CALCULUL PRIMITIVELOR RAȚIONALE PRIN ÎMPĂRȚIRE

Calculați:

1) $\int \frac{x^3+2x^2+x}{x^2+x-1}\,dx$

2) $\int \frac{x^3+x+1}{x^2-x+2}\,dx$

3) $\int \frac{x^3+2x^2-x}{x^2+x-1}\,dx$

4) $\int \frac{x^3+4x^2-2x+1}{x^2+3x-2}\,dx$

5) $\int \frac{3x^3-x^2+2}{x^2+x+1}\,dx$

6) $\int \frac{2x^3+x^2+x+1}{2x^2-x-1}\,dx$

7) $\int \frac{3x^4-x^2+5x+2}{x^2+2x}\,dx$

8) $\int \frac{5x^4-x^3+2x-1}{x+2}\,dx$

9) $\int \frac{3x^2-4x+1}{x-3}\,dx$

10) $\int \frac{2x^5-4x^3+2x^2-5}{x^3-6x^2+11x-6}\,dx$

Prof. Ovidiu Bădescu

Calculați

$$I=\int \frac{x^3+2x^2+x}{x^2+x-1}\,dx$$

E1) $x^3+2x^2+x \;\big|\; x^2+x-1$

$-x^3-x^2+x$ | $x+1$

x^2+2x

$-x^2-x+1$

$x+1$

E2) Soluția $\frac{P(x)}{Q(x)}=C(x)+\frac{R(x)}{Q(x)}$

E3) $I=\underbrace{\int (x+1)\,dx}_{I_1}+\underbrace{\int \frac{x+1}{x^2+x-1}\,dx}_{I_2}$

E4) $I_1=\frac{x^2}{2}+x+c$

E5) $I_2=\frac{1}{2}\int \frac{2x+2}{x^2+x-1}\,dx=\frac{1}{2}\int \frac{2x+1+1}{x^2+x-1}\,dx$

$=\frac{1}{2}\left(\int \frac{2x+1}{x^2+x-1}\,dx+\int \frac{1}{x^2+x-1}\,dx\right)$

$=\frac{1}{2}\left(\ln|x^2+x-1|+\int \frac{dx}{(x+\frac{1}{2})^2-\frac{5}{4}}\right)$

$=\frac{1}{2}\left(\ln|x^2+x-1|+\frac{1}{2\sqrt{\frac{5}{4}}}\ln\left|\frac{x+\frac{1}{2}-\frac{\sqrt5}{2}}{x+\frac{1}{2}+\frac{\sqrt5}{2}}\right|\right)+c$

E6) $I=\frac{x^2}{2}+x+\frac{1}{2}\left(\ln|x^2+x-1|+\frac{\sqrt5}{5}\ln\left|\frac{2x+1-\sqrt5}{2x+1+\sqrt5}\right|\right)+c$

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Calculați:

$$I=\int \frac{x^3+2x^2-x}{x^2+x-1}\,dx$$

$E_1)$ $x^3+2x^2-x \;\big|\; x^2+x-1$

$-x^3-x^2+x \quad\quad x+1$

x^2

$-x^2-x+1$

$-x+1$

$$\Rightarrow \frac{x^3+2x^2-x}{x^2+x-1}=x+1+\frac{-x+1}{x^2+x-1}$$

$E_2)$ $I=\int\left(x+1+\frac{-x+1}{x^2+x-1}\right)dx=$

$$=\frac{x^2}{2}+x-\underbrace{\int\frac{x-1}{x^2+x-1}\,dx}_{I_1} \quad \text{și } I_1,\ (x^2+x+1)'=2x+1$$

$E_3)$ $I_1=\frac{1}{2}\int\frac{2x-2}{x^2+x-1}\,dx=\frac{1}{2}\int\frac{2x+1-3}{x^2+x-1}\,dx=$

$$=\frac{1}{2}\left(\int\frac{2x+1}{x^2+x+1}\,dx-3\int\frac{1}{x^2+x-1}\,dx\right)=$$

$$=\frac{1}{2}\left(\int\frac{(x^2+x-1)'}{x^2+x-1}\,dx-3\int\frac{1}{1(x+\frac{1}{2})^2-\frac{5}{4}}\,dx\right)$$

$$=\frac{1}{2}\left(\ln|x^2+x-1|-3I_2\right)$$

$E_4)$ I_2 $\quad t=x+\frac{1}{2}\,|()'\Rightarrow dt=dx\Rightarrow I_2=\int\frac{1}{t^2-\frac{5}{4}}\,dt\Rightarrow$

