INECUAȚII TRIGONOMETRICE

Rezolvați:

1) $\cos x < -\frac{1}{2}$

2) $\sin x \geq \frac{\sqrt{2}}{2}$

3) $\operatorname{tg} x \geq \sqrt{3}$

4) $\operatorname{ctg} x < 1$

5) $\cos x > -1$

6) $\sin x \leq 1$

7) $\operatorname{tg} x > -\sqrt{3}$

8) $\operatorname{ctg} x \leq -\frac{1}{\sqrt{3}}$

9) $\frac{\left(\sin x - \frac{\sqrt{3}}{2}\right)\left(\cos x + \frac{\sqrt{2}}{2}\right)}{(\operatorname{tg} x - 1)(\sqrt{3} - \operatorname{ctg} x)} \geq 0$

10) $\frac{\left(\sin\left(x + \frac{\pi}{6}\right) - \frac{\sqrt{3}}{2}\right)\left(\cos\left(x + \frac{\pi}{6}\right) - \frac{\sqrt{2}}{2}\right)}{\operatorname{tg}\left(x + \frac{\pi}{6}\right) - \frac{1}{\sqrt{3}}} \geq 0$

11) $12(\arcsin x)^2 - \pi \arcsin x - \pi^2 \leq 0$

Dorin Ordiu Bădescu

Rezolvați: $\cos x < -\frac{1}{2}$

E_1) ec. atașată: $\cos x = -\frac{1}{2} \Rightarrow$

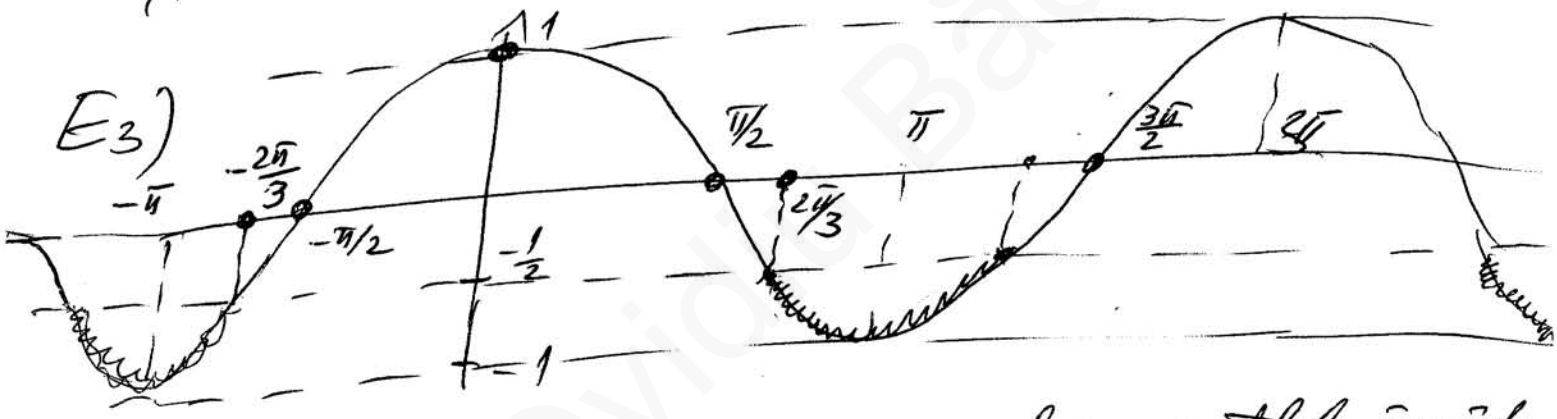
$\Rightarrow x \in \{\arccos\left(-\frac{1}{2}\right) + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\arccos\left(-\frac{1}{2}\right) + 2k\pi \mid k \in \mathbb{Z}\}$

$x \in \{\pi - \arccos\frac{1}{2} + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-(\pi - \arccos\frac{1}{2}) + 2k\pi \mid k \in \mathbb{Z}\}$

$x \in \{\frac{2\pi}{3} + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\frac{2\pi}{3} + 2k\pi \mid k \in \mathbb{Z}\}$

E_2) $k = 0 \Rightarrow x \in \{\frac{2\pi}{3}, -\frac{2\pi}{3}\}$

E_3)



