LIMITE NONSTANDARD DE FUNCȚII

Calculați:

1) $\lim\limits_{x\to 0} x\cdot[x]$ și $\lim\limits_{x\to 0} x\cdot\left[\frac{1}{x}\right]$

2) $\lim\limits_{x\to\infty} x\left(\frac{\pi}{4}-\operatorname{arctg}\frac{x}{x+1}\right)$

3) $\lim\limits_{x\to\pi/2} (2x-\pi)\cdot \operatorname{tg} x$

4) $\lim\limits_{x\to\infty} x^2\left(e^{\frac{1}{x+1}}-e^{\frac{1}{x}}\right)$

5) $\lim\limits_{x\to 0} \dfrac{x\cdot e^{-\frac{1}{x^2}}}{\operatorname{tg}^2 x}$

6) $\lim\limits_{x\to 0} \dfrac{\ln x}{x^3-1}$

7) $\lim\limits_{x\to 0} \left(1+\sum\limits_{k=1}^{n} \ln(1+kx)\right)^{\frac{1}{x}}$

8) $\lim\limits_{x\to 3} (7-2x)^{\operatorname{tg}\frac{\pi x}{6}}$

9) $\lim\limits_{x\to\infty} \left(\cos\frac{\pi}{x}\right)^{x}$

10) $\lim\limits_{x\to\pi/3} \dfrac{1-2\cos x}{\pi-3x}$

11) $\lim\limits_{\substack{x\to\pi\\ x<\pi}} \dfrac{\sin x}{\sqrt{\pi-x}}$

12) $\lim\limits_{x\to 0} \dfrac{\cos 2x-\cos 3x}{1-\cos x}$

13) $\lim\limits_{x\to 0} \dfrac{1-\cos^n x}{x\sin x}$

14) $\lim\limits_{x\to 0} \dfrac{(1-\cos x)^n}{x^{2n}}$

15) $\lim\limits_{x\to 0} \dfrac{1-\cos^3 x}{x\cdot\sin 2x}$

Prof. Ovidiu Bădescu

Calculați: a) $\lim\limits_{x\to 0} x\cdot[x]$

b) $\lim\limits_{x\to 0} x\cdot\left[\frac{1}{x}\right]$

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a) Folosesc $x-1$ $[x]$ x

$\Rightarrow x-1<[x]\leq x$

$E_2)\ l_s(0)=\lim\limits_{\substack{x\to 0\\ x<0}} x\cdot[x]$,, $x-1<[x]\leq x \mid \cdot x<0$

$x^2\leq x[x]<x^2-x$

$\to 0$ $\quad$ 0

$\Rightarrow l_s(0)=0$

$l_d(0)=\lim\limits_{\substack{x\to 0\\ x>0}} x\cdot[x]$,, $x-1<[x]\leq x \mid \cdot x>0$

$x^2-x<x[x]<x^2$

$\to 0$ $\quad$ 0

$\Rightarrow l_d(0)=0$

$E_3)\ l_s(0)=l_d(0)=0\Rightarrow \boxed{\lim\limits_{x\to 0} x\cdot[x]=0}$

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b) $E_1)$ deoarece $\frac{1}{x}-1<\left[\frac{1}{x}\right]\leq\frac{1}{x}$, avem cazuri

$E_2)\ l_s(0)=\lim\limits_{\substack{x\to 0\\ x<0}} x\cdot\left[\frac{1}{x}\right]$,, $\frac{1}{x}-1<\left[\frac{1}{x}\right]\leq\frac{1}{x} \mid \cdot x<0$

$\Rightarrow 1\leq x\left[\frac{1}{x}\right]<1-x \Rightarrow l_s(0)=1$

$\to 1$ $\quad$ 1

$l_d(0)=\lim\limits_{\substack{x\to 0\\ x>0}} x\cdot\left[\frac{1}{x}\right]$,, $\frac{1}{x}-1<\left[\frac{1}{x}\right]\leq\frac{1}{x} \mid \cdot x>0$

$\Rightarrow 1-x<x\left[\frac{1}{x}\right]\leq 1 \Rightarrow l_d(0)=1 \Rightarrow \boxed{\lim\limits_{x\to 0} x\left[\frac{1}{x}\right]=1}$

