LIMITE DE FUNCȚII ELEMENTARE

(NUMITOR CE TINDE LA ZERO, FACTOR FORȚAT, RAȚIONALIZARE, LIMITE REMARCABILE)

Calculați limitele următoare:

1) $\lim\limits_{\substack{x\to 2\\ x<2}} \frac{x+2}{4-x^2}$ și $\lim\limits_{\substack{x\to 3\\ x<3}} \frac{8+x}{x^2-3}$ și $\lim\limits_{\substack{x\to 3\\ x>3}} \frac{3-x}{9-x^2}$

2) $\lim\limits_{x\to\infty} \frac{2x+1}{-x^2+3}$ și $\lim\limits_{x\to-\infty} \frac{3x+1}{-x^3+4}$

3) $\lim\limits_{x\to-\infty} \frac{2x-x^2}{3x+1}$ și $\lim\limits_{x\to\infty} \frac{3x+x^2}{2x-1}$

4) $\lim\limits_{x\to\infty} \frac{2x-5}{4x+2}$ și $\lim\limits_{x\to-\infty} \frac{2x-5}{4x+2}$

5) $\lim\limits_{\substack{x\to 1\\ x<1}} \operatorname{arctg} \frac{1}{x^2-1}$ și $\lim\limits_{\substack{x\to 2\\ x<2}} \operatorname{arctg} \frac{1}{4-x^2}$

6) $\lim\limits_{x\to\infty} \sqrt[3]{\frac{x^2+2x}{8x^2+2x+1}}$ și $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2+5}}{3x+2}$

7) $\lim\limits_{x\to\infty} \frac{\sqrt{x^2+2}}{5x+1}$ și $\lim\limits_{x\to-\infty} \frac{\sqrt{4x^4+3x^3}}{4x^2+6x}$

8) $\lim\limits_{x\to-\infty} \frac{\sqrt{2x^6+x^5}}{3x^2+x}$ și $\lim\limits_{x\to-\infty} \frac{2x+1}{\sqrt{4x^2+x}}$

9) $\lim\limits_{x\to\infty} \left(\sqrt{x+1}-\sqrt{x}\right)$ și $\lim\limits_{x\to-\infty} \left(\sqrt{x^2+x}+x\right)$

10) $\lim\limits_{x\to\infty} \left(\sqrt[3]{x^3+x^2}-\sqrt[3]{x^3+1}\right)$ și $\lim\limits_{x\to-\infty} \left(\sqrt[3]{x^3+x^2}+x\right)$

11) $\lim\limits_{x\to\infty} \left(\sqrt[3]{x^3+2x}-\sqrt{x^2+1}\right)$

12) $\lim\limits_{x\to\infty} \left(\sqrt{2x+1}-\sqrt{x+1}\right)$

13) $\lim\limits_{x\to\infty} \left(\sqrt[3]{x^3+x^2+1}-\sqrt[3]{x^3+x}\right)$

14) $\lim\limits_{x\to\infty} \left(\sqrt{x+2}+\sqrt{x+1}-2\sqrt{x}\right)$

15) $\lim\limits_{x\to 0} \left(\frac{1+4x+x^2}{1+x}\right)^{\frac{1}{x}}$

16) $\lim\limits_{x\to\infty} (x-\ln x)$ și $\lim\limits_{x\to\infty} (2^x-x^3)$

17) $\lim\limits_{x\to\infty} \frac{\ln(x^4+1)}{\ln(x^2+2)}$ și $\lim\limits_{x\to\infty} \frac{3^x-5^x}{4^x+x^2}$

18) $\lim\limits_{x\to\infty} \frac{\ln(1+e^{5x})}{\ln(1+e^{2x})}$ și $\lim\limits_{x\to\infty} \frac{2^{x+1}+x+1}{3^{x+1}-5^x}$

19) $\lim\limits_{x\to\infty} \frac{\ln(x^2+e^x)}{\ln(x^4+e^{2x})}$ și $\lim\limits_{x\to\infty} \frac{3^x+4^x}{2^x-5^x}$

20) $\lim\limits_{x\to\infty} (3^x-x)$ și $\lim\limits_{x\to-\infty} \frac{e^x}{x^2+x}$

21) $\lim\limits_{x\to 0} \frac{e^x-1+\ln(1+2x)}{3x}$

22) $\lim\limits_{x\to 2} \frac{\sqrt{x+2}-\sqrt{3x-2}}{\sqrt{4x+1}-\sqrt{5x-1}}$

