(x^2+1)^7 dx$

2) $\int \frac{x^3}{(x-1)^{10}} dx$ și $\int \frac{x^2 dx}{2x^3+1}$

3) $\int \sqrt{2x+1}\, dx$ și $\int \frac{x\,dx}{x^4+6x^2+5}$, $t=x^2$

4) $\int x\sqrt{1+x}\, dx$ și $\int x\sqrt{x^2+3}\, dx$

5) $\int \frac{x}{e^{x^2}} dx$ și $\int \frac{e^x}{e^{2x}+4} dx$

6) $\int \frac{\sin(\ln x)}{x} dx$ și $\int \frac{dx}{x \ln x \ln(\ln x)}$

7) $\int \sin^2 x \cdot \cos x\, dx$ și $\int \frac{\sin x + \cos x}{\cos x - \sin x} dx$

8) $\int (\operatorname{tg} x + \operatorname{tg}^3 x)\, dx$ și $\int \frac{\arcsin x}{\sqrt{1-x^2}} dx$

9) $\int x^2 \sqrt{x^3+1}\, dx$ și $\int \frac{\ln(\arccos x)\, dx}{\sqrt{1-x^2}\cdot \arccos x}$

10) $\int \frac{1}{x(1+\ln^2 x)} dx$ și $\int \frac{x}{\sqrt{x^4-1}} dx$

11) $\int \frac{x^2}{1+x^6} dx$ și $\int \frac{8x^3+6x}{2x^4+3x^2+10} dx$

12) $\int x^2 e^{x^3} dx$ și $\int (x^2+4) e^{x^3+12x} dx$

13) $\int (2x+1)^3 \cdot \sqrt{1+5(2x+1)^4}\, dx$ și $\int \frac{\operatorname{arctg}^2 x}{1+x^2} dx$

14) $\int \frac{(1+\ln x)^7}{x} dx$ și $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

15) $\int \frac{x}{e^{x^2}} dx$ și $\int \left(\frac{x}{x^5+7}\right)^4 dx$, $t=x^5+7$

16) $\int (\operatorname{tg}^n x + \operatorname{tg}^{n+2} x)\, dx$

Prof. Ciprian Bădescu

Calculați: a) $\int \sqrt{2x+1}\,dx$ b) $\int \frac{x}{x^4+6x^2+5}\,dx$

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a) $I=\int \sqrt{2x+1}\,dx$

$t=2x+1 \mid ()' \Rightarrow dt=2dx$

$I(t)=\int \sqrt{t}\cdot\frac{dt}{2}=\frac{1}{2}\int t^{\frac{1}{2}}dt=\frac{1}{2}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$

$I(t)=\frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C \Rightarrow I(x)=\frac{1}{3}(2x+1)^{\frac{3}{2}}+C$

$I(x)=\frac{1}{3}\cdot\sqrt{(2x+1)^3}+C$

b) $I=\int \frac{x}{x^4+6x^2+5}\,dx$

$t=x^2 \mid ()' \Rightarrow dt=2x\,dx \Rightarrow x\,dx=\frac{dt}{2}$

$I(t)=\int \frac{\frac{dt}{2}}{t^2+6t+5}=\frac{1}{2}\int \frac{1}{t^2+6t+5}\,dt$ forma canonică

$=\frac{1}{2}\int \frac{1}{(t+3)^2-4}\,dt$

$=\frac{1}{2}\int \frac{1}{(t+\frac{6}{2})^2-\frac{16}{4}}\,dt$

not $t+3=u \mid ()' \Rightarrow dt=du$

$I(u)=\frac{1}{2}\int \frac{1}{u^2-4}\,du=\frac{1}{2}\cdot\frac{1}{2\cdot 2}\ln\left|\frac{u-2}{u+2}\right|+C$

$I(t)=\frac{1}{8}\ln\left|\frac{t+1}{t+5}\right|+C \Rightarrow \boxed{I(x)=\frac{1}{8}\ln\left|\frac{x^2+1}{x^2+5}\right|+C}$

