ASIMPTOTE

1) ASIMPTOTE ORIZONTALE - se pot calcula doar dacă D_f nemărginit

$\lim_{x\to-\infty} f(x)=l$, $l\in\mathbb{R}$ (număr) → $y=l$ as. oriz spre $-\infty$

$l=\pm\infty$ sau $\nexists l$ → $\nexists$ as. oriz spre $-\infty$

Analog as. oriz spre ∞

2) ASIMPTOTE OBLICE - se pot calcula doar dacă D_f nemărginit și dacă $\nexists$ as. oriz în direcția respectivă

$m=\lim_{x\to-\infty}\frac{f(x)}{x}$, $m\in\mathbb{R}$ → $n=\lim_{x\to-\infty}(f(x)-mx)$, $n\in\mathbb{R}$ → $y=mx+n$ as. oblică spre $-\infty$

$n=\pm\infty$ sau $\nexists n$ → $\nexists$ as. oblică spre $-\infty$

$m=\pm\infty$ sau $\nexists m$ → $\nexists$ as. oblică spre $-\infty$

Analog spre ∞

3) ASIMPTOTE VERTICALE - se pot calcula doar în capetele deschise și finite ale D_f, se vor calcula limite laterale

$l=\lim_{\substack{x\to\alpha\\ x<\alpha}} f(x)$, $l=\pm\infty$ → $x=\alpha$ as. verticală la stânga pt G_f

$l\in\mathbb{R}$ (număr) sau $\nexists l$ → $\nexists$ as. verticală la stg pt G_f

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Exp 1: $f(x)=\ln(x-1)$, $f:D\to\mathbb{R}$

E_1) Cond: $x-1>0\Rightarrow x>1\Rightarrow D_f=(1;\infty)$

E_2) as. oriz:

spre ∞: $\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\ln(x-1)=\frac{\infty\cdot\infty}{}=\infty\notin\mathbb{R}\Rightarrow G_f$ nu are as. oriz spre ∞

spre $-\infty$ nu e voie să calculăm deoarece $D_f=(1;\infty)$

E_3) as. oblice:

spre ∞: $m=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{\ln(x-1)}{x}=$

$=\lim_{x\to\infty}\frac{\ln(x-1)}{x-1}\cdot\frac{x-1}{x}\overset{0\cdot1}{=\!=}0$, s-a folosit $\lim_{x_n\to\infty}\frac{\ln x_n}{x_n}=0$

$n=\lim_{x\to\infty}(f(x)-mx)=\lim_{x\to\infty}\ln(x-1)=\infty$

$\Rightarrow G_f$ nu are as. oblică spre ∞

E_4) as. verticală: $D_f=(1;\infty)$

$\lim_{\substack{x\to1\\ x>1}} f(x)=\lim_{\substack{x\to1\\ x>1}}\ln(x-1)\overset{\ln0_+}{=\!=}\infty$

$\Rightarrow x=1$ as. vert la dreapta

5 ASIMPTOTE ORIZONTALE, OBLICE, VERTICALE

Calculați asimptotele pentru $f: D \to \mathbb{R}$, unde

1) $f(x) = \frac{x}{2x-6}$

2) $f(x) = \frac{x^2}{x+5}$

3) $f(x) = \sqrt{x^2-x} - x$

4) $f(x) = \frac{\sqrt{x^2+1}}{x-2}$

5) $f(x) = \frac{x-1}{\sqrt{x^2+4}}$

6) $f(x) = x \ln(x-1)$

7) $f(x) = \sqrt{x^2+4x}$

8) $f(x) = x \cdot e^{\frac{1}{x}}$

9) $f(x) = \frac{x^3}{x-1}$

10) $f(x) = \frac{\sqrt{x^2-1}}{x-4}$

11) $f(x) = x \ln(x-1)$

12) $f(x) = \frac{1}{2} \ln^2 x$

13) $f(x) = \frac{2x+1}{x^2(x+1)^2}$

14) $f(x) = \frac{x+2}{1-x^2}$

15) $f(x) = \frac{1}{e^{\frac{1}{x}} - e}$

16) Găsiți $a, b \in \mathbb{R}$ dacă $f(x) = \frac{4x^2+ax+1}{bx+1}$ are $+\infty$ asimptotă pe $y = 2x+3$

17) $a = ?$ dacă G_f are exact o as. verticală unde $f(x) = \frac{x^2-x+a}{x^2-1}$

18) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt{|x^2-x|}$, arătați că G_f are o asimpt $-\infty$

19) $f(x) = \frac{x^2+ax+2}{x^2+2x+b}$, găsiți $b \in \mathbb{R}$ dacă G_f are o singură as. verticală și în acest caz găsiți $a \in \mathbb{R}$ dacă G_f nu intersectează as. orizontală

20) $f: D \to \mathbb{R}$, $f(x) = \operatorname{arctg} \frac{1}{x^2-1}$, x cer asimpt.

