# CALCULUL DETERMINANȚILOR SUB FORMĂ DE PRODUS

Calculați, scriind rezultatul sub formă de produs:

1) $\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ a^3 & b^3 & c^3 \end{vmatrix}$

2) $\begin{vmatrix} a-1 & a+1 & a^2-1 \\ b-1 & b+1 & b^2-1 \\ c-1 & c+1 & c^2-1 \end{vmatrix}$

3) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$

4) $\begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 \\ b^2 & (b+1)^2 & (b+2)^2 \\ c^2 & (c+1)^2 & (c+2)^2 \end{vmatrix}$

5) $\begin{vmatrix} 2x-1 & 2x+1 & x \\ 2y-1 & 2y+1 & y \\ 2z-1 & 2z+1 & z \end{vmatrix}$

6) $\begin{vmatrix} x-y & x+2y & 2x-y \\ y-z & y+2z & 2y-z \\ z-x & z+2x & 2z-x \end{vmatrix}$

7) $\begin{vmatrix} \sin^2 x & \sin^2 y & \sin^2 z \\ 2 & 2 & 2 \\ \cos^2 x & \cos^2 y & \cos^2 z \end{vmatrix}$

8) $\begin{vmatrix} \cos 2x & \sin^2 x & \sin x \\ \cos 2y & \sin^2 y & \sin y \\ \cos 2z & \sin^2 z & \sin z \end{vmatrix}$

9) $\begin{vmatrix} \sin^2 x & \cos^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & \sin 2x \\ 1+\sin 2x & -1 & 1 \end{vmatrix}$

10) $\begin{vmatrix} 1+\cos x & 1+\sin x & 1 \\ 1-\sin x & 1+\cos x & 1 \\ 1 & 1 & 1 \end{vmatrix}$

11) arătați că $\det A=(a-b)(b-c)(c-a)$ dacă $A=\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$

12) dacă $a, b, c$ sunt lungimile laturilor unui triunghi și dacă $\det A(a,b,c)=0$, să. $\Delta$ e isoscel unde $A(a,b,c)=\begin{pmatrix} 1 & 1 & 1 \\ 2a & 2b & 2c \\ 3a^2 & 3b^2 & 3c^2 \end{pmatrix}$

13) rezolvați:

$$\begin{vmatrix} 2^x & 3^x & 5^x \\ 5^x & 2^x & 3^x \\ 3^x & 5^x & 2^x \end{vmatrix}=0$$

14) dacă $x_1, x_2, x_3$ sunt rād. ec. $x^3+5x+2=0$, calculați $d$ unde

$$d=\begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix}$$

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Scrieți sub formă de produs

$$\Delta = \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

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E1) $\Delta \xrightarrow[c_3 - c_1 \to c_3]{c_2 - c_1 \to c_2} \begin{vmatrix} a & b-a & c-a \\ 1 & 0 & 0 \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$

$$= (-1)^{2+1} \cdot 1 \cdot \begin{vmatrix} b-a & c-a \\ b^3-a^3 & c^3-a^3 \end{vmatrix}$$

E2) $\Delta = -1 \cdot \begin{vmatrix} b-a & c-a \\ (b-a)(b^2+ba+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$

$$\Delta = (-1)\cdot(b-a)\cdot(c-a) \begin{vmatrix} 1 & 1 \\ b^2+ba+a^2 & c^2+ca+a^2 \end{vmatrix}$$

$$\Delta = (a-b)(c-a)\left(c^2+ca+a^2-b^2-ba-a^2\right)$$

$$\Delta = (a-b)(c-a)\left((c-b)(c+b)+a(c-b)\right)$$

$$\Delta = (a-b)(c-a)(c-b)(a+b+c)$$

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Calculați sub formă de produs

$$\begin{vmatrix} a-1 & a+1 & a^2-1 \\ b-1 & b+1 & b^2-1 \\ c-1 & c+1 & c^2-1 \end{vmatrix}$$

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$E_1)$ $\Delta \xrightarrow{c_1 - c_2 \to c_1} \begin{vmatrix} -2 & a+1 & a^2-1 \\ -2 & b+1 & b^2-1 \\ -2 & c+1 & c^2-1 \end{vmatrix}$

$E_2)$ $\Delta \xrightarrow{\text{factor de pe } c_1} -2 \cdot \begin{vmatrix} 1 & a+1 & a^2-1 \\ 1 & b+1 & b^2-1 \\ 1 & c+1 & c^2-1 \end{vmatrix}$

