ECUAȚII TRIGONOMETRICE DATE PRIN EGALITĂȚI DE FUNCȚII

Rezolvați în $\mathbb{R}$:

1) $\sin 7x = \sin 2x$

2) $\cos 12x = \cos 5x$

3) $\operatorname{tg} 5x = \operatorname{tg} 3x$

4) $\operatorname{ctg} 8x = \operatorname{ctg} 2x$

5) $\sin\left(2x + \frac{\pi}{6}\right) = \sin\left(x + \frac{\pi}{3}\right)$

6) $\operatorname{tg}\left(5x - \frac{\pi}{3}\right) = \operatorname{tg}\left(7x + \frac{\pi}{8}\right)$

7) $\operatorname{ctg}\left(2x + \frac{\pi}{4}\right) = \operatorname{ctg}\left(5x + \frac{\pi}{3}\right)$

8) $\cos\left(5x - \frac{\pi}{2}\right) = \cos\left(3x - \frac{\pi}{6}\right)$

9) $\cos\left(x - \frac{\pi}{7}\right) = \sin\left(3x + \frac{\pi}{3}\right)$

10) $\operatorname{tg}\left(5x + \frac{\pi}{3}\right) = \operatorname{ctg}\left(2x - \frac{\pi}{4}\right)$

Rezolvați în $[0; 2\pi]$

11) $\sin 2x = \sin x$

12) $\cos\left(3x + \frac{\pi}{7}\right) = \cos\left(x + \frac{\pi}{3}\right)$

13) $\cos\left(4x + \frac{\pi}{3}\right) = \cos\left(3x + \frac{\pi}{5}\right)$

14) $\sin\left(x - \frac{\pi}{4}\right) = \sin 2x$

15) $\operatorname{tg}\left(5x + \frac{\pi}{3}\right) = \operatorname{tg}\left(2x + \frac{\pi}{7}\right)$

16) $\operatorname{ctg}\left(x - \frac{\pi}{7}\right) = \operatorname{ctg}\left(3x + \frac{\pi}{3}\right)$

Rezolvați în $[-2\pi; 0]$:

17) $\sin\left(x + \frac{\pi}{6}\right) = \cos\left(2x + \frac{\pi}{3}\right)$

18) $\cos\left(3x + \frac{\pi}{3}\right) = \sin\left(x + \frac{\pi}{4}\right)$

19) $\operatorname{tg}\left(x + \frac{\pi}{4}\right) = \operatorname{ctg}\left(x + \frac{\pi}{3}\right)$

20) $\cos 5x = \cos 2x$ în $(0; \pi)$

21) $\sin\left(3x + \frac{\pi}{4}\right) = -\sin\left(2x + \frac{\pi}{6}\right)$

22) $\cos\left(3x + \frac{\pi}{3}\right) = -\cos\left(7x + \frac{\pi}{6}\right)$

23) $\operatorname{tg}\left(4x + \frac{\pi}{3}\right) = -\operatorname{tg}\left(x + \frac{\pi}{4}\right)$

24) $-\operatorname{ctg}\left(x + \frac{\pi}{5}\right) = \operatorname{ctg}\left(2x + \frac{\pi}{3}\right)$

25) $\cos\left(-3x + \frac{\pi}{4}\right) = -\cos\left(x + \frac{\pi}{6}\right)$

26) $-\cos\left(2x + \frac{\pi}{3}\right) = \sin\left(7x + \frac{\pi}{4}\right)$

27) $-\operatorname{ctg}\left(2x + \frac{\pi}{4}\right) = \operatorname{tg}\left(3x + \frac{\pi}{3}\right)$

28) $\sin\left(3x + \frac{\pi}{4}\right) = \sin\left(2x + \frac{\pi}{6}\right)$, $x \in [-4\pi; -2\pi]$

29) $\cos\left(3x + \frac{\pi}{3}\right) = -\cos\left(x + \frac{\pi}{4}\right)$, $x \in [0; 2\pi]$

30) $-\operatorname{tg}\left(x + \frac{\pi}{4}\right) = \operatorname{ctg}\left(x + \frac{\pi}{6}\right)$, $x \in [2\pi; 4\pi]$

31) $-\operatorname{ctg}\left(x + \frac{\pi}{3}\right) = \operatorname{ctg}\left(2x + \frac{\pi}{6}\right)$, $x \in [2\pi; 4\pi]$

32) $\sin\left(5x + \frac{\pi}{6}\right) = \sin\left(x - \frac{\pi}{3}\right)$

Rezolvați: $\sin 7x = \sin 2x$

E_1) folosim $\sin f(x) = \sin g(x) \Leftrightarrow$

$f(x) = g(x) + 2k\pi$ sau $f(x) = \pi - g(x) + 2k\pi$

E_2) în cazul nostru,

$7x = 2x + 2k\pi$ sau $7x = \pi - 2x + 2k\pi \Leftrightarrow$

$5x = 2k\pi$ sau $9x = \pi + 2k\pi$

$\Rightarrow x = \frac{2k\pi}{5}$ sau $x = \frac{\pi}{9} + \frac{2k\pi}{9}$

E_3) $x \in \left\{ \frac{2k\pi}{5} \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{9} + \frac{2k\pi}{9} \mid k \in \mathbb{Z} \right\}$

