MORFISME ȘI IZOMORFISME DE GRUPURI

FUNCȚII CA MATRICI

Studiați dacă grupurile următoare sunt izomorfe în fiecare din situațiile date:

1) $G=\left\{\begin{pmatrix}1 & x\\ 0 & 1\end{pmatrix} \mid x\in\mathbb{R}\right\}$, se cere $(\mathbb{R},+)\simeq(G,\cdot)$

2) $G_1=\{x+y\sqrt{5} \mid x,y\in\mathbb{Q}, x^2-5y^2=1\}$

$G_2=\left\{\begin{pmatrix}x & 2y\\ \frac{5y}{2} & x\end{pmatrix} \mid x,y\in\mathbb{Q}, x^2-5y^2=1\right\}$,

se cere $(G_1,\cdot)\simeq(G_2,\cdot)$

3) $G=\left\{\begin{pmatrix}1 & 0\\ x & 1\end{pmatrix} \mid x\in\mathbb{R}\right\}$, se cere $(\mathbb{R},+)\simeq(G,\cdot)$

4) $G_1=\{x+y\sqrt{7} \mid x,y\in\mathbb{Q}, x^2-7y^2=1\}$

$G_2=\left\{\begin{pmatrix}x & 3y\\ \frac{7y}{3} & x\end{pmatrix} \mid x,y\in\mathbb{Q}, x^2-7y^2=1\right\}$

se cere $(G_1,\cdot)\simeq(G_2,\cdot)$

5) $G=\left\{\begin{pmatrix}2^x & 0 & 0\\ 0 & 3^x & 0\\ 0 & 0 & 5^x\end{pmatrix} \mid x\in\mathbb{R}\right\}$,

se cere $(\mathbb{R},+)\simeq(G,\cdot)$

6) $G_1=\{x+y\sqrt{3} \mid x,y\in\mathbb{Q}, x^2-3y^2=1\}$

$G_2=\left\{\begin{pmatrix}x & 2y\\ \frac{3y}{2} & x\end{pmatrix} \mid x,y\in\mathbb{Q}, x^2-3y^2=1\right\}$

se cere $(G_1,\cdot)\simeq(G_2,\cdot)$

7) $G=\left\{\begin{pmatrix}1 & \ln x & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \mid x>0\right\}$

Se cere $(\mathbb{R}_+^*,\cdot)\simeq(G,\cdot)$

8) $G_1=\{x+y\sqrt{11} \mid x,y\in\mathbb{Q}, x^2-11y^2=1\}$

$G_2=\left\{\begin{pmatrix}x & 7y\\ \frac{11y}{7} & x\end{pmatrix} \mid x,y\in\mathbb{Q}, x^2-11y^2=1\right\}$

Se cere $(G_1,\cdot)\simeq(G_2,\cdot)$

9) $G_1=\{x+y\sqrt{7} \mid x,y\in\mathbb{Q}, x^2-7y^2=1\}$

$G_2=\left\{\begin{pmatrix}x & 5y\\ \frac{7y}{5} & x\end{pmatrix} \mid x,y\in\mathbb{Q}, x^2-7y^2=1\right\}$

Se cere $(G_1,\cdot)\simeq(G_2,\cdot)$

10) $G=\left\{\begin{pmatrix}1 & x & 0\\ 0 & 1 & 0\\ 0 & 0 & 3^x\end{pmatrix} \mid x\in\mathbb{R}\right\}$

Se cere $(\mathbb{R},+)\simeq(G,\cdot)$