21) Arătați că G_f nu are asimpt. pe ∞ unde $f(x) = x - \sin x$

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Fie $f(x)=\sqrt{x^2-x}-x$, găsiți ec. asimptotelor

E1) cond.: $x^2-x\geq 0$ $\quad x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 1\cdot 0}}{2\cdot 1}=$

x	$-\infty$		0		1		$+\infty$
x^2-x	$+$	$+$	0	$-$	0	$+$	$+$

$=\frac{1\pm 1}{2}\begin{matrix}1\\ <\\ 0\end{matrix}$

$D_f=(-\infty;0]\cup[1;\infty)$

E2) as. oriz:

spre $-\infty$: $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}(\sqrt{x^2-x}-x)\overset{\infty+\infty}{=\!=\!=}\infty$

$\Rightarrow \nexists$ as. oriz spre $-\infty$

spre ∞: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}(\sqrt{x^2-x}-x)\overset{\infty-\infty}{=\!=\!=}$

$=\lim\limits_{x\to\infty}\frac{(\sqrt{x^2-x}-x)(\sqrt{x^2-x}+x)}{\sqrt{x^2-x}+x}=\lim\limits_{x\to\infty}\frac{x^2-x-x^2}{x(\sqrt{1-\frac{1}{x}}+1)}=-\frac{1}{2}$

$\Rightarrow y=-\frac{1}{2}$ as. oriz ord. spre ∞

E3) as. oblică: deoarece $\exists$ as. oriz spre $\infty\Rightarrow$ $\nexists$ as. oblică spre ∞, calc. as. oblică spre $-\infty$

$m=\lim\limits_{x\to-\infty}\frac{f(x)}{x}=\lim\limits_{x\to-\infty}\frac{\sqrt{x^2-x}-x}{x}\overset{\frac{\infty}{-\infty}}{=\!=\!=}$

$=\lim\limits_{x\to-\infty}\frac{|x|\sqrt{1-\frac{1}{x}}-x}{x}=\lim\limits_{x\to-\infty}\frac{-x\sqrt{1-\frac{1}{x}}-x}{x}$

$=\lim\limits_{x\to-\infty}\frac{x(-\sqrt{1-\frac{1}{x}}-1)}{x}=-2\in\mathbb{R}$

$n=\lim\limits_{x\to\infty}(f(x)-mx)=\lim\limits_{x\to\infty}(\sqrt{x^2-x}-x+2x)=$

$=\lim\limits_{x\to+\infty}(\sqrt{x^2-x}+x)\overset{\infty-\infty}{=\!=\!=}\lim\limits_{x\to-\infty}\frac{x^2-x-x^2}{\sqrt{x^2-x}-x}=\frac{1}{2}$

$\Rightarrow \boxed{y=-2x+\frac{1}{2}}$

... laterale: $D_f=(-\infty;0]\cup[1;\infty)\Rightarrow \nexists$ as. verticale

Fie $f: D \to \mathbb{R}$, $f(x) = \frac{\sqrt{x^2+1}}{x-2}$, a cer asimptotele

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E_1) Cond.: $\begin{cases} x^2+1 \geq 0 \\ x-2 \neq 0 \end{cases}$

$x^2+1 \geq 0 \Rightarrow$ inec de grad II $\Rightarrow$ tabel de semne

$\Delta = 0-4 < 0$
$\Rightarrow \nexists x_1, x_2 \in \mathbb{R}$

x	$-\infty$		$+\infty$
x^2+1	$+$	$+$	$+$

$\Rightarrow \forall x \in \mathbb{R}$ a.c. $x^2+1 \geq 0 \Leftrightarrow x \in \mathbb{R} = I_1$

$x-2 \neq 0 \Rightarrow x \neq 2 \Rightarrow x \in (-\infty; 2) \cup (2; \infty) = I_2$

$I_c = I_1 \cap I_2 = (-\infty; 2) \cup (2; \infty)$

E_2) as. orizontale

spre $-\infty$: $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \frac{\sqrt{x^2+1}}{x-2} \overset{\frac{\infty}{-\infty}}{=\!=} \lim\limits_{x \to -\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{x(1-\frac{2}{x})} =$

$= \lim\limits_{x \to -\infty} \frac{-x\sqrt{1+\left(\frac{1}{x^2}\right)^{\to 0}}}{x(1-\left(\frac{2}{x}\right)_{\to 0})} = -1 \in \mathbb{R} \Rightarrow y = -1$ as. oriz spre $-\infty$

spre ∞: $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \frac{\sqrt{x^2+1}}{x-2} \overset{\frac{\infty}{\infty}}{=\!=} \lim\limits_{x \to \infty} \frac{x\sqrt{1+\frac{1}{x^2}}}{x(1-\frac{2}{x})} = 1$