$E_3)$ $\Delta \xrightarrow[l_3 - l_1 \to l_3]{l_2 - l_1 \to l_2} -2 \cdot \begin{vmatrix} 1 & a+1 & a^2-1 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$

$$\Delta = -2 \cdot (-1)^{1+1} \cdot \begin{vmatrix} b-a & (b-a)(b+a) \\ c-a & (c-a)(c+a) \end{vmatrix}$$

$E_4)$ $\Delta \xrightarrow[\text{factor pe } l_2]{\text{factor pe } l_1} -2 \cdot (b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$

$$\Delta = -2(b-a)(c-a)(c-b)$$

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Calculați sub formă de produs

$$\Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

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$E_1)\ \Delta \xrightarrow[c_3 - c_1 \to c_3]{c_2 - c_1 \to c_2} \begin{vmatrix} a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \\ bc & c(a-b) & b(a-c) \end{vmatrix}$

$E_2)\ \Delta = \begin{vmatrix} a & b-a & c-a \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \\ bc & -c(b-a) & -b(c-a) \end{vmatrix}$

$E_3)\ \Delta \xrightarrow[\text{factor pe } c_3]{\text{factor pe } c_2} (b-a)(c-a) \begin{vmatrix} a & 1 & 1 \\ a^2 & b+a & c+a \\ bc & -c & -b \end{vmatrix}$

$E_4)\ \Delta \xrightarrow[c_2 - c_3 \to c_2]{c_1 - a \cdot c_3 \to c_1} (b-a)(c-a) \begin{vmatrix} 0 & 0 & 1 \\ a^2-ac-a^2 & b-c & c+a \\ bc+ab & -c+b & -b \end{vmatrix}$

$E_5)\ \Delta = (b-a)(c-a)\cdot(-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} -ac & b-c \\ b(a+c) & b-c \end{vmatrix}$

$E_6)\ \Delta \xrightarrow{\text{factor}} (b-a)(c-a)(b-c) \begin{vmatrix} -ac & 1 \\ b(a+c) & 1 \end{vmatrix}$

$$\Delta = (b-a)(c-a)(b-c)(-ac-ab-bc)$$

$$\Delta = -(b-a)(c-a)(b-c)(ac+ab+bc)$$

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Scrieți sub formă de produs:

$$\begin{vmatrix} x-y & x+2y & 2x-y \\ y-z & y+2z & 2y-z \\ z-x & z+2x & 2z-x \end{vmatrix}$$

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$E_1)$ $\Delta \xrightarrow{C_1-C_2\to C_1} \begin{vmatrix} -3y & x+2y & 2x-y \\ -3z & y+2z & 2y-z \\ -3x & z+2x & 2z-x \end{vmatrix}$

$\xrightarrow{C_3-2C_2\to C_3} \begin{vmatrix} -3y & x+2y & -5y \\ -3z & y+2z & -5z \\ -3x & z+2x & -5x \end{vmatrix}$

$E_2)$ $\Delta \xrightarrow[\text{factor pe } C_3]{\text{factor pe } C_1} (-3)\cdot(-5)\cdot \begin{vmatrix} y & x+2y & y \\ z & y+2z & z \\ x & z+2x & x \end{vmatrix}$

$E_3)$ $\Delta = 0$ (pt că $C_1 = C_3$)

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Calculați $\Delta = \begin{vmatrix} \cos 2x & \sin^2 x & \sin x \\ \cos 2y & \sin^2 y & \sin y \\ \cos 2z & \sin^2 z & \sin z \end{vmatrix}$

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$E_1)$ deoarece $\cos^2 a = 1 - 2\sin^2 a \Rightarrow$

$$\Delta = \begin{vmatrix} 1-2\sin^2 x & \sin^2 x & \sin x \\ 1-2\sin^2 y & \sin^2 y & \sin y \\ 1-2\sin^2 z & \sin^2 z & \sin z \end{vmatrix} \quad \begin{matrix} \\ \\ \\ \end{matrix} \; C_1 + 2C_2 \to C_1$$

$$= \begin{vmatrix} 1 & \sin^2 x & \sin x \\ 1 & \sin^2 y & \sin y \\ 1 & \sin^2 z & \sin z \end{vmatrix} \quad \begin{matrix} \\ \\ \\ \end{matrix} \; \begin{matrix} L_2 - L_1 \to L_2 \\ L_3 - L_1 \to L_3 \end{matrix}$$

$E_2)$ $$\Delta = \begin{vmatrix} 1 & \sin^2 x & \sin x \\ 0 & \sin^2 y - \sin^2 x & \sin y - \sin x \\ 0 & \sin^2 z - \sin^2 x & \sin z - \sin x \end{vmatrix}$$

$$= (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} (\sin y - \sin x)(\sin y + \sin x) & \sin y - \sin x \\ (\sin z - \sin x)(\sin z + \sin x) & \sin z - \sin x \end{vmatrix}$$