23) $\lim\limits_{x\to 2} \frac{\arcsin(x^2-5x+6)}{x^2-x-2}$

24) $\lim\limits_{x\to 3} \frac{2^x-8}{x-3}$ și $\lim\limits_{x\to 0} \frac{\sqrt{1+x}-\sqrt[3]{1-x}}{x}$

25) $\lim\limits_{x\to\frac{1}{2}} \frac{\arcsin(1-2x)}{4x^2-1}$

Prof. Ovidiu Bâcu

Calculați: a) $\lim\limits_{\substack{x\to 2\\ x<2}} \frac{x+2}{4-x^2}$

b) $\lim\limits_{\substack{x\to 3\\ x<3}} \frac{8+x}{x^2-9}$

c) $\lim\limits_{\substack{x\to 3\\ x>3}} \frac{3-x}{9-x^2}$

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a) $\lim\limits_{\substack{x\to 2\\ x<2}} \frac{x+2}{4-x^2} \Rightarrow \frac{4}{0}$

Ne a vedea semnul lui zero, $4-x^2=0 \to x^2=4$

$\Rightarrow x=\pm 2$

x		-2		2	
$4-x^2$	$---$	0	$+++$	0	$---$

$\Rightarrow l=\lim\limits_{\substack{x\to 2\\ x<2}} \frac{x+2}{4-x^2} \Rightarrow \frac{4}{0+} \Rightarrow \infty$

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b) $\lim\limits_{\substack{x\to 3\\ x<3}} \frac{8+x}{x^2-9} \Rightarrow \frac{11}{0}$

Ne a vedea semnul numitorului, $x^2-9=0 \to x=\pm 3$

x		-3		3	
x^2-9	$+$	0	$---$	0	$+++$

$\Rightarrow l=\lim\limits_{\substack{x\to 3\\ x<3}} \frac{8+x}{x^2-9} \Rightarrow \frac{11}{0-} \Rightarrow -\infty$

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c) $\lim\limits_{\substack{x\to 3\\ x>3}} \frac{3-x}{9-x^2} \Rightarrow \frac{0}{0}$ caz exceptat $\lim\limits_{\substack{x\to 3\\ x>3}} \frac{3-x}{(3-x)(3+x)} = \frac{1}{6}$

Calculati:

$$\lim_{x\to-\infty} \frac{\sqrt{4x^4+3x^3}}{4x^2+6x}$$

E1) $\ell = \lim_{x\to-\infty} \frac{|x^2|\sqrt{4+\frac{3}{x}}}{x^2\left(4+\frac{6}{x}\right)}$

E2) dar, $x\to-\infty \Rightarrow x<0 \Rightarrow x^2>0 \Rightarrow |x^2| = x^2$

E3) $\ell = \lim_{x\to-\infty} \frac{x^2\sqrt{4+\left(\frac{3}{x}\right)^{\to 0}}}{x^2\left(4+\left(\frac{6}{x}\right)_{\to 0}\right)} = \frac{\sqrt{4}}{4} = \frac{2}{4} = \frac{1}{2}$

Prof. Ovidiu Sadescu

Calculați:

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+5}}{3x+2}$$

$$E_1)\ l = \lim_{x \to -\infty} \frac{|x|\sqrt{1+\frac{5}{x^2}}}{x\left(3+\frac{2}{x}\right)}$$

$E_2)$ dar, $x \to -\infty \Rightarrow x < 0 \Rightarrow |x| = -x$

$$E_3)\ l = \lim_{x \to -\infty} \frac{-x\sqrt{1+\left(\frac{5}{x^2}\right)}}{x\left(3+\left(\frac{2}{x}\right)\right)} = -\frac{1}{3}$$

($\frac{5}{x^2} > 0$, $\frac{2}{x} > 0$)

Prof. Ovidiu Pădescu

Calculați: $\lim\limits_{x\to-\infty} \dfrac{\sqrt{2x^6+x^5}}{3x^2+x}$

$E_1)\ l=\lim\limits_{x\to-\infty} \dfrac{|x^3|\sqrt{2+\frac{1}{x}}}{x^2\left(3+\frac{1}{x}\right)}$