Prof. Ovidiu Bădescu

Calculați a) $\int x\sqrt{1+x}\,dx$

b) $\int x\sqrt{x^2+3}\,dx$

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a) E_1) $I=\int x\sqrt{1+x}\,dx$

not $t=\sqrt{1+x}\,|()^2 \Rightarrow t^2=1+x\,|()' \Rightarrow 2t\,dt=dx$

din $t^2=1+x \Rightarrow x=t^2-1$

E_2) $I=\int (t^2-1)\cdot t\cdot 2t\,dt=2\int(t^4-t^2)dt=$

$=2\left(\frac{t^5}{5}-\frac{t^3}{3}\right)+C$

E_3) $I=2\left(\frac{(\sqrt{1+x})^5}{5}-\frac{(\sqrt{1+x})^3}{3}\right)+C$

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b) $I=\int x\sqrt{x^2+3}\,dx$

E_1) not $t=x^2+3\,|()' \Rightarrow dt=2x\,dx \Rightarrow x\,dx=\frac{dt}{2}$

E_2) $I=\int\sqrt{t}\cdot\frac{dt}{2}=\frac{1}{2}\int t^{\frac{1}{2}}dt=\frac{1}{2}\,\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=$

$=\frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{1}{3}\sqrt{t^3}+C$

E_3) $I(x)=\frac{1}{3}\sqrt{(x^2+3)^3}+C$

Prof. Ovidiu Sadescu

Calculați: a) $I=\int \frac{\sin(\ln x)}{x}dx$

b) $I=\int \frac{1}{x\ln x\ln(\ln x)}dx$

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a) $I=\int \frac{\sin(\ln x)}{x}dx$

$t=\ln x \mid (\,)' \Rightarrow dt=\frac{1}{x}dx$

$I(t)=\int \sin t\,dt=-\cos t+C$

$\boxed{I(x)=-\cos(\ln x)+C}$

b) $I=\int \frac{1}{x\cdot\ln x\cdot\ln(\ln x)}dx$

$t=\ln(\ln x) \mid (\,)' \Rightarrow dt=\frac{1}{\ln x}\cdot(\ln x)'dx$

$\Rightarrow dt=\frac{1}{\ln x}\cdot\frac{1}{x}dx$

$I(t)=\int \frac{1}{t}\cdot dt=\ln|t|+C$

$\boxed{I(x)=\ln|\ln(\ln x)|+C}$

Prof. Ovidiu Bădescu

Calculați: a) $\int \sin^2 x \cos x \, dx$

b) $\int \frac{\sin x + \cos x}{\cos x - \sin x} dx$

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a) $I = \int \sin^2 x \cdot \cos x \, dx$

$t = \sin x \mid ()' \Rightarrow dt = \cos x \, dx$

$I(t) = \int t^2 dt = \frac{t^3}{3} + C$

$I(x) = \frac{\sin^3 x}{3} + C$

b) $I = \int \frac{\sin x + \cos x}{\cos x - \sin x} dx$

$t = \cos x - \sin x \mid ()' \Rightarrow dt = (-\sin x - \cos x) dx$

$\Rightarrow (\sin x + \cos x) dx = -dt$

$I(t) = \int \frac{-dt}{t} = -\ln|t| + C$

$I(x) = -\ln|\cos x - \sin x| + C$

Prof. Ovidiu Bădescu

Calculați: a) $\int (\operatorname{tg} x + \operatorname{tg}^3 x)\,dx$

b) $\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

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a) $I = \int \operatorname{tg} x (1 + \operatorname{tg}^2 x)\,dx =$

$= \int \operatorname{tg} x \left(\frac{\cos^2 x}{1 + \frac{\sin^2 x}{\cos^2 x}} \right) dx = \int \operatorname{tg} x \cdot \frac{1}{\cos^2 x}\,dx$