$\Rightarrow y = 1$ as. oriz spre ∞

E_3) as. oblice: $\nexists$ a.c. $\exists$ as. oriz spre $\pm\infty$

E_4) as. verticale: $D_f = (-\infty; 2) \cup (2; \infty)$

$\lim\limits_{\substack{x \to 2 \\ x<2}} f(x) = \lim\limits_{\substack{x \to 2 \\ x<2}} \frac{\sqrt{x^2+1}}{x-2} \overset{\frac{\sqrt{5}}{0_-}}{=\!=} -\infty \Rightarrow x = 2$ a.v. spre stg

x	$-\infty$		2		$+\infty$
$x-2$	$-$	$-$	0	$+$	$+$

$\lim\limits_{\substack{x \to 2 \\ x>2}} f(x) = \lim\limits_{\substack{x \to 2 \\ x>2}} \frac{\sqrt{x^2+1}}{x-2} \overset{\frac{\sqrt{5}}{0_+}}{=\!=} \infty \Rightarrow x = 2$ as spre dr.

Calculați asimptotele pentru

$$f(x) = \sqrt{x^2 - 4x}$$

E_1) Cond.: $x^2 - 4x \geq 0$, $x^2 - 4x = 0 \Rightarrow x_{1,2} = \frac{4 \pm \sqrt{16}}{2 \cdot 1}$

$\Rightarrow \begin{cases} x_1 = 4 \\ x_2 = 0 \end{cases}$

x		0		4	
$x^2 - 4x$	$+ +$	0	$- - -$	0	$+ +$

$x^2 - 4x \geq 0 \Leftrightarrow x \in (-\infty; 0] \cup [4; \infty)$

E_2) as. orizontale

spre $-\infty$: $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \sqrt{x^2 - 4x} =$

$= \lim\limits_{x \to -\infty} |x| \sqrt{1 - \frac{4}{x}} = \lim\limits_{x \to -\infty} -x\sqrt{1 - \frac{4}{x}} = \infty$

$\Rightarrow \nexists$ as. oriz spre $-\infty$

spre ∞: $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \sqrt{x^2 - 4x} = \lim\limits_{x \to \infty} x\sqrt{1 - \frac{4}{x}} = \infty$

$\Rightarrow \nexists$ as. oriz spre ∞

E_3) as. oblică:

spre $-\infty$: $m = \lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \frac{\sqrt{x^2 - 4x}}{x} = \lim\limits_{x \to -\infty} \frac{-x\sqrt{1 - \frac{4}{x}}}{x} = -1$

$n = \lim\limits_{x \to -\infty} (f(x) - mx) = \lim\limits_{x \to -\infty} (\sqrt{x^2 - 4x} + x) \overset{\infty - \infty}{=\!=\!=}$

$= \lim\limits_{x \to -\infty} \frac{x^2 - 4x - x^2}{\sqrt{x^2 - 4x} - x} = \lim\limits_{x \to -\infty} \frac{-4x}{x(-\sqrt{1 - \frac{4}{x}} - 1)} = 2$

$\Rightarrow y = mx + n \Rightarrow \boxed{y = -x + 2}$ as. oblică spre $-\infty$

spre ∞: $m = \lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} \frac{\sqrt{x^2 - 4x}}{x} = \lim\limits_{x \to \infty} \frac{x\sqrt{1 - \frac{4}{x}}}{x} = 1$

$n = \lim\limits_{x \to \infty} (f(x) - mx) = \lim\limits_{x \to \infty} (\sqrt{x^2 - 4x} - x) = \lim\limits_{x \to \infty} \frac{x^2 - 4x - x^2}{\sqrt{x^2 - 4x} + x} = -2$

$\boxed{y = x - 2}$

$D = (-\infty; 2] \cup [2; \infty) \Rightarrow \nexists$ as. verticale

Fie $f(x) = x \cdot e^{\frac{1}{x}}$, să se afle asimptotele

E_1) cond: $x \neq 0 \Rightarrow D_f = (-\infty; 0) \cup (0; \infty)$

E_2) as. orizontale

spre $-\infty$: $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} x e^{\frac{1}{x}} \overset{-\infty \cdot e^{\frac{1}{-\infty}}}{=\!=\!=} -\infty$

$-\infty \cdot 1$

$\Rightarrow \nexists$ as. oriz spre $-\infty$

spre ∞: $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} x e^{\frac{1}{x}} \overset{\infty \cdot e^{\frac{1}{\infty}}}{=\!=\!=} \infty$

$\Rightarrow \nexists$ as. oriz spre ∞

E_3) as. oblice:

spre $-\infty$: $m = \lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \frac{x e^{\frac{1}{x}}}{x} =$

$= \lim\limits_{x \to -\infty} e^{\frac{1}{x}} \overset{e^{\frac{1}{-\infty}}}{=\!=\!=} 1 \in \mathbb{R}$

e^0

$n = \lim\limits_{x \to -\infty} (f(x) - mx) = \lim\limits_{x \to -\infty} (x e^{\frac{1}{x}} - x)$