$E_2)\ x\to-\infty \Rightarrow x^3<0 \Rightarrow |x^3|=-x^3$

$E_3)\ l=\lim\limits_{x\to-\infty} \dfrac{-x^3\sqrt{2+\frac{1}{x}}}{x^2\left(3+\frac{1}{x}\right)}=$

$=\lim\limits_{x\to-\infty} \dfrac{-x\sqrt{2+\underbrace{\frac{1}{x}}_{\to 0}}}{3+\underbrace{\frac{1}{x}}_{\to 0}} = \infty$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{x\to\infty}\left(\sqrt[3]{x^3+2x}-\sqrt{x^2+1}\right)$$

E1) $l=\lim_{x\to\infty}\left(\sqrt[3]{x^3+2x}-\sqrt{x^2+1}\right)\overset{\infty-\infty}{=\!=}$

nu există conjugat pentru $\sqrt[3]{\ }-\sqrt{\ }$

E2) $l=\lim_{x\to\infty}\left(\sqrt[3]{x^3+2x}-x+x-\sqrt{x^2+1}\right)$

$$=\lim_{x\to\infty}\left(\frac{x^3+2x-x^3}{\sqrt[3]{(x^3+2x)^2}+\sqrt[3]{(x^3+2x)}\cdot x+x^2}+\frac{x^2-(x^2-1)}{x+\sqrt{x^2+1}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{2x}{x^2\left(\sqrt[3]{\left(1+\frac{2}{x^2}\right)^2}+\sqrt[3]{1+\frac{2}{x^2}}+1\right)}+\frac{1}{x+\sqrt{x^2+1}}\right)$$

$$\frac{2}{\infty(1+1+1)}+\frac{1}{\infty}\qquad 0+0=0$$

Prof. Vivian Bădescu

Calculați: $\lim\limits_{x\to\infty}\left(\sqrt{2x+1}-\sqrt{x+1}\right)$

$E_1)\ l \overset{\infty-\infty}{=\!=} \lim\limits_{x\to\infty} \dfrac{2x+1-x-1}{\sqrt{2x+1}+\sqrt{x+1}} =$

$= \lim\limits_{x\to\infty} \dfrac{x}{\sqrt{2x+1}+\sqrt{x+1}}$

$E_2)\ l \overset{\frac{\infty}{\infty}}{=\!=} \lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x}\left(\sqrt{2+\frac{1}{x}}+\sqrt{1+\frac{1}{x}}\right)}$

$= \lim\limits_{x\to\infty} \dfrac{\cancel{\sqrt{x}}\cdot\sqrt{x}}{\cancel{\sqrt{x}}\left(\sqrt{2+\frac{1}{x}}+\sqrt{1+\frac{1}{x}}\right)}$

$\dfrac{\infty}{\sqrt{2}+1} \overset{=\!=}{} \infty$

Prof. Ovidiu Bădescu

Calculați: $\lim_{x\to\infty}\left(\sqrt[3]{x^3+x^2+1}-\sqrt[3]{x^3+x}\right)$

$E_1)\ l=\lim_{x\to\infty}\left(\sqrt[3]{x^3+x^2+1}-\sqrt[3]{x^3+x}\right)\overset{\infty-\infty}{=\!=}$

$E_2)$ conjugatul lui $\sqrt[3]{a}-\sqrt[3]{b}$ este $\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}$

$$E_3)\ l=\lim_{x\to\infty}\frac{(x^3+x^2+1)-(x^3+x)}{\sqrt[3]{(x^3+x^2+1)^2}+\sqrt[3]{(x^3+x^2+1)(x^3+x)}+\sqrt[3]{(x^3+x)^2}}$$

$$=\lim_{x\to\infty}\frac{x^2-x+1}{x^2\sqrt[3]{\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^2}+x^2\sqrt[3]{\left(1+\frac{1}{x}+\frac{1}{x^3}\right)\left(1+\frac{1}{x^2}\right)}+x^2\sqrt[3]{\left(1+\frac{1}{x^2}\right)^2}}$$

$$=\lim_{x\to\infty}\frac{x^2\left(1-\left(\frac{1}{x}+\frac{1}{x^2}\right)^{\to 0}\right)}{x^2\left(\sqrt[3]{\left(1+\left(\frac{1}{x}+\frac{1}{x^3}\right)^{\to 0}\right)^2}+\sqrt[3]{\left(1+\left(\frac{1}{x}+\frac{1}{x^3}\right)^{\to 0}\right)\left(1+\left(\frac{1}{x^2}\right)^{\to 0}\right)}+\sqrt[3]{\left(1+\left(\frac{1}{x^2}\right)^{\to 0}\right)^2}\right)}$$