Prof. Ovidiu Bădescu

Calculați $\lim_{x\to\infty} x\left(\frac{\pi}{4} - \operatorname{arctg}\frac{x}{x+1}\right)$

E_1) deoarece $\operatorname{arctg} 1 = \frac{\pi}{4}$ și formula

$$\operatorname{arctg} a - \operatorname{arctg} b = \operatorname{arctg}\frac{a-b}{1+ab} \Rightarrow$$

E_2) $l = \lim_{x\to\infty} x(\operatorname{arctg} 1 - \operatorname{arctg}\frac{x}{x+1}) =$

$$= \lim_{x\to\infty} x \cdot \operatorname{arctg}\frac{1 - \frac{x}{x+1}}{1 + 1\cdot\frac{x}{x+1}} = \lim_{x\to\infty} x \cdot \operatorname{arctg}\frac{\frac{1}{x+1}}{\frac{2x+1}{x+1}}$$

$$= \lim_{x\to\infty} x \cdot \operatorname{arctg}\frac{1}{2x+1} = \lim_{x\to\infty} x \left(\frac{\operatorname{arctg}\frac{1}{2x+1}}{\frac{1}{2x+1}} \cdot \frac{1}{2x+1}\right)$$

(the factor $\frac{\operatorname{arctg}\frac{1}{2x+1}}{\frac{1}{2x+1}} \to 1$)

$$= \lim_{x\to\infty} \frac{x}{x(2+\frac{1}{x})} = \frac{1}{2}$$

Prof. Ovidiu Badescu

Calculați: $\lim_{x \to \pi/2} (2x - \pi) \cdot \operatorname{tg} x$

Ex 1) $\ell \begin{matrix} x - \frac{\pi}{2} = t \\ \hline x = t + \frac{\pi}{2} \end{matrix}$ $\lim_{t \to 0} \left(2\left(t + \frac{\pi}{2}\right) - \pi\right) \operatorname{tg}\left(t + \frac{\pi}{2}\right) =$

$$= \lim_{t \to 0} (2t + \pi - \pi) \cdot \frac{\sin\left(t + \frac{\pi}{2}\right)}{\cos\left(t + \frac{\pi}{2}\right)} =$$

$$= \lim_{t \to 0} 2t \cdot \frac{\left(\sin t \underset{0}{\cos \frac{\pi}{2}} + \cos t \underset{1}{\sin \frac{\pi}{2}}\right)}{\left(\cos t \underset{0}{\cos \frac{\pi}{2}} - \sin t \underset{1}{\sin \frac{\pi}{2}}\right)} =$$

$$= \lim_{t \to 0} 2t \cdot \frac{\cos t}{-\sin t} = \lim_{t \to 0} -2 \underbrace{\left(\frac{t}{\sin t}\right)}_{\to 1} \cdot \underbrace{\cos t}_{\to 1}$$

$$= -2$$

Prof. Ovidiu Băjescu

Calculați: $\lim\limits_{x\to\infty} x^2\left(e^{\frac{1}{x+1}}-e^{\frac{1}{x}}\right)$

E1) $l \overset{\infty(1-1)}{\underset{\infty\cdot 0}{=}} \lim\limits_{x\to\infty} x^2\cdot e^{\frac{1}{x}}\left(e^{\frac{\frac{x}{1}-\frac{x+1}{1}}{x+1}\cdot\frac{1}{x}}-1\right)$

$$l=\lim_{x\to\infty} x^2\cdot e^{\frac{1}{x}}\cdot\left(e^{-\frac{1}{x(x+1)}}-1\right)$$

E2) deoarece $\frac{1}{x(x+1)}\to 0$, ne gândim la $\lim\limits_{x_n\to 0}\frac{a^{x_n}-1}{x_n}=\ln a$

$$\Rightarrow l=\lim_{x\to\infty} x^2\cdot\underbrace{e^{\frac{1}{x}}}_{\to 1}\cdot\underbrace{\frac{e^{-\frac{1}{x(x+1)}}-1}{-\frac{1}{x(x+1)}}}_{\ln e=1}\cdot\left(-\frac{1}{x(x+1)}\right)$$

E3) $l=\lim\limits_{x\to\infty} x^2\cdot\left(-\frac{1}{x(x+1)}\right)=\lim\limits_{x\to\infty}-\frac{x}{x+1}=-1$