$= \lim\limits_{x \to -\infty} \frac{x(e^{\frac{1}{x}} - 1)}{\frac{1}{x}} \cdot \frac{1}{x} = 1 \in \mathbb{R} \Rightarrow y = mx + n$

$\Rightarrow y = x + 1$ as oblică spre $-\infty$

$\ln e = 1$

spre ∞: analog $\Rightarrow$ tot $y = x + 1$

E_4) as. verticale: $\lim\limits_{\substack{x \to 0 \\ x < 0}} x \cdot e^{\frac{1}{x}} \overset{0 \cdot e^{\frac{1}{0^-}}}{=\!=\!=} \overset{0 \cdot e^{-\infty} = 0 \cdot 0}{0} \Rightarrow \exists$ verticală

$\lim\limits_{\substack{x \to 0 \\ x > 0}} x e^{\frac{1}{x}} \overset{0 \cdot e^{\infty}}{=\!=\!=} \overset{0 \cdot \infty \to \frac{1}{x} = t}{\infty} \lim\limits_{t \to \infty} \frac{1}{t} e^t = \lim\limits_{t \to \infty} \frac{e^t}{t} \overset{\frac{\infty}{\infty}}{=\!=\!=} \overset{l'H}{} \lim \frac{e^t}{1} = \infty$

$\Rightarrow x = 0$ as. vert la dreapta

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Calculați asimptotele pentru $f:A\to\mathbb{R}$,

$$f(x)=\frac{\sqrt{x^2-1}}{x-4}$$

$E_1)$ cond: $\begin{cases} x^2-1\geq 0 \\ x-4\neq 0\Rightarrow x\in(-\infty;4)\cup(4;\infty)=I_2 \end{cases}$

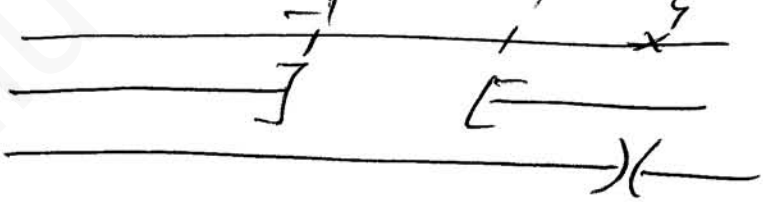
$\det x^2-1\geq 0\Rightarrow x^2-1=0\Rightarrow x^2=1\Rightarrow x=\pm 1$

x	$-\infty$		-1		1		∞
x^2-1		$+$	0	$-$	0	$+$	

$\not$ ca $x^2-1\geq 0\Leftrightarrow x\in(-\infty;-1]\cup[1;\infty)=I_1$

$I_C=I_1\cap I_2=$

$=(-\infty;-1]\cup[1;4)\cup(4;\infty)$



$E_2)\ f:(-\infty;-1]\cup[1;4)\cup(4;\infty)\to\mathbb{R},\ f(x)=\frac{\sqrt{x^2-1}}{x-4}$

$E_3)$ as. oriz: spre $-\infty$: $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2-1}}{x-4}=\lim\limits_{x\to-\infty}\frac{-x\sqrt{1-\frac{1}{x^2}}}{x(1-\frac{4}{x})}=-1\in\mathbb{R}$

$\lim\limits_{x\to\infty}\frac{\sqrt{x^2-1}}{x-4}=\lim\limits_{x\to\infty}\frac{x\sqrt{1-\frac{1}{x^2}}}{x(1-\frac{4}{x})}=1\in\mathbb{R}\Rightarrow y=-1$ as. oriz spre $-\infty$ și $y=1$ as. oriz spre ∞

$E_4)$ as. oblice: deoarece G_f are as. oriz spre $-\infty$ și spre ∞ $\Rightarrow G_f$ nu are as. oblice spre $-\infty$ și nici spre ∞

$E_5)$ as. verticale; x pot calcula doar în $x=4$

x	4	
$x-4$	0	$+$

$\lim\limits_{\substack{x\to 4\\ x<4}}\frac{\sqrt{x^2-1}}{x-4}\ \frac{\sqrt{15}}{0-}\ -\infty\Rightarrow x=4$ as. vert la stg

$\lim\limits_{x\to 4}\frac{\sqrt{x^2-1}}{x-4}\ \frac{\sqrt{15}}{0+}\ \infty\Rightarrow x=4$ as. vert la dr.