$$I_2=\frac{1}{2\cdot\frac{\sqrt5}{2}}\ln\left|\frac{t-\frac{\sqrt5}{2}}{t+\frac{\sqrt5}{2}}\right|+C=\frac{1}{\sqrt5}\ln\frac{2t-\sqrt5}{2t+\sqrt5}+C=\frac{1}{\sqrt5}\ln\frac{2x+1-\sqrt5}{2x+1+\sqrt5}+C$$

$$F_1)\ I=\frac{x^2}{2}+x-\frac{1}{2}\ln|x^2+x-1|+\frac{3}{2\sqrt5}\ln\frac{2x+1-\sqrt5}{2x+1+\sqrt5}+C$$

Prof. Ovidiu Bădescu

Calculați: $\int \frac{3x^4 - x^2 + 5x + 2}{x^2 + 2x}\,dx$

$E_1)$ $3x^4 - x^2 + 5x + 2 \;\big|\; x^2 + 2x$

$-3x^4 - 6x^3 \quad\quad 3x^2 - 6x + 11$

$\backslash\; -6x^3 - x^2 + 5x + 2$

$6x^3 + 12x^2$

$\backslash\; 11x^2 + 5x + 2$

$-11x^2 - 22x$

$\backslash\; -17x + 2$

$E_2)$ $I = \int (3x^2 - 6x + 11)\,dx - \int \frac{17x - 2}{x^2 + 2x}\,dx$

$I = 3 \cdot \frac{x^3}{3} - 6 \cdot \frac{x^2}{2} + 11x - 17 \int \frac{x - \frac{2}{17}}{x^2 + 2x}\,dx$

$I = x^3 - 3x^2 + 11x - \frac{17}{2} \int \frac{2x - \frac{4}{17} + 2 - \frac{17}{2}}{x^2 + 2x}\,dx$

$I = x^3 - 3x^2 + 11x - \frac{17}{2}\left(\int \frac{2x + 2}{x^2 + 2x}\,dx - \frac{38}{17} \int \frac{1}{x^2 + 2x}\,dx \right)$

$I = x^3 - 3x^2 + 11x - \frac{17}{2}\left(\ln|x^2 + 2x| - \frac{38}{17} \int \frac{1}{(x+1)^2 - \frac{4}{4}}\,dx \right)$

$I = x^3 - 3x^2 + 11x - \frac{17}{2}\ln(x^2 + 2x) + 19 \underbrace{\int \frac{1}{(x+1)^2 - 1}\,dx}_{I_1}$

pt I_1, $x + 1 = t \;|\;(\,)' \Rightarrow dx = dt$

$I_1 = \int \frac{1}{t^2 - 1} = \frac{1}{2 \cdot 1} \ln\left|\frac{t-1}{t+1}\right| + C = \frac{1}{2} \ln\left|\frac{x+1-1}{x+1+1}\right| + C$

$$\boxed{I = x^3 - 3x^2 + 11x - \frac{17}{2}\ln|x^2 + 2x| + \frac{19}{2}\ln\left|\frac{x}{x+2}\right| + C}$$

Prof Ovidiu Bădescu

Calculați: $I=\int \frac{2x^5-4x^3+2x^2-5}{x^3-6x^2+11x-6}\,dx$

E1) $2x^5-4x^3+2x^2-5 \;\big|\; x^3-6x^2+11x-6$

$-2x^5+12x^4-22x^3+12x^2 \quad 2x^2+12x+46$

$12x^4-26x^3+14x^2-5$

$-12x^4+72x^3-132x^2+72x$

$46x^3-118x^2+72x-5$

$-46x^3+276x^2-506x+264$

$=158x^2-434x+259$

E2) $I=\int(2x^2+12x+46)\,dx-\underbrace{\int \frac{158x^2-434x+259}{x^3-6x^2+11x-6}\,dx}_{I_1}$

E3)

	x^3	x^2	x	x^0
	1	−6	11	−6
1	1	−5	6	0

$\Rightarrow (x-1)(x^2-5x+6)$

E4) $\frac{158x^2-434x+259}{(x-1)(x^2-5x+6)}=\frac{A}{x-1}+\frac{Bx+C}{x^2-5x+6}$

iar de aici se continuă

Prof. Ovidiu Bârsescu