$$\frac{1-0+0}{1+1+1}\qquad\frac{1}{3}$$

Prof. Ovidiu Bârsan

Calculați:

$$\lim_{x\to\infty}\left(\sqrt{x+2}+\sqrt{x+1}-2\sqrt{x}\right)$$

$$E_1)\ l=\lim_{x\to\infty}\left(\left(\sqrt{x+2}-\sqrt{x}\right)+\left(\sqrt{x+1}-\sqrt{x}\right)\right)$$

$$E_2)\ l=\lim_{x\to\infty}\left(\frac{x+2-x}{\sqrt{x+2}+\sqrt{x}}+\frac{x+1-x}{\sqrt{x+1}+\sqrt{x}}\right)$$

$$\frac{2}{\infty}+\frac{1}{\infty}\quad 0$$

Prof. Ovidiu Bădescu

Calculați $\lim\limits_{x\to 0}\left(\frac{1+4x+x^2}{1+x}\right)^{\frac{1}{x}}$

E1) $l \overset{1^{\pm\infty}}{=\!=} \lim\limits_{x\to 0}\left(1+\frac{1+4x+x^2}{1+x}-1\right)^{\frac{1}{x}} =$

$= \lim\limits_{x\to 0}\left(1+\frac{1+4x+x^2-1-x}{1+x}\right)^{\frac{1}{x}} =$

$= \lim\limits_{x\to 0}\left[\left(1+\frac{x^2+3x}{x+1}\right)^{\frac{x+1}{x^2+3x}}\right]^{\frac{x^2+3x}{x+1}\cdot\frac{1}{x}}$

$= e^{\lim\limits_{x\to 0}\frac{x^2+3x}{x(x+1)}}$

E2) $l_1 = \lim\limits_{x\to 0}\frac{x(x+3)}{x(x+1)} = 3$

$\Rightarrow l = e$

Δ-a folosit: $\lim\limits_{x_n\to 0}(1+x_n)^{\frac{1}{x_n}} = e$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{x \to \infty} (x - \ln x)$$

E1) $l \overset{\infty - \infty}{=\!=} \lim_{x \to \infty} x\left(1 - \frac{\ln x}{x}\right) \overset{\infty(1-0)}{=\!=}$

$\Rightarrow l = \infty$

Obs.: s-a folosit $\lim_{x_n \to \infty} \frac{\ln x_n}{x_n} = 0$

Prof. Ovidiu Badescu

Calculați

$$\lim_{x\to\infty} \frac{\ln(x^4+1)}{\ln(x^2+2)}$$

E1) $$l = \lim_{x\to\infty} \frac{\ln\left(x^4\left(1+\frac{1}{x^4}\right)\right)}{\ln\left(x^2\left(1+\frac{2}{x^2}\right)\right)} =$$

$$= \lim_{x\to\infty} \frac{\ln x^4 + \ln\left(1+\frac{1}{x^4}\right)}{\ln x^2 + \ln\left(1+\frac{2}{x^2}\right)} =$$

$$= \lim_{x\to\infty} \frac{4\ln x + \ln\left(1+\frac{1}{x^4}\right)}{2\ln x + \ln\left(1+\frac{2}{x^2}\right)} \quad \frac{\infty+0}{\infty+0}=\frac{\infty}{\infty}$$

E2) $$l \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{\cancel{\ln x}\left(4+\frac{1}{\ln x}\cdot\ln\left(1+\frac{1}{x^4}\right)\right)}{\cancel{\ln x}\left(2+\frac{1}{\ln x}\cdot\ln\left(1+\frac{2}{x^2}\right)\right)}$$

$$\frac{4+0\cdot 0}{2+0\cdot 0} \quad \frac{4}{2} = 2$$

Prof. Ovidiu Badescu

Calculați:

$$\lim_{x\to\infty} \frac{3^x - 5^x}{4^x + x^2}$$

$$E_1)\ l = \lim_{x\to\infty} \frac{5^x\left(\left(\frac{3}{5}\right)^x - 1\right)}{4^x\left(1 + \frac{x^2}{4^x}\right)}$$