Prof. Ovidiu Bădescu

Calculați: $\lim\limits_{x\to 0} \dfrac{x\cdot e^{-\frac{1}{x^2}}}{tg^2 x}$

E1) $l = \lim\limits_{x\to 0} \dfrac{x\cdot e^{-\frac{1}{x^2}}}{tg^2 x} = \lim\limits_{x\to 0} \dfrac{x\cdot e^{-\frac{1}{x^2}}}{\left(\frac{tg x}{x}\right)^2 \cdot x^2}$, $\left(\frac{tg x}{x}\right)^2 \to 1$

E2) $l = \lim\limits_{x\to 0} \dfrac{e^{-\frac{1}{x^2}}}{x} = \lim\limits_{x\to 0} \dfrac{1}{x\, e^{\frac{1}{x^2}}}$

$\overset{\frac{1}{0\cdot\infty}}{=\!=} \lim\limits_{x\to 0} \dfrac{1}{x\cdot \frac{e^{\frac{1}{x^2}}}{\frac{1}{x^2}}\cdot \frac{1}{x}} = \lim\limits_{x\to 0} \dfrac{x}{\frac{e^{\frac{1}{x^2}}}{\frac{1}{x^2}}} \overset{\frac{0}{\infty}}{=\!=} 0$, $\dfrac{e^{\frac{1}{x^2}}}{\frac{1}{x^2}} \to \infty$

1 - a folosit: $\lim\limits_{x_n\to\infty} \dfrac{e^{x_n}}{x_n} = \infty$

Prof. Ovidiu Băsescu

Calculați:

$$\lim_{x\to 1} \frac{\ln x}{x^3-1}$$

$E_1)$ $\lim\limits_{x\to 1} \dfrac{\ln x}{x^3-1} \overset{\frac{0}{0}}{=\!=}$

$E_2)$ $t = x-1 \Rightarrow x = t+1$

$x \to 1 \Rightarrow t \to 0$

$E_3)$ $\ell = \lim\limits_{t\to 0} \dfrac{\ln(1+t)}{(t+1)^3-1} = \lim\limits_{t\to 0} \dfrac{\ln(1+t)}{t^3+3t^2+3t+1-1}$

$$= \lim_{t\to 0} \underbrace{\frac{\ln(1+t)}{t}}_{1} \cdot \frac{t}{t(t^2+3t+3)} = \frac{1}{3}$$

Obs.: s-a folosit $\lim\limits_{x_n\to 0} \dfrac{\ln(1+x_n)}{x_n} = 1$

Prof. Oniu Bădescu

Calculați:

$$\lim_{x\to 0}\left(1+\sum_{k=1}^{n}\ln(1+kx)\right)^{\frac{1}{x}}$$

E1) $l \overset{1^\infty}{=} \lim_{x\to 0}\left[\left(1+\sum_{k=1}^{n}\ln(1+kx)\right)^{\frac{1}{\sum\ln(1+kx)}}\right]^{\left(\sum\ln(1+kx)\right)\cdot\frac{1}{x}}$

$= e^{\lim\limits_{x\to 0}\frac{\sum\ln(1+kx)}{x}}$

E2) $l_1 = \lim_{x\to 0}\dfrac{\sum_{k=1}^{n}\ln(1+kx)}{x} =$

$= \lim_{x\to 0}\dfrac{\ln(1+x)+\ln(1+2x)+\dots+\ln(1+nx)}{x}$

$= \lim_{x\to 0}\left(\dfrac{\ln(1+x)}{x}+\dfrac{\ln(1+2x)}{x}+\dots+\dfrac{\ln(1+nx)}{x}\right)$

$= \lim_{x\to 0}\left(\underbrace{\dfrac{\ln(1+x)}{x}}_{\to 1}+\underbrace{\dfrac{\ln(1+2x)}{2x}}_{\to 1}\cdot 2+\dots+\underbrace{\dfrac{\ln(1+nx)}{nx}}_{\to 1}\cdot n\right)$

$= 1+2+\dots+n = \dfrac{n(n+1)}{2}$

E3) $l = e^{\frac{n(n+1)}{2}}$

Prof. Ovidiu Badescu

Calculați:

$$\lim_{x\to 3}(7-2x)^{\operatorname{tg}\frac{\pi x}{6}}$$

E1) $l = 1^{\infty}$

E2) $\begin{cases} t = x-3 \Rightarrow x = t+3 \\ x\to 3 \Rightarrow x-3 \to 0 \Rightarrow t\to 0 \end{cases}$