Factori $(\sin y - \sin x)(\sin z - \sin x) \cdot \begin{vmatrix} \sin y + \sin x & 1 \\ \sin z + \sin x & 1 \end{vmatrix}$

$$= (\sin y - \sin x)(\sin z - \sin x)(\sin z - \sin y)$$

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Calculați $\Delta = \begin{vmatrix} \sin^2 x & \cos^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & \sin 2x \\ 1+\sin 2x & -1 & 1 \end{vmatrix}$ 9

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$E_1)\ \Delta \xrightarrow{C_3+C_2+C_1 \to C_1} \begin{vmatrix} 1+\sin 2x & \cos^2 x & \sin 2x \\ 1+\sin 2x & \sin^2 x & \sin 2x \\ 1+\sin 2x & -1 & 1 \end{vmatrix}$

Factor pe $C_1$ $(1+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & \sin^2 x & \sin 2x \\ 1 & -1 & 1 \end{vmatrix}$

$E_2)\ \Delta \xrightarrow[l_2-l_3 \to l_2]{l_1-l_3 \to l_1} (1+\sin 2x)\begin{vmatrix} 0 & \cos^2 x+1 & \sin 2x+1 \\ 0 & \sin^2 x+1 & \sin 2x-1 \\ 1 & -1 & 1 \end{vmatrix} =$

$= (1+\sin 2x)(-1)^{3+1} \cdot 1 \cdot \begin{vmatrix} \cos^2 x+1 & \sin 2x-1 \\ \sin^2 x+1 & \sin 2x-1 \end{vmatrix}$

$E_3)\ \Delta \xrightarrow{\text{Factor pe } C_2} (1+\sin 2x)(\sin 2x-1)\begin{vmatrix} \cos^2 x+1 & 1 \\ \sin^2 x+1 & 1 \end{vmatrix}$

$= \left((\sin 2x)^2-1\right)(\cos^2 x+1-\sin^2 x-1) =$

$= -\cos^2 2x \cdot \cos 2x = -\cos^3(2x)$

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Calculați $\begin{vmatrix} 1+\cos\alpha & 1+\sin\alpha & 1 \\ 1-\sin\alpha & 1+\cos\alpha & 1 \\ 1 & 1 & 1 \end{vmatrix}$ 10

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E1) $\Delta \overset{\substack{l_1-l_3\to l_1 \\ l_2-l_3\to l_2}}{=} \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{vmatrix} =$

$= \cos^2\alpha + \sin^2\alpha = 1$

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Calculați, scriind rezultatul sub formă de produs: $\Delta = \begin{vmatrix} 2x-1 & 2x+1 & x \\ 2y-1 & 2y+1 & y \\ 2z-1 & 2z+1 & z \end{vmatrix}$

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$E_1)\ \Delta \overset{C_1-C_2\to C_1}{=\!=} \begin{vmatrix} -2 & 2x+1 & x \\ -2 & 2y+1 & y \\ -2 & 2z+1 & z \end{vmatrix} \overset{\text{factor pe } c_1}{=\!=}$

$= -2 \cdot \begin{vmatrix} 1 & 2x+1 & x \\ 1 & 2y+1 & y \\ 1 & 2z+1 & z \end{vmatrix}$

$E_2)\ \Delta \overset{\substack{l_2-l_1\to l_2 \\ l_3-l_1\to l_3}}{=\!=} \begin{vmatrix} 1 & 2x+1 & x \\ 0 & 2(y-x) & y-x \\ 0 & 2(z-x) & z-x \end{vmatrix}$

$E_3)\ \Delta = (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 2(y-x) & y-x \\ 2(z-x) & z-x \end{vmatrix} =$

$\overset{\text{factor}}{=\!=} (y-x)(z-x) \cdot \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} \overset{l_1=l_2}{=\!=}$

$= (y-x)(z-x) \cdot 0 = 0$

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Fie $A=\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$

Arătați că $\det A=(a-b)(b-c)(c-a)$

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$E_1$) Formăm zerouri

$E_2$) $\det A=\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{matrix} \\ l_2-l_1 \\ l_3-l_1 \end{matrix} \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$

$E_3$) $\det A=(-1)^{1+1}\cdot 1\cdot \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix}=$

$= \begin{vmatrix} b-a & (b-a)(b+a) \\ c-a & (c-a)(c+a) \end{vmatrix} \Rightarrow$