$E_2)$ dar, $\lim_{x\to\infty} \frac{P(x)}{a^x} = 0,\ \forall a > 1$

$$\Rightarrow \lim_{x\to\infty} \frac{x^2}{4^x} = 0$$

$$E_3)\ l = \lim_{x\to\infty} \frac{5^x\left(\left(\frac{3}{5}\right)^x - 1\right)}{4^x\left(1 + \frac{x^2}{4^x}\right)} =$$

$$= \lim_{x\to\infty} \left(\frac{5}{4}\right)^x \cdot \frac{\left(\frac{3}{5}\right)^x - 1}{1 + \frac{x^2}{4^x}} \overset{\infty \cdot \frac{0-1}{1+0}}{=\!=\!=} -\infty$$

Obs.: S-a folosit $\lim_{x\to\infty} q^x = \begin{cases} \nexists, & q \le -1 \\ 0, & q \in (-1;1) \\ 1, & q = 1 \\ \infty, & q > 1 \end{cases}$

Prof. Ovidiu Badescu

Calculați: a) $\lim\limits_{x\to\infty} \frac{\ln(x^2+e^x)}{\ln(x^4+e^{2x})}$

b) $\lim\limits_{x\to\infty} \frac{3^x+4^x}{2^x-5^x}$

a) E1) $l \overset{\frac{\infty}{\infty}}{=\!=} \lim\limits_{x\to\infty} \frac{\ln\left(e^x\left(\frac{x^2}{e^x}+1\right)\right)}{\ln\left(e^{2x}\left(\frac{x^4}{e^{2x}}+1\right)\right)} =$

$= \lim\limits_{x\to\infty} \frac{\ln e^x + \ln\left(\frac{x^2}{e^x}+1\right)}{\ln e^{2x} + \ln\left(\frac{x^4}{e^{2x}}+1\right)} = \lim\limits_{x\to\infty} \frac{x+\ln\left(\frac{x^2}{e^x}+1\right)}{2x+\ln\left(\frac{x^4}{e^{2x}}+1\right)}$ ($\frac{x^2}{e^x}\to 0$)

E2) $l \overset{\frac{\infty+0}{\infty+0}}{=\!=} \lim\limits_{x\to\infty} \frac{x\left(1+\frac{1}{x}\ln\left(\frac{x^2}{e^x}+1\right)\right)}{x\left(2+\frac{1}{x}\ln\left(\frac{x^4}{e^{2x}}+1\right)\right)} \overset{\frac{1+0\cdot 0}{2+0\cdot 0}}{=\!=} \frac{1}{2}$

s-a folosit $\lim\limits_{x\to\infty} \frac{P(x)}{a^x} = 0, \forall a>1$

b) $l \overset{\infty-\infty}{=\!=} \lim\limits_{x\to\infty} \frac{4^x\left(\left(\frac{3}{4}\right)^x+1\right)}{5^x\left(\left(\frac{2}{5}\right)^x-1\right)} = \lim\limits_{x\to\infty} \left(\frac{4}{5}\right)^x \cdot \frac{\left(\frac{3}{4}\right)^x+1}{\left(\frac{2}{5}\right)^x-1}$

$0\cdot\frac{0+1}{0-1} \overset{}{=\!=} 0$

s-a folosit $\lim\limits_{x\to\infty} q^x = \begin{cases} \nexists, & q\le -1 \\ 0, & q\in(-1,1) \\ 1, & q=1 \\ \infty, & q>1 \end{cases}$

în limbajul nostru, $q\in(-1,1)$

Calculați:

a) $\lim\limits_{x\to\infty} \dfrac{\ln(x^2+e^x)}{\ln(x^4+e^{2x})}$ și b) $\lim\limits_{x\to\infty} \dfrac{3^x+4^x}{2^x-5^x}$

a)

E.1) $\lim\limits_{x\to\infty} \dfrac{\ln(x^2+e^x)}{\ln(x^4+e^{2x})} \overset{\frac{\infty}{\infty}}{=\!=} \lim\limits_{x\to\infty} \dfrac{\ln\left(e^x\left(\frac{x^2}{e^x}+1\right)\right)}{\ln\left(e^{2x}\left(\frac{x^4}{e^{2x}}+1\right)\right)}$

$= \lim\limits_{x\to\infty} \dfrac{\ln e^x + \ln\left(\frac{x^2}{e^x}+1\right)}{\ln e^{2x} + \ln\left(\frac{x^4}{e^{2x}}+1\right)} = \lim\limits_{x\to\infty} \dfrac{x+\ln\left(\frac{x^2}{e^x}+1\right)}{2x+\ln\left(\frac{x^4}{e^{2x}}+1\right)} =$