E3) $l = \lim_{t\to 0}(7-2(t+3))^{\operatorname{tg}\frac{\pi(t+3)}{6}}$

$= \lim_{t\to 0}(1-2t)^{\operatorname{tg}\left(\frac{\pi t}{6}+\frac{3\pi}{6}\right)} =$

$= \lim_{t\to 0}\left[(1+(-2t))^{\frac{1}{-2t}}\right]^{-2t\cdot \operatorname{tg}\left(\frac{\pi t}{6}+\frac{\pi}{2}\right)}$

$= e^{\lim_{t\to 0} -2t\operatorname{tg}\left(\frac{\pi t}{6}+\frac{\pi}{2}\right)}$

E4) $l_1 = \lim_{t\to 0} -2t\operatorname{tg}\left(\frac{\pi t}{6}+\frac{\pi}{2}\right)$

E5) $\operatorname{tg}(a+\frac{\pi}{2}) = \frac{\operatorname{tg}a+\operatorname{tg}\frac{\pi}{2}}{1-\operatorname{tg}a\operatorname{tg}\frac{\pi}{2}}$ [illegible]

$\operatorname{tg}(a+\frac{\pi}{2}) = \frac{\sin(a+\frac{\pi}{2})}{\cos(a+\frac{\pi}{2})} = \frac{\sin a\cos\frac{\pi}{2}+\cos a\sin\frac{\pi}{2}}{\cos a\cos\frac{\pi}{2}-\sin a\sin\frac{\pi}{2}} = \frac{\cos a}{-\sin a}$

$\Rightarrow l_1 = \lim_{t\to 0} -2t\cdot\frac{\cos\frac{\pi t}{6}}{\sin\frac{\pi t}{6}} = \lim_{t\to 0} 2\cdot\frac{\frac{\pi t}{6}}{\sin\frac{\pi t}{6}}\cdot\frac{t}{\frac{\pi t}{6}}\cdot\cos\frac{\pi t}{6} = \frac{12}{\pi}$

$\Rightarrow \boxed{l = e^{\frac{12}{\pi}}}$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{x\to\infty}\left(\cos\frac{\pi}{x}\right)^{x}$$

E1) $l \overset{1^{\infty}}{=\!=} \lim_{x\to\infty}\left(1+\cos\frac{\pi}{x}-1\right)^{x} =$

$$= \lim_{x\to\infty}\left[\underbrace{\left(1+\left(\cos\frac{\pi}{x}-1\right)\right)^{\frac{1}{\cos\frac{\pi}{x}-1}}}\right]^{\left(\cos\frac{\pi}{x}-1\right)\cdot x}$$

$$\lim_{x\to\infty}\left(\cos\frac{\pi}{x}-1\right)\cdot x$$

$$= e$$

E2) $l_1 = \lim_{x\to\infty}\left(\cos\frac{\pi}{x}-1\right)\cdot x \overset{0\cdot\infty}{=\!=}$

Folosim $1-\cos a = 2\sin^2\frac{a}{2} \Rightarrow \cos a - 1 = -2\sin^2\frac{a}{2}$

$$l_1 = \lim_{x\to\infty}\left(-2\sin^2\frac{\pi}{2x}\right)\cdot x =$$

$$= \lim_{x\to\infty} -2\cdot\left(\frac{\sin\frac{\pi}{2x}}{\frac{\pi}{2x}}\right)^{2}\cdot\left(\frac{\pi}{2}\right)^{2}\cdot\left(\frac{1}{x}\right)\cdot(x) = 0$$

$\hookrightarrow 1$

E3) $l_1 = e^{0} = 1$

Prof. Ovidiu Badescu

Calculați:

$$\lim_{x \to \frac{\pi}{3}} \frac{1 - 2\cos x}{\pi - 3x}$$

E1) $\ell = \lim_{x \to \frac{\pi}{3}} \frac{1 - 2\cos x}{\pi - 3x} \overset{\frac{0}{0}}{=}$