$t = \operatorname{tg} x \,|\,()' \Rightarrow dt = \frac{1}{\cos^2 x}\,dx$

$\Rightarrow I(t) = \int t\,dt = \frac{t^2}{2} + C$

$$\boxed{I(x) = \frac{\operatorname{tg}^2 x}{2} + C}$$

b) $I = \int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$

$t = \arcsin x \,|\,()' \Rightarrow dt = \frac{1}{\sqrt{1-x^2}}\,dx$

$I(t) = \int t \cdot dt = \frac{t^2}{2} + C$

$$\boxed{I(x) = \frac{\arcsin^2 x}{2} + C}$$

Prof. Ovidiu Badescu

Calculați: a) $\int x^2 \cdot \sqrt{x^3+1}\, dx$

b) $\int \frac{\ln(\arccos x)}{\sqrt{1-x^2} \cdot \arccos x}\, dx$

a) $I = \int x^2 \sqrt{x^3+1}\, dx$

$t = x^3+1 \mid ()' \Rightarrow dt = 3x^2 dx \Rightarrow x^2 dx = \frac{dt}{3}$

$I(t) = \int \sqrt{t} \cdot \frac{dt}{3} = \frac{1}{3}\int t^{\frac{1}{2}} dt = \frac{1}{3} \cdot \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{1}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$

$\Rightarrow I(x) = \frac{1}{3} \cdot (x^3+1)^{\frac{3}{2}} \cdot \frac{2}{3} + C \Rightarrow \boxed{I(x) = 2(x^3+1)^{\frac{3}{2}} + C}$

b) $I = \int \frac{\ln(\arccos x)}{\sqrt{1-x^2} \cdot \arccos x}\, dx$

$t = \ln(\arccos x) \mid ()' \Rightarrow dt = \frac{1}{\arccos x} (\arccos x)' dx$

$\Rightarrow dt = \frac{1}{\arccos x} \left(-\frac{1}{\sqrt{1-x^2}}\right) dx$

$\Rightarrow \frac{1}{\sqrt{1-x^2} \cdot \arccos x}\, dx = -dt$

$I(t) = \int t(-dt) = -\frac{t^2}{2} + C$

$\boxed{I(x) = -\frac{\ln^2(\arccos x)}{2} + C}$

Prof. Ovidiu Badescu

Calculați: a) $\int \frac{1}{x(1+\ln^2 x)}\,dx$

b) $\int \frac{x}{\sqrt{x^4-1}}\,dx$

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a) $I = \int \frac{1}{x(1+\ln^2 x)}\,dx$

$t = \ln x \,|()' \Rightarrow dt = \frac{1}{x}dx$

$I(t) = \int \frac{1}{1+t^2}dt = \frac{1}{1}\operatorname{arctg}\frac{t}{1} + C$

$\boxed{I(x) = \operatorname{arctg}(\ln x) + C}$

b) $I = \int \frac{x}{\sqrt{x^4-1}}\,dx$

$t = x^2 \,|()' \Rightarrow dt = 2x\,dx \Rightarrow x\,dx = \frac{dt}{2}$

$I(t) = \int \frac{\frac{dt}{2}}{\sqrt{t^2-1}} = \frac{1}{2}\int \frac{dt}{\sqrt{t^2-1}} = \frac{1}{2}\cdot \ln\left|t+\sqrt{t^2-1}\right| + C$

$\boxed{I(x) = \frac{1}{2}\ln\left|x^2+\sqrt{x^2-1}\right| + C}$

Prof. Ovidiu Badescu

Calculați $F(x)=\int (tg^n x + tg^{n+2} x)\,dx$

$E_1)\ F(x)=\int tg^n x(1+tg^2 x)\,dx =$

$=\int tg^n x\left(1+\frac{\sin^2 x}{\cos^2 x}\right)dx=$

$=\int tg^n x\cdot\frac{1}{\cos^2 x}\,dx$

$E_2)$ Fie $F(x)$, notăm $t=tg\,x$ $(\ )'$

$dt=\frac{1}{\cos^2 x}\,dx$

$E_3)\ F(t)=\int t^n\cdot dt=\frac{t^{n+1}}{n+1}+C$

$\Rightarrow F(x)=\frac{tg^{n+1} x}{n+1}+C$

Prof. Dumitru Bagescu