Determinați asimptotele pentru $f: D \to \mathbb{R}$,

$$f(x) = x \cdot \ln(x-1)$$

E_1) Cond.: $x-1>0 \Rightarrow x>1 \Rightarrow x \in (1;\infty)$

$\Rightarrow f: (1;\infty) \to \mathbb{R}$

E_2) as. oriz: putem calcula doar spre ∞

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x \cdot \ln(x-1) \overset{\infty \cdot \ln\infty}{=\!=\!=} \infty \notin \mathbb{R}$$

$\Rightarrow G_f$ nu are as. oriz spre ∞

E_3) as. oblică: putem calcula doar spre ∞

$$m = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x \ln(x-1)}{x} =$$

$$= \lim_{x\to\infty} \ln(x-1) = \infty \notin \mathbb{R}$$

$\Rightarrow G_f$ nu are as. oblică spre ∞

E_4) as. verticale: deoarece $D_f = (1;\infty)$, studiem dacă G_f are as verticale în $x=1$

$$\lim_{\substack{x\to 1 \\ x>1}} f(x) = \lim_{\substack{x\to 1 \\ x>1}} x \cdot \ln(x-1) \overset{1 \cdot \ln 0}{=\!=\!=} -\infty$$

$\Rightarrow x=1$ as. vert la dreapta

Fie $f:(0;\infty)\to\mathbb{R}$, $f(x)=\frac{1}{2}\ln^2 x$, găsiți ec. asimptotelor

$E_1)$ as. oriz: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to 0} \frac{1}{2}\ln^2 x \overset{\frac{1}{2}\ln^2 0}{=\!=\!=} \infty\notin\mathbb{R}$

$\Rightarrow \nexists$ as. oriz spre ∞

$E_2)$ as. oblice: au forma $y=mx+n$

$$m=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{\frac{1}{2}\ln^2 x}{x}\overset{\frac{\infty}{\infty}}{\underset{l'H}{=\!=}}$$

$$=\lim_{x\to\infty}\frac{\frac{1}{2}\cdot 2\ln x\cdot\frac{1}{x}}{1}=\lim_{x\to\infty}\frac{\ln x}{x}=$$

$$\overset{\frac{\infty}{\infty}}{\underset{l'H}{=\!=}}\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=0\in\mathbb{R}$$

$$n=\lim_{x\to\infty}\left(f(x)-mx\right)=\lim_{x\to\infty}\left(\frac{1}{2}\ln^2 x-0\right)=\infty\notin\mathbb{R}$$

$\Rightarrow \nexists$ as. oblică spre ∞

$E_3)$ as. verticale: $D_f=(0;\infty)$, calc asimpt în $x=0$ (dacă există)

$$\lim_{\substack{x\to 0\\ x>0}} f(x)=\lim_{\substack{x\to 0\\ x>0}}\frac{1}{2}\ln^2 x\overset{(\ln 0)^2}{=\!=\!=}\infty$$

$\Rightarrow x=0$ as. vert la dreapta

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299/92 Fie $f: \mathbb{R}\setminus\{-1;0\} \to \mathbb{R}$, $f(x) = \frac{2x+1}{x^2(x+1)^2}$

T38

Se cer asimptotele lui G_f

E1) as. orizontale

$x \to -\infty$; $\lim\limits_{x\to-\infty} \frac{2x+1}{x^2(x+1)^2} \overset{gr P < gr Q}{=\!=} 0 \in \mathbb{R} \Rightarrow$

$y = 0$ as. oriz spre $-\infty$

$x \to \infty$; $\lim\limits_{x\to\infty} \frac{2x+1}{x^2(x+1)^2} \overset{gr P < gr Q}{=\!=} 0 \in \mathbb{R} \Rightarrow$

$y = 0$ as. oriz spre ∞

E2) as. ~~verticale~~ oblice: deoarece $\exists$ as. oriz spre $-\infty$ și spre $\infty \Rightarrow \nexists$ as. oblice spre ∞ și $-\infty$

E3) as. verticale: $f: (-\infty;-1)\cup(-1;0)\cup(0;\infty) \to \mathbb{R}$

$\lim\limits_{\substack{x\to-1\\x<-1}} \frac{2x+1}{x^2(x+1)^2} \overset{\frac{-1}{1\cdot 0_+}}{=\!=} -\infty \Rightarrow x = -1$ as. vert la stg

$\lim\limits_{\substack{x\to-1\\x>-1}} \frac{2x+1}{x^2(x+1)^2} \overset{\frac{-1}{1\cdot 0_+}}{=\!=} -\infty \Rightarrow x = -1$ as. vert la dr.

$\lim\limits_{\substack{x\to0\\x<0}} \frac{2x+1}{x^2(x+1)^2} \overset{\frac{1}{0_+\cdot 1}}{=\!=} \infty \Rightarrow x = 0$ as. vert la stg

$\lim\limits_{\substack{x\to0\\x>0}} \frac{2x+1}{x^2(x+1)^2} \overset{\frac{1}{0_+\cdot 1}}{=\!=} \infty \Rightarrow x = 0$ as. vert la dr.