$= \lim\limits_{x\to\infty} \dfrac{x\left(1+\underbrace{\frac{1}{x}}_{\to 0}\cdot \underbrace{\ln\left(\frac{x^2}{e^{2x}}+1\right)}_{\to 0}\right)}{x\left(2+\frac{1}{x}\ln\left(\frac{x^4}{e^{2x}}+1\right)\right)} = \dfrac{1}{2}$

b)

E.1) $\lim\limits_{x\to\infty} \dfrac{3^x+4^x}{2^x-5^x} \overset{\frac{\infty}{\infty-\infty}}{=\!=} \lim\limits_{x\to\infty} \dfrac{4^x\left(\left(\frac{3}{4}\right)^x+1\right)}{5^x\left(\left(\frac{2}{5}\right)^x-1\right)} =$

$= \lim\limits_{x\to\infty} \left(\dfrac{4}{5}\right)^x \cdot \dfrac{\left(\frac{3}{4}\right)^x+1}{\left(\frac{2}{5}\right)^x-1} \overset{0\cdot\frac{0+1}{0-1}}{=\!=} 0$

Prof. Ovidiu Bățeș

Calculați: $\lim\limits_{x\to-\infty} \dfrac{e^x}{x^2+x}$

$E_1)\ l = \lim\limits_{x\to-\infty} \dfrac{e^x}{x^2+x} \overset{}{=\!=\!=}$

$E_2)\ l_1 = \lim\limits_{x\to-\infty} e^x \overset{e^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0}{=\!=\!=} 0$

$E_3)\ l_2 = \lim\limits_{x\to-\infty} (x^2+x) \overset{\infty-\infty}{=\!=\!=}$

$= \lim\limits_{x\to-\infty} x^2\left(1+\dfrac{1}{x}\right) \overset{\infty(1+0)}{=\!=\!=} \infty$

$E_4)\ l = \lim\limits_{x\to-\infty} \dfrac{e^x}{x^2+x} \overset{\frac{0}{\infty}}{=\!=\!=} 0$

Prof. Ovidiu Bădescu

Calculați

$$\lim_{x\to 0} \frac{e^x - 1 + \ln(1+2x)}{3x}$$

E1) $l = \lim_{x\to 0} \left(\frac{e^x - 1}{3x} + \frac{\ln(1+2x)}{3x} \right)$

$= \lim_{x\to 0} \left(\underbrace{\frac{e^x - 1}{x}}_{\to 1} \cdot \frac{1}{3} + \underbrace{\frac{\ln(1+2x)}{2x}}_{\to 1} \cdot \frac{2}{3} \right)$

$= \frac{1}{3} + \frac{2}{3} = 1$

Obs.: s-a folosit

$\lim_{x_n\to 0} \frac{a^{x_n} - 1}{x_n} = \ln a$ și

$\lim_{x_n\to 0} \frac{\ln(1+2x_n)}{2x_n} = 1$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{x\to 2}\frac{\sqrt{x+2}-\sqrt{3x-2}}{\sqrt{4x+1}-\sqrt{5x-1}}$$

E1) $l\ \frac{2-2}{3-3}=\frac{0}{0}$

E2) $\lim_{x\to 2}\left(\frac{(\sqrt{x+2}-\sqrt{3x-2})(\sqrt{x+2}+\sqrt{3x-2})}{\sqrt{x+2}+\sqrt{3x-2}}\cdot\right.$

$$\left.\cdot\frac{\sqrt{4x+1}+\sqrt{5x-1}}{(\sqrt{4x+1}-\sqrt{5x-1})(\sqrt{4x+1}+\sqrt{5x-1})}\right)$$

$$=\lim_{x\to 2}\frac{x+2-3x+2}{\sqrt{x+2}+\sqrt{3x-2}}\cdot\frac{\sqrt{4x+1}+\sqrt{5x-1}}{4x+1-5x+4}$$

$$=\lim_{x\to 2}\frac{-2x+4}{\sqrt{x+2}+\sqrt{3x-2}}\cdot\frac{\sqrt{4x+1}+\sqrt{5x-1}}{-x+2}$$

$$=\lim_{x\to 2}\frac{2(-x+2)}{\sqrt{x+2}+\sqrt{3x-2}}\cdot\frac{\sqrt{4x+1}+\sqrt{5x-1}}{-x+2}$$

$$=\frac{2\cdot(3+3)}{\quad}=\frac{12}{4}=3$$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{x\to 2} \frac{\arcsin(x^2-5x+6)}{x^2-x-2}$$