E_4) Soluția particulară se alege astfel încât să îl conțină pe 0 sau imediat după 0

Un capăt al soluției este $\frac{2\pi}{3}$, celălalt capăt

$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3} \Rightarrow S_p = \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

$\Rightarrow S_g = \left(\frac{2\pi}{3} + 2k\pi; \frac{4\pi}{3} + 2k\pi\right)$

Prof. Ovidiu Bădescu

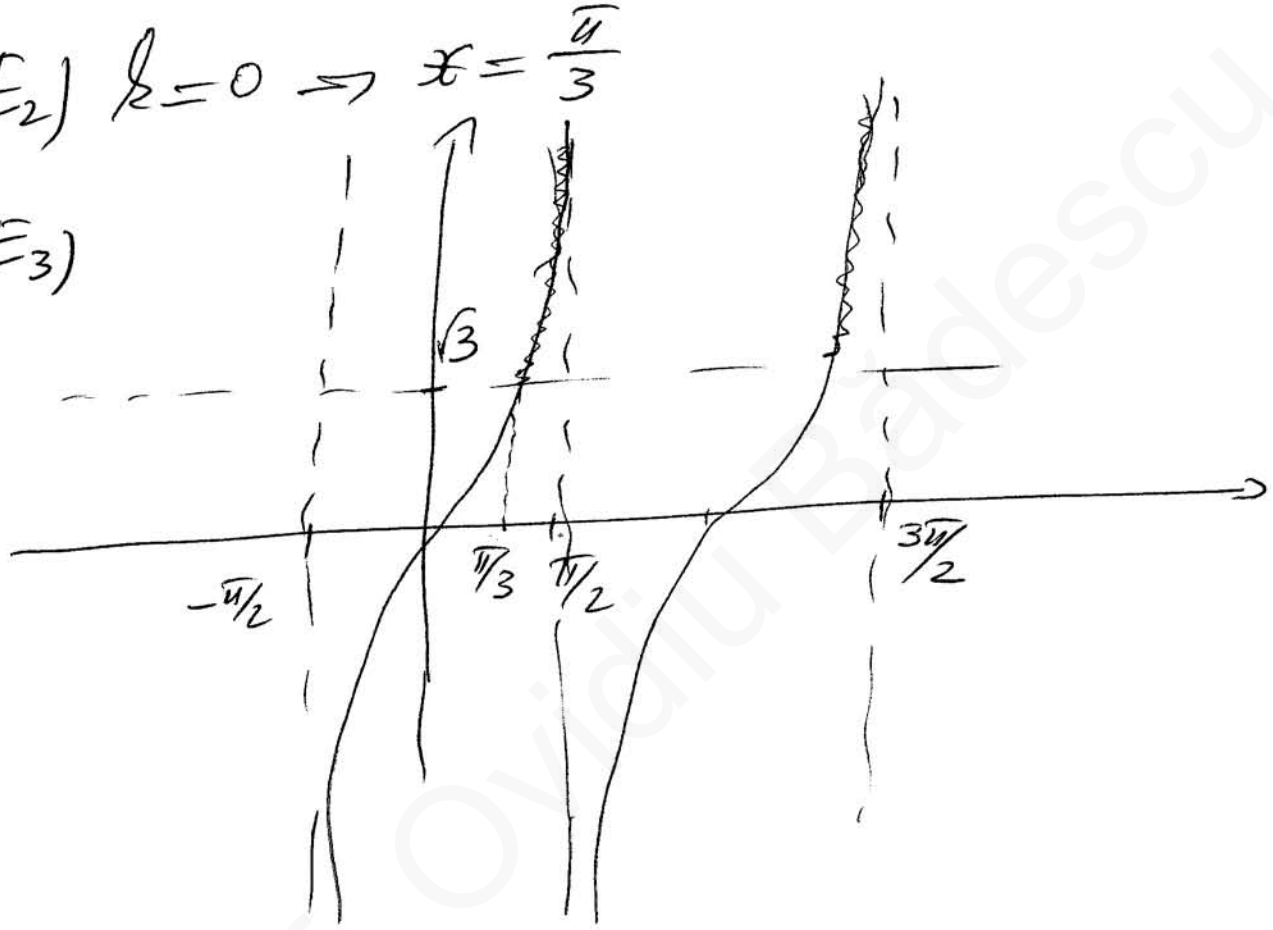
Rezolvați: $\operatorname{tg} x \geq \sqrt{3}$