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Calculați asimptotele lui $f:\Delta\to\mathbb{R}$, $f(x)=\dfrac{1}{e^{\frac{1}{x}}-e}$

$E_1)$ $\Delta_f=?$, Cond.: $\begin{cases} x\neq 0 \Leftrightarrow x\neq 0 \\ e^{\frac{1}{x}}-e\neq 0 \quad \frac{1}{x}\neq 1 \Leftrightarrow x\neq 1 \end{cases}$

$\Rightarrow \Delta_f=(-\infty;0)\cup(0;1)\cup(1;\infty)$

$E_2)$ as. oriz:

spre $-\infty$: $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{1}{e^{\frac{1}{x}}-e}=\dfrac{1}{e^0-e}=\dfrac{1}{1-e}\in\mathbb{R}$

$\Rightarrow y=\frac{1}{1-e}$ as. oriz spre $-\infty$

spre ∞: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{1}{e^{\frac{1}{x}}-e}=\dfrac{1}{e^0-e}=\dfrac{1}{1-e}\in\mathbb{R}$

$\Rightarrow y=\frac{1}{1-e}$ as. oriz spre ∞

$E_3)$ as. oblică: deoarece $\exists$ as. oriz spre $-\infty \Rightarrow \nexists$ as. oblică spre $-\infty$, analog deoarece $\exists$ as. oriz spre $\infty \Rightarrow \nexists$ oblică spre ∞

$E_4)$ as. verticale:

$\lim\limits_{\substack{x\to 0\\ x<0}}\dfrac{1}{e^{\frac{1}{x}}-e}=\dfrac{1}{0-e}=-\dfrac{1}{e}\in\mathbb{R}\Rightarrow \nexists$ as. vert la stg în $x=0$

$\lim\limits_{\substack{x\to 0\\ x>0}}\dfrac{1}{e^{\frac{1}{x}}-e}=\dfrac{1}{e^{\infty}-e}=\dfrac{1}{\infty}=0\in\mathbb{R}\Rightarrow \nexists$ as. vert la dr în $x=0$

$\lim\limits_{\substack{x\to 1\\ x<1}}\dfrac{1}{e^{\frac{1}{x}}-e}=\dfrac{1}{0_+}=\infty \Rightarrow x=1$ as. vert la stg

$\lim\limits_{\substack{x\to 1\\ x>1}}\dfrac{1}{e^{\frac{1}{x}}-e}=\dfrac{1}{0_-}=-\infty \Rightarrow x=1$ as. vert la dr

semnul lui $e^{\frac{1}{x}}-e$ e din tabelul următ

		$\frac{1}{2}$		1		2
$e^{\frac{1}{x}}-e$		+	0	−	−	

Găsiți $a, b \in \mathbb{R}$ dacă $f(x) = \frac{4x^2+ax+1}{bx+1}$ are ca asimptotă dreapta $y = 2x+3$

E_1) $y = 2x+3$ are formă de as. oblică $\Rightarrow m = 2$, $n = 3$

E_2) $m = \lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} \frac{4x^2+ax+1}{bx^2+x} =$

$= \lim\limits_{x\to\infty} \frac{x^2(4 + \frac{a}{x} + \frac{1}{x^2})}{x^2(b + \frac{1}{x})} = \frac{4}{b}$

Dar, $m = 2 \Rightarrow \frac{4}{b} = 2 \Rightarrow \boxed{b = 2}$

E_3) $f(x) = \frac{4x^2+ax+1}{2x+1}$

E_4) $n = \lim\limits_{x\to\infty} (f(x) - mx) = \lim\limits_{x\to\infty} \left(\frac{4x^2+ax+1}{2x+1} - 2x\right) =$

$= \lim\limits_{x\to\infty} \frac{4x^2+ax+1-4x^2-2x}{2x+1} = \lim\limits_{x\to\infty} \frac{x(a-2+\frac{1}{x})}{x(2+\frac{1}{x})} =$

$= \frac{a-2}{2}$, dar $n = 3 \Rightarrow \frac{a-2}{2} = 3 \Rightarrow a-2 = 6$

$\Rightarrow \boxed{a = 8}$

E_5) e bine să verificăm dacă $f(x) = \frac{4x^2+8x+1}{2x+1}$ într-adevăr ca asimpt pe $y = 2x+3$, pt că am obținut valori posibile, iar dacă astea nu mergeau $\Rightarrow \nexists a, b$

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Fie $f: D \to \mathbb{R}$, $f(x) = \frac{x^2 - x + a}{x^2 - 1}$. Se cere

$a = ?$ dacă G_f are extra o as. verticală

E_1) Cond.: $x^2 - 1 \neq 0 \Leftrightarrow x^2 \neq 1 \Leftrightarrow x \neq \pm 1$