E2) $t = x - \frac{\pi}{3} \Rightarrow x = t + \frac{\pi}{3}$

$x \to \frac{\pi}{3} \Rightarrow t \to 0$

E3) $\ell = \lim_{t \to 0} \frac{1 - 2\cos\left(t + \frac{\pi}{3}\right)}{\pi - 3\left(t + \frac{\pi}{3}\right)} =$

$= \lim_{t \to 0} \frac{1 - 2\left(\cos t \cos\frac{\pi}{3} - \sin t \sin\frac{\pi}{3}\right)}{\pi - 3t - \pi}$

$= \lim_{t \to 0} \frac{1 - 2\left(\cos t \cdot \frac{1}{2} - \sin t \cdot \frac{\sqrt{3}}{2}\right)}{-3t}$

$= \lim_{t \to 0} \frac{1 - \cos t + \sqrt{3}\sin t}{-3t} = \lim_{t \to 0}\left(\frac{1 - \cos t}{-3t} + \frac{\sqrt{3}\sin t}{-3t}\right)$

$= -\lim \left(\frac{2\sin^2 \frac{t}{2}}{} - \frac{\sqrt{3}}{3}\right) = \lim_{t \to 0}\left(-\frac{2}{3} \cdot \left(\frac{\sin \frac{t}{2}}{\frac{t}{2}}\right)^2 \cdot \frac{t^2}{4} - \frac{\sqrt{3}}{3}\right) = 0 -$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{\substack{x\to\pi\\ x<\pi}} \frac{\sin x}{\sqrt{\pi - x}}$$

$E_1)$ $l = \lim\limits_{\substack{x\to\pi\\ x<\pi}} \frac{\sin x}{\sqrt{\pi - x}}$ $\frac{0}{0}$

$E_2)$ $t = x - \pi \Rightarrow x = t + \pi$

$x \to \pi \Rightarrow t \to 0$

$x < \pi \Rightarrow t < 0$

$E_3)$ $l = \lim\limits_{\substack{t\to 0\\ t<0}} \frac{\sin(t+\pi)}{\sqrt{\pi - (t+\pi)}} =$

$\sin t \cos\pi + \cos t \sin\pi$ ($\cos\pi = -1$, $\sin\pi = 0$)

$$= \lim_{\substack{t\to 0\\ t<0}} \frac{\sin t \cos\pi + \cos t \sin\pi}{\sqrt{-t}} =$$

$$= \lim_{\substack{t\to 0\\ t<0}} \frac{-\sin t}{\sqrt{-t}} = \lim_{\substack{t\to 0\\ t<0}} -\frac{\sin t}{t} \cdot \frac{t}{\sqrt{-t}} =$$

($\frac{\sin t}{t} \to 1$, $t = \sqrt{-t}\cdot\sqrt{-t}$)

$$= \lim_{t\to 0} -\frac{t\sqrt{-t}}{-t} = 0$$

Prof Ovidiu Bădescu

Calculați: $\lim\limits_{x\to 0} \dfrac{\cos 2x - \cos 3x}{1-\cos x}$

E1) $l \overset{\frac{1-1}{0}=\frac{0}{0}}{=\!=} \lim\limits_{x\to 0} \dfrac{(\cos 2x - 1)+(1-\cos 3x)}{1-\cos x}$

E2) Deoarece $1-\cos x = 2\sin^2\dfrac{x}{2} \Rightarrow$

$\Rightarrow 1-\cos 2x = 2\sin^2\dfrac{2x}{2} = 2\sin^2 x$

$1-\cos 3x = 2\sin^2\dfrac{3x}{2}$

E3) $l = \lim\limits_{x\to 0} \dfrac{-2\sin^2 x + 2\sin^2\frac{3x}{2}}{2\sin^2\frac{x}{2}}$

$= \lim\limits_{x\to 0} \dfrac{-\sin^2 x + \sin^2\frac{3x}{2}}{\sin^2\frac{x}{2}} = \lim\limits_{x\to 0}\left(\dfrac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2 \cdot \dfrac{\sin^2\frac{3x}{2} - \sin^2 x}{\left(\frac{x}{2}\right)^2}$ ($\sin\frac{x}{2} \sim \frac{x}{2}$, $\to 1$)