E_1) Ec. atașată: $\operatorname{tg} x = \sqrt{3}$

$x \in \{\operatorname{arctg}\sqrt{3} + k\pi \mid k \in \mathbb{Z}\} \Rightarrow x \in \{\frac{\pi}{3} + k\pi \mid k \in \mathbb{Z}\}$

E_2) $k = 0 \Rightarrow x = \frac{\pi}{3}$

E_3)



E_4) $S_f = \left[\frac{\pi}{3}; \frac{\pi}{2}\right)$

$S_g = \left[\frac{\pi}{3} + k\pi; \frac{\pi}{2} + k\pi\right)$

Prof. Ovidiu Badescu

Rezolvați inecuația:

$12(\arcsin x)^2 - \pi \arcsin x - \pi^2 \leq 0$

E_1) not. $\arcsin x = t \Rightarrow$

$12t^2 - \pi t - \pi^2 \leq 0$

E_2) $t_{1,2} = \frac{\pi \pm \sqrt{\pi^2 - 4 \cdot 12 \cdot (-\pi^2)}}{2 \cdot 12} = \frac{\pi \pm \sqrt{49\pi^2}}{24} = \frac{\pi \pm 7\pi}{24}$

$t_1 = \frac{8\pi}{24} = \frac{\pi}{3}$

$t_2 = \frac{-6\pi}{24} = -\frac{\pi}{4}$

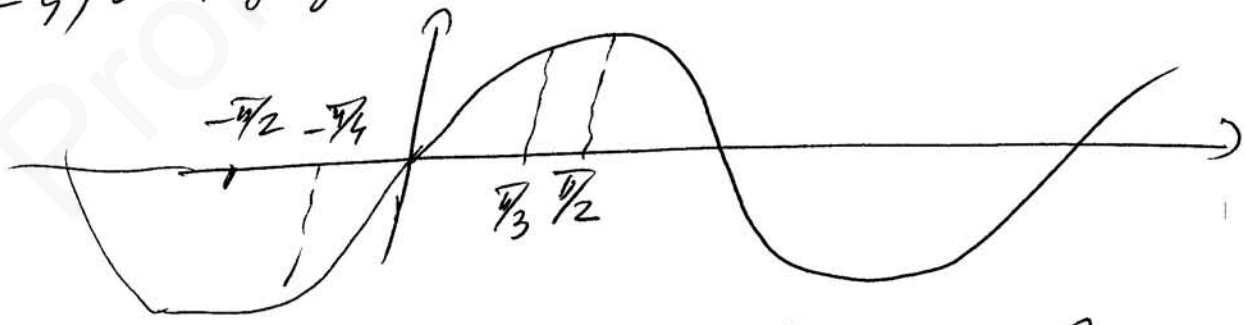
E_3)

t		$-\pi/4$		$\pi/3$	
$12t^2 - \pi t - \pi^2$	$+ + +$	0	$- - -$	0	$+ + + + +$

Rț ca $12t^2 - \pi t - \pi^2 \leq 0 \Rightarrow t \in [-\pi/4; \pi/3]$

$\Rightarrow -\pi/4 \leq \arcsin x \leq \pi/3$

E_4) Din graficul lui $\sin x \Rightarrow \sin$ este $\nearrow$ pe $[-\pi/4; \pi/3]$



E_5) $-\pi/4 \leq \arcsin x \leq \pi/3$ / $\sin \nearrow$

$\Rightarrow \sin(-\pi/4) \leq \sin(\arcsin x) \leq \sin \pi/3$

$\Rightarrow -\sin\frac{\pi}{4} \leq x \leq \sin\frac{\pi}{3} \Rightarrow -\frac{\sqrt{2}}{2} \leq x \leq \frac{\sqrt{3}}{2}$

$x \in [-\frac{\sqrt{2}}{2}; \frac{\sqrt{3}}{2}]$

Prof. Ovidiu Pădescu