$\Rightarrow D_f = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

E_2) as. verticale pot fi calculate în $x = -1$, $x = 1$

E_3) În $x = -1$: $\lim\limits_{\substack{x \to -1 \\ x < -1}} \frac{x^2 - x + a}{x^2 - 1} \overset{\frac{2+a}{0+}}{=\!=} \pm\infty$ pt $a \neq -2$

x		-1		1	
$x^2 - 1$	$+$	0	$-$	0	$+$

În $x = 1$: $\lim\limits_{\substack{x \to 1 \\ x < 1}} \frac{x^2 - x + a}{x^2 - 1} \overset{\frac{a}{0-}}{=\!=} \pm\infty$ pt $a \neq 0$

E_4) dacă $a \in \mathbb{R} \setminus \{-2, 0\} \Rightarrow x = -1$, $x = 1$ as verticale $\Rightarrow$ are 2 as verticale, fals

E_5) $a = -2 \Rightarrow f(x) = \frac{x^2 - x + 2}{x^2 - 1} = \frac{(x+1)(x-2)}{(x-1)(x+1)}$

$\Rightarrow f(x) = \frac{x-2}{x-1}$ $\Rightarrow$ are doar pe $x = 1$ as vert. ok.

E_6) $a = 0 \Rightarrow f(x) = \frac{x^2 - x}{x^2 - 1} = \frac{x(x-1)}{(x-1)(x+1)} = \frac{x}{x+1}$

$\Rightarrow$ are doar as. $x = -1 \Rightarrow$ ok.

S_f: $\boxed{a \in \{-2, 0\}}$

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Fie $f:\mathbb{R}\to\mathbb{R}$ $f(x)=\sqrt{|x^2-x|}$

Arătați că G_f are o asimptotă spre $-\infty$

$E_1)$ $x^2-x=0 \Rightarrow x(x-1)=0 \Leftarrow \begin{matrix} x=0 \\ x=1 \end{matrix}$

x		0		1	
x^2-x	$+\,+$	0	$-\,-\,-$	0	$+\,+\,+$

$E_2)$ $f(x)=\begin{cases} \sqrt{x^2-x}, & x\in(-\infty,0] \\ \sqrt{-x^2+x}, & x\in(0,1] \\ \sqrt{x^2-x}, & x\in(1,\infty) \end{cases}$

$E_3)$ Asimptotă spre $-\infty$, luăm doar

$f(x)=\sqrt{x^2-x}$

$E_4)$ as. oriz.: $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\sqrt{x^2-x}=\infty$

$\Rightarrow \nexists$ as. oriz spre $-\infty$

$E_5)$ as. oblică: $m=\lim\limits_{x\to-\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2-x}}{x}=$

$=\lim\limits_{x\to-\infty}\dfrac{|x|\sqrt{1-\frac{1}{x}}}{x}=\lim\limits_{x\to-\infty}\dfrac{-x\sqrt{1-\frac{1}{x}}}{x}=-1\in\mathbb{R}$

$E_6)$ $n=\lim\limits_{x\to-\infty}(f(x)-mx)=\lim\limits_{x\to-\infty}(\sqrt{x^2-x}+x)\overset{\infty-\infty}{=}$

$=\lim\limits_{x\to-\infty}\dfrac{x^2-x-x^2}{\sqrt{x^2-x}-x}=\lim\limits_{x\to-\infty}\dfrac{-x}{|x|\sqrt{1-\frac{1}{x}}-x}=$

$=\dfrac{-x}{-x-x}=\dfrac{1}{2}\Rightarrow y=mx+n\Rightarrow y=-x+\dfrac{1}{2}$

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Fie $f(x)=\frac{x^2+ax+2}{x^2+2x+b}$, $a,b\in\mathbb{R}$. Găsiți $b\in\mathbb{R}$ dacă G_f are o singură as. verticală și, în acest caz, găsiți $a\in\mathbb{R}$ dacă G_f nu intersectează as. orizontală

E_1) <u>as. verticale</u> le pot calcula în capetele deschise și finite ale D_f, deci ar trebui ca D_f să aibă un singur capăt deschis și finit

E_2) Cond: $x^2+2x+b\neq 0$ și dacă $\Delta>0\Rightarrow D_f=(-\infty,x_1)\cup(x_2,\infty)$ $\Rightarrow$ 2 capete deschise finite, fals

dacă $\Delta=0\Rightarrow D_f=(-\infty,x_1)\cup(x_1,\infty)$ ok

dacă $\Delta<0\Rightarrow D_f=\mathbb{R}$ fals

$\Rightarrow$ cond NECESARĂ $\Delta=0\Rightarrow 4-4b=0\Rightarrow \boxed{b=1}$

E_3) $f(x)=\frac{x^2+ax+2}{x^2+2x+1}$, as. oriz la $\pm\infty$ se calculează ușor și iese $y=1$

E_4) $G_f\not\parallel$ as. oriz $\Rightarrow$ sist $\begin{cases} y=\frac{x^2+ax+2}{x^2+2x+1} \\ y=1 \end{cases}$ nu are sol.