$E_4$) $\det A=(b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}=$

$=(b-a)(c-a)[c+a-b-a]$

$=(b-a)(c-a)(c-b)$

$E_5$) $\det A=-(a-b)(c-a)\cdot[-(b-c)]$

$=(a-b)(c-a)(b-c)$

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Fie $D(a,b,c)$ determinantul matricei

$$A(a,b,c)=\begin{pmatrix} 1 & 1 & 1 \\ 2a & 2b & 2c \\ 3a^2 & 3b^2 & 3c^2 \end{pmatrix}$$

Arătați că dacă $a,b,c$ sunt lungimile laturilor unui $\Delta$ și $\Delta(a,b,c)=0$ atunci $\Delta ABC$ isoscel

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E1) $D(a,b,c)=0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2a & 2b & 2c \\ 3a^2 & 3b^2 & 3c^2 \end{vmatrix}=0 \xRightarrow{\text{Lat. } l_2 \text{ Lat. } l_3}$

$\Rightarrow 2\cdot 3 \cdot \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}=0 \xRightarrow{\text{VANDERMONDE}}$

$\Rightarrow 6\cdot(b-a)(c-a)(c-b)=0$

$\Rightarrow b-a=0 \Rightarrow a=b$

sau

$c-a=0 \Rightarrow a=c$

sau

$c-b=0 \Rightarrow b=c$

E2) Deci în orice caz $\Rightarrow \Delta ABC$ isoscel

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Rezolvați: $\begin{vmatrix} 2^x & 3^x & 5^x \\ 5^x & 2^x & 3^x \\ 3^x & 5^x & 2^x \end{vmatrix} = 0$

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$E_1)$ $\Delta \overset{l_3+l_2+l_1 \to l_1}{=\!=\!=} \begin{vmatrix} 2^x+5^x+3^x & 3^x+2^x+5^x & 5^x+3^x+2^x \\ 5^x & 2^x & 3^x \\ 3^x & 5^x & 2^x \end{vmatrix}$

$E_2)$ $\Delta = (2^x+5^x+3^x) \begin{vmatrix} 1 & 1 & 1 \\ 5^x & 2^x & 3^x \\ 3^x & 5^x & 2^x \end{vmatrix}$

$E_3)$ $\Delta = (2^x+5^x+3^x)\left(2^{2x}+5^{2x}+3^{2x}-2^x\cdot 3^x-2^x\cdot 5^x-3^x\cdot 5^x\right)$

$E_4)$ $\Delta = 0 \Rightarrow$ fals pt că
$2^x+5^x+3^x=0$
$a>0 \Rightarrow a^x>0$
sau
$2^{2x}+5^{2x}+3^{2x}-2^x\cdot 3^x-2^x\cdot 5^x-3^x\cdot 5^x=0$

$E_5)$ $2^x=a,\ 3^x=b,\ 5^x=c \Rightarrow$
$a^2+b^2+c^2-ab-bc-ca=0$
$a^2+b^2+c^2=ab+bc+ca \mid \cdot 2$

$E_6)$ $a^2+b^2-2ab+b^2+c^2-2ac+c^2+a^2-2ca=0$
$\Leftrightarrow (a-b)^2+(b-c)^2+(c-a)^2=0 \Rightarrow$ $a=b$ și $b=c$ și $c=a$

$\Leftrightarrow x \to x=0$

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Dacă $x_1, x_2, x_3$ sunt rădăcinile ecuației $x^3+5x+2=0$, calculați valoarea determinantului $d=\begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix}$

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$E_1)\ d \xrightarrow{l_3+l_2+l_1} \begin{vmatrix} x_1+x_2+x_3 & x_2+x_3+x_1 & x_3+x_1+x_2 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix}$

$E_2)\ d=(x_1+x_2+x_3)\begin{vmatrix} 1 & 1 & 1 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix}$

$E_3)\ d=(x_1+x_2+x_3)(x_2x_3+x_1x_2+x_1x_3-x_3^2-x_2^2-x_1^2)$

$E_4)$ Din VIETE $\Rightarrow S_1=x_1+x_2+x_3=-\frac{b}{a}=0$

$S_2=x_1x_2+x_1x_3+x_2x_3=\frac{c}{a}=\frac{5}{1}=5$

$S_3=x_1x_2x_3=-\frac{d}{a}=-2$

$E_5)$ în $d$ apare $x_1^2+x_2^2+x_3^2=(S_1)^2-2S_2=$

$=0^2-2\cdot 5=-10$

$E_6)\ d=0\cdot(5-(-10))=0\cdot 15=0$

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