$= \lim\limits_{x\to 0}\left[\left(\dfrac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)^2 \cdot \left(\dfrac{3x}{2}\right)^2 - \left(\dfrac{\sin x}{x}\right)^2 \cdot x^2\right] \cdot \dfrac{1}{\left(\frac{x}{2}\right)^2}$

$= \lim\limits_{x\to 0}\left(\dfrac{9x^2}{4} - x^2\right) \cdot \dfrac{4}{x^2} = \lim\limits_{x\to 0} \dfrac{5x^2}{4} \cdot \dfrac{4}{x^2}$

$= 5$

Prof. Ovidiu Bădescu

Calculați: $\lim_{x\to 0} \frac{1-\cos^n x}{x \sin x}$

E1) folosim $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$

E2) $l \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x+\cos^2 x+\dots+\cos^{n-1} x)}{x \sin x}$

$$l = \lim_{x\to 0} \frac{\frac{1-\cos x}{x \sin x}}{1+\cos x} \cdot \lim_{x\to 0} (1+\cos x+\dots+\cos^{n-1} x)$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x \sin x (1+\cos x)} \cdot n$$

$$= n \lim_{x\to 0} \frac{\sin^2 x}{x \sin x} \cdot \lim_{x\to 0} \frac{1}{1+\cos x}$$

$$= n \cdot \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{2}$$

$$= \frac{n}{2} \cdot 1 = \frac{n}{2}$$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{x\to 0} \frac{(1-\cos x)^n}{x^{2n}}$$

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met I:

E1) $l = \lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)^n \overset{\frac{0}{0}}{=\!=} \lim_{x\to 0}\left(\frac{1-\cos^2 x}{x^2(1+\cos x)}\right)^n$ ($1+\cos x$)

$$= \lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)^n \cdot \left(\frac{1}{1+\cos x}\right)^n =$$

$$= \lim_{x\to 0}\left[\left(\frac{\sin x}{x}\right)^2\right]^n \cdot \left(\frac{1}{1+\cos x}\right)^n = 1^n \cdot \left(\frac{1}{2}\right)^n$$

($\frac{\sin x}{x} \to 1$)

E2) $\Rightarrow l = \frac{1}{2^n}$

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met II: E1) $\exists$ formula $\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow$

$\Rightarrow 1-\cos 2x = 2\sin x$, de aici $1-\cos x$

$\Rightarrow 1-\cos x = 2\sin^2 \frac{x}{2}$

E2) $l = \lim_{x\to 0}\left(\frac{2\sin^2 \frac{x}{2}}{x^2}\right)^n = \lim_{x\to 0}\left(2\cdot\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 \cdot \frac{\frac{x^2}{4}}{x^2}\right)^n =$

($\frac{\sin \frac{x}{2}}{\frac{x}{2}} \to 1$)

$$= \lim_{x\to 0}\left(2\cdot 1\cdot \frac{1}{4}\right)^n = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$$

Prof. Ovidiu Bădescu

Calculați

$$\lim_{x\to 0} \frac{1-\cos^3 x}{x\cdot \sin 2x}$$

$$E_1)\ l = \lim_{x\to 0} \frac{1-\cos^3 x}{x\cdot \sin 2x} \overset{\frac{0}{0.0}}{=\!=} \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x\cdot 2\sin x\cos x}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{x\cdot 2\sin x}\cdot \frac{1+\cos x+\cos^2 x}{\cos x}$$

$$E_2)\ l_1 = \lim_{x\to 0} \frac{1-\cos x}{x\cdot 2\sin x}$$

$$\exists \text{ formula } \sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 1-\cos 2x = 2\sin^2 x$$

$$\Rightarrow 1-\cos x = 2\sin^2 \frac{x}{2}$$

$$E_3)\ l_1 = \lim_{x\to 0} \frac{2\sin^2 \frac{x}{2}}{x\cdot 2\sin x} \overset{\frac{0}{0}}{=\!=}$$

$$= \lim_{x\to 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 \cdot \frac{x^2}{4}\cdot \frac{1}{x\cdot \left(\frac{\sin x}{x}\right)\cdot x} = \frac{1}{4}$$

($\frac{\sin \frac{x}{2}}{\frac{x}{2}} \to 1$, $\frac{\sin x}{x} \to 1$)

$$E_4)\ l = \frac{1}{4}\cdot \frac{3}{1} = \frac{3}{4}$$

Prof. Ovidiu Bădescu