$\Rightarrow \frac{x^2+ax+2}{x^2+2x+1}=1$ nu are sol $\Rightarrow x^2+ax+2=x^2+2x+1$

nu are sol. $\Rightarrow (a-2)x=-1$ nu are sol.

$a\neq 2\Rightarrow x=-\frac{1}{a-2}$, are sol fals

$a=2\Rightarrow 0\cdot x=-1\Rightarrow 0=-1$ deci $\nexists x$ care să nu are soluție. Așadar, $\boxed{\begin{matrix} a=2 \\ b=1 \end{matrix}}$

Fie $f: D \to \mathbb{R}$, $f(x) = \operatorname{arctg} \frac{1}{x^2-1}$

Se cer asimptotele

E_1) Găsim D, cond: $x^2-1 \neq 0 \Rightarrow x^2 \neq 1$

$\Rightarrow x \neq \pm 1 \Rightarrow x \in (-\infty;-1) \cup (-1;1) \cup (1;\infty)$

E_2) as. orizontale

spre $-\infty$: $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \operatorname{arctg} \frac{1}{x^2-1} \overset{\operatorname{arctg}\frac{1}{\infty}}{=\!=\!=} \overset{\operatorname{arctg} 0}{=\!=\!=} 0 \in \mathbb{R}$

$\Rightarrow y = 0$ as. oriz spre $-\infty$

spre ∞: $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \operatorname{arctg} \frac{1}{x^2-1} \overset{\operatorname{arctg}\frac{1}{\infty}}{=\!=\!=} \overset{\operatorname{arctg} 0}{=\!=\!=} 0 \in \mathbb{R}$

$\Rightarrow y = 0$ as. oriz spre ∞

E_3) as. oblice: deoarece $\exists$ as. oriz spre $-\infty \Rightarrow \nexists$ as. oblice spre $-\infty$

deoarece $\exists$ as. oriz spre $\infty \Rightarrow \nexists$ as. oblice spre ∞

E_4) as. verticale:

$\lim\limits_{\substack{x\to-1\\ x<-1}} f(x) = \lim\limits_{\substack{x\to-1\\ x<-1}} \operatorname{arctg} \frac{1}{x^2-1} \overset{\operatorname{arctg}\frac{1}{0_+}}{=\!=\!=} \overset{\operatorname{arctg}\infty}{=\!=\!=} \frac{\pi}{2} \in \mathbb{R}$

x	$-\infty$		-1		1		∞
x^2-1	$+$	0	$-$	$-$	0	$+$	$+$

$\Rightarrow \exists$ as. vert la stg

$\lim\limits_{\substack{x\to-1\\ x>-1}} f(x) = \lim\limits_{\substack{x\to-1\\ x>-1}} \operatorname{arctg} \frac{1}{x^2-1} \overset{\operatorname{arctg}\frac{1}{0_-}}{=\!=\!=} \overset{\operatorname{arctg}(-\infty)}{=\!=\!=} -\frac{\pi}{2} \Rightarrow \exists$ as. vert la dr.

$\lim\limits_{\substack{x\to1\\ x<1}} f(x) = \lim\limits_{\substack{x\to1\\ x<1}} \operatorname{arctg} \frac{1}{x^2-1} = -\frac{\pi}{2} \Rightarrow \exists$ as. vert la stg

$\lim\limits_{\substack{x\to1\\ x>1}} f(x) = -\frac{\pi}{2} \Rightarrow \exists$ as. vert la dr.

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Arătați că graficul lui $f(x) = x - \sin x$ nu are asimptotă spre ∞

$E_1)$ <u>as. orizontală</u>

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (x - \sin x) = \lim_{x\to\infty} x\left(1 - \frac{\sin x}{x}\right) =$$

$$\underline{\underline{\infty(1-0)}} \infty \Rightarrow G_f \text{ nu are as. oriz spre } \infty$$

$E_2)$ <u>as. oblică</u>

$$m = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x - \sin x}{x} = \lim_{x\to\infty} \frac{x\left(1 - \frac{\sin x}{x}\right)}{x}$$

$$= \lim_{x\to\infty} \left(1 - \frac{\sin x}{x}\right) \underline{\underline{1-0}} \; 1 \in \mathbb{R}$$

$$E_3)\; n = \lim_{x\to\infty} \left(f(x) - mx\right) = \lim_{x\to\infty} (x - \sin x - x)$$

$$= -\lim_{x\to\infty} \sin x \;\nexists \text{ deoarece}$$

$$E_4)\; x_n = \frac{\pi}{2} + 2n\pi \to \infty \Rightarrow \sin x_n = \sin\frac{\pi}{2} = 1 \to 1$$

$$x'_n = \pi + 2n\pi \to \infty \Rightarrow \sin x'_n = \sin\pi = 0 \to 0$$

$$\Rightarrow \nexists \lim_{x\to\infty} \sin x \Rightarrow \nexists\, n$$

$$\Rightarrow \nexists \text{ as. oblică spre } \infty$$

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