E1) folosim $\lim_{x_n\to 0} \frac{\arcsin x_n}{x_n} = 1$

E2) $l = \lim_{x\to 2} \frac{\arcsin(x^2-5x+6)}{x^2-5x+6} \cdot \frac{x^2-5x+6}{x^2-x-2}$

↓

1 pt că $x_n = x^2-5x+6 \to 0$

E3) $l = \lim_{x\to 2} \frac{x^2-5x+6}{x^2-x-2} = \frac{0}{0}$

folosim $ax^2+bx+c \overset{\Delta>0}{=\!=} a(x-x_1)(x-x_2)$

$x^2-5x+6=0 \Rightarrow x_{1,2} = \frac{5\pm\sqrt{25-24}}{2\cdot 1} = \frac{5\pm 1}{2} \begin{matrix} 3 \\ 2 \end{matrix}$

$x^2-x-2=0 \Rightarrow x_{1,2} = \frac{1\pm\sqrt{1+8}}{2\cdot 1} = \frac{1\pm 3}{2} \begin{matrix} 2 \\ -1 \end{matrix}$

E4) $l = \lim_{x\to 2} \frac{1(x-3)(x-2)}{1(x-2)(x+1)} = \frac{-1}{3} = -\frac{1}{3}$

Obs.: $x^2-5x+6 = x^2-2x-3x+6 = x(x-2)-3(x-2)$
$= (x-2)(x-3)$

și $x^2-x-2 = x^2-2x+x-2 = x(x-2)+(x-2)$
$= (x-2)(x+1)$

Prof. Ovidiu Bădescu

Calculați:

$$\lim_{x \to 3} \frac{2^x - 8}{x - 3}$$

E1) $l = \lim_{x \to 3} \frac{2^x - 8}{x - 3} \overset{\frac{0}{0}}{=\!=}$

E2) $t = x - 3 \Rightarrow x = t + 3$

$x \to 3 \Rightarrow t \to 0$

E3) $l = \lim_{t \to 0} \frac{2^{t+3} - 8}{t} = \lim_{t \to 0} \frac{2^t \cdot 8 - 8}{t} =$

$$= \lim_{t \to 0} \underbrace{\frac{8(2^t - 1)}{t}}_{\to \ln 2} = 8 \ln 2$$

Obs.: s-a folosit: $\lim_{x_n \to 0} \frac{a^{x_n} - 1}{x_n} = \ln a$

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Calculați:

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt[3]{1-x}}{x}$$

$$E_1)\ l=\lim_{x\to 0}\frac{\sqrt{1+x}-1+1-\sqrt[3]{1-x}}{x}=$$

$$=\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}+\frac{1-\sqrt[3]{1-x}}{x}\right)$$

$$E_2)\ l=\lim_{x\to 0}\left(\frac{1+x-1}{x(\sqrt{1+x}+1)}+\frac{1-(1-x)}{x(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2})}\right)$$

$$=\lim_{x\to 0}\left(\frac{x}{x(\sqrt{1+x}+1)}+\frac{x}{x(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2})}\right)$$

$$=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$$

Obs.: atunci când apar radicali de ordin diferit, căutăm să aducem la același candidat

Prof. Ovidiu Bădescu

Calculați: $\lim\limits_{x\to\frac{1}{2}} \dfrac{\arcsin(1-2x)}{4x^2-1}$

E1) $l = \lim\limits_{x\to\frac{1}{2}} \dfrac{\arcsin(1-2x)}{4x^2-1} \overset{\frac{0}{0}}{=}$

$= \lim\limits_{x\to\frac{1}{2}} \underbrace{\dfrac{\arcsin(1-2x)}{1-2x}}_{\to 1} \cdot \dfrac{1-2x}{4x^2-1} =$

$= \lim\limits_{x\to\frac{1}{2}} \dfrac{1-2x}{(2x-1)(2x+1)} \overset{\frac{-1}{1+1}}{=} -\dfrac{1}{2}$

s-a folosit $\lim\limits_{x_n\to 0} \dfrac{\arcsin x_n}{x_n} = 1$

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