ECUAȚII TRIGONOMETRICE NONSTANDARD

Rezolvați în $\mathbb{R}$:

1) $5\sin^2 x + 5\sin x \cos x = 3$

2) $\cos\frac{x}{2}\cos\frac{3x}{2} - \sin x \sin 3x - \sin 2x \sin 3x = 0$

3) $\operatorname{tg} x - \sin x = 2\sin^2\frac{x}{2}$

Prof. Ovidiu Bădescu

Rezolvați:

$5\sin^2 x + 5\sin x \cos x = 3$

E_1) deoarece $\sin^2 x + \cos^2 x = 1 \Rightarrow 3 = 3(\sin^2 x + \cos^2 x)$

E_2) $5\sin^2 x + 5\sin x \cos x = 3(\sin^2 x + \cos^2 x)$

E_3) $2\sin^2 x + 5\sin x \cos x - 3\cos^2 x = 0$

E_4) ~~ema~~ ec omogenă $\Rightarrow \cos x = t \sin x$

$\Rightarrow 2\cdot \sin^2 x + 5t\sin^2 x - 3t^2 \sin^2 x = 0$

$\Rightarrow \sin^2 x (2 + 5t - 3t^2) = 0$

E_5) $\begin{cases} \sin^2 x = 0 \Rightarrow \sin x$ înlocuim în ec inițiale $\Rightarrow 0 = 3$ fals \\ \text{sau} \\ 2 + 5t - 3t^2 = 0 \mid \cdot(-1) \Rightarrow 3t^2 - 5t - 2 = 0 \end{cases}$

$t_{1,2} = \dfrac{5 \pm \sqrt{25+24}}{2\cdot 3} = \dfrac{5 \pm 7}{6} \begin{matrix} 2 \\ \swarrow \\ -\frac{1}{3} \end{matrix}$

E_6) $t_1 = 2 \Rightarrow \cos x = 2\sin x \mid : \sin x \Rightarrow \operatorname{ctg} x = 2$

$\Rightarrow x \in \{\operatorname{arcctg} 2 + k\pi \mid k \in \mathbb{Z}\}$

$t_2 = -\frac{1}{2} \Rightarrow \cos x = -\frac{1}{3}\sin x \mid : \sin x \Rightarrow$

$\operatorname{ctg} x = -\frac{1}{3} \Rightarrow x \in \{\operatorname{arcctg}(-\frac{1}{3}) + k\pi \mid k \in \mathbb{Z}\}$

E_7) S_f: $x \in \{\operatorname{arcctg} 2 + k\pi \mid k \in \mathbb{Z}\} \cup \{\pi - \operatorname{arcctg}\frac{1}{3} + k\pi \mid k \in \mathbb{Z}\}$

Prof. Virgil Bălescu

Rezolvați:

$$\cos\frac{x}{2}\cos\frac{3x}{2}-\sin x\sin 3x-\sin 2x\sin 3x=0$$

E_1) folosim $\cos a\cos b=\frac{1}{2}[\cos(a+b)+\cos(a-b)]$ și

$\sin a\sin b=-\frac{1}{2}[\cos(a+b)-\cos(a-b)]\Rightarrow$ ec devine

E_2) $\frac{1}{2}[\cos 2x+\cos(-x)]+\frac{1}{2}[\cos 4x-\cos(-2x)]+\frac{1}{2}[\cos 5x-\cos(-x)]=0$

$1\cdot 2$

$\Leftrightarrow \cos 2x+\cos x+\cos 4x-\cos 2x+\cos 5x-\cos x=0$

$\Leftrightarrow \cos 4x+\cos 5x=0\Leftrightarrow \cos 4x=-\cos 5x$

E_3) $\cos 4x=\cos(\pi-5x)\Leftrightarrow$

$4x=\pi-5x+2k\pi$ sau $4x=-(\pi-5x)+2k\pi$

$\Leftrightarrow 9x=\pi+2k\pi$ sau $-x=-\pi+2k\pi$

$\Leftrightarrow x=\frac{\pi+2k\pi}{9}$ sau $x=\pi-2k\pi$, $k\in\mathbb{Z}$

E_4) $S_f: x\in\left\{\frac{\pi+2k\pi}{9}\,|\,k\in\mathbb{Z}\right\}\cup\{\pi-2k\pi\,|\,k\in\mathbb{Z}\}$

Prof. Ovidiu Badescu

Rezolvați:

$$\operatorname{tg} x - \sin x = 2\sin^2 \frac{x}{2}$$

E_1) din $\sin^2 a = \frac{1-\cos 2a}{2} \Rightarrow \sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$

$\Rightarrow 2\sin^2 \frac{x}{2} = 1 - \cos x$

E_2) ec devine: $\frac{\sin x}{\cos x} - \sin x = 1 - \cos x \,/\cdot \cos x$

$\Leftrightarrow \sin x - \sin x \cos x = \cos x (1 - \cos x)$

$\Leftrightarrow \sin x (1 - \cos x) = \cos x (1 - \cos x)$

$\Leftrightarrow (1 - \cos x)(\sin x - \cos x) = 0$

E_3) Caz I: $1 - \cos x = 0 \Leftrightarrow \cos x = 1 \Leftrightarrow$

$x \in \{\arccos 1 + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\arccos 1 + 2k\pi \mid k \in \mathbb{Z}\}$

$x \in \{0 + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-0 + 2k\pi \mid k \in \mathbb{Z}\}$

$\Leftrightarrow x \in \{0 + 2k\pi \mid k \in \mathbb{Z}\} = S_I$

Caz II: $\sin x - \cos x = 0 \Leftrightarrow \sin x = \cos x \,/: \cos x \neq 0$

$\Leftrightarrow \operatorname{tg} x = 1 \Rightarrow x \in \{\operatorname{arctg} 1 + k\pi \mid k \in \mathbb{Z}\} \Leftrightarrow$

$x \in \{\frac{\pi}{4} + k\pi \mid k \in \mathbb{Z}\} = S_{II}$

E_4) $S_f = S_I \cup S_{II} = \{2k\pi \mid k \in \mathbb{Z}\} \cup \{\frac{\pi}{4} + k\pi \mid k \in \mathbb{Z}\}$

Prof. Ovidiu Bădescu