Prof. Ovidiu Badescu

Rezolvați: $\cos 12x = \cos 5x$

E_1) Folosim: $\cos f(x) = \cos g(x) \Leftrightarrow$

$f(x) = g(x) + 2k\pi$ sau $f(x) = -g(x) + 2k\pi$

E_2) $12x = 5x + 2k\pi$ sau $12x = -5x + 2k\pi$

$\Rightarrow 7x = 2k\pi$ sau $17x = 2k\pi$

$\Rightarrow x = \frac{2k\pi}{7}$ sau $x = \frac{2k\pi}{17}$

E_3) $x \in \left\{ \frac{2k\pi}{7}, \frac{2k\pi}{17} \right\}$

Prof. Ovidiu Bădescu

Rezolvați: $\operatorname{tg} 5x = \operatorname{tg} 3x$

E_1) folosim: $\operatorname{tg} f(x) = \operatorname{tg} g(x) \Leftrightarrow$

$f(x) = g(x) + k\pi$

E_2) $5x = 3x + k\pi \Rightarrow 2x = k\pi$

$\Rightarrow x = \frac{k\pi}{2}$

E_3) $x \in \left\{ \frac{k\pi}{2} \mid k \in \mathbb{Z} \right\}$

Prof. Ovidiu Bădescu

Rezolvați: $\operatorname{ctg} 8x = \operatorname{ctg} 2x$

E_1) folosim $\operatorname{ctg} f(x) = \operatorname{ctg} g(x) \Leftrightarrow$

$f(x) = g(x) + k\pi, \quad k \in \mathbb{Z}$

E_2) În cazul nostru,

$$8x = 2x + k\pi \Leftrightarrow 6x = k\pi$$

$$\Leftrightarrow x = \frac{k\pi}{6}, \quad k \in \mathbb{Z}$$

E_3) $x \in \left\{ \frac{k\pi}{6} \mid k \in \mathbb{Z} \right\}$

Prof. Ovidiu Bădescu

Rezolvați

$$\sin\left(2x+\frac{\pi}{6}\right)=\sin\left(x+\frac{\pi}{3}\right)$$

E_1) Folosim $\sin f(x)=\sin g(x) \Rightarrow$

$\Rightarrow f(x)=g(x)+2k\pi$ sau $f(x)=\pi-g(x)+2k\pi$

E_2) Din ecuație $\Rightarrow$

$2x+\frac{\pi}{6}=x+\frac{\pi}{3}+2k\pi$ sau $2x+\frac{\pi}{6}=\pi-\left(x+\frac{\pi}{3}\right)+2k\pi$

$\Rightarrow x=\frac{\pi}{3}-\frac{\pi}{6}+2k\pi$ sau $3x=\pi-\frac{\pi}{3}-\frac{\pi}{6}+2k\pi$

$\Rightarrow x=\frac{\pi}{6}+2k\pi$ sau $3x=\frac{3\pi}{6_2}+2k\pi \mid :3$

$x=\frac{\pi}{6}+2k\pi$ sau $x=\frac{\pi}{6}+\frac{2k\pi}{3}$

$E_3) x\in\left\{\frac{\pi}{6}+2k\pi \mid k\in\mathbb{Z}\right\}\cup\left\{\frac{\pi}{6}+\frac{2k\pi}{3} \mid k\in\mathbb{Z}\right\}$

Prof. Ovidiu Băăscu

Rezolvați: $\cos\left(x-\frac{\pi}{7}\right)=\sin\left(3x+\frac{\pi}{3}\right)$

E_1) folosim $\cos t=\sin\left(\frac{\pi}{2}-t\right)$

$$\Rightarrow \cos\left(x-\frac{\pi}{7}\right)=\sin\left(\frac{7}{\frac{\pi}{2}}-x+\frac{2}{\frac{\pi}{7}}\right)=$$

$$=\sin\left(\frac{9\pi}{14}-x\right)$$

E_2) Ec.: $\sin\left(\frac{9\pi}{14}-x\right)=\sin\left(3x+\frac{\pi}{3}\right)$

$$\Rightarrow \frac{9\pi}{14}-x=3x+\frac{\pi}{3}+2k\pi \text{ sau}$$

$$\frac{9\pi}{14}-x=\pi-\left(3x+\frac{\pi}{3}\right)+2k\pi$$

E_3) $-4x=\frac{\pi}{3}-\frac{9\pi}{14}+2k\pi$ sau

$$2x=\pi-\frac{\pi}{3}-\frac{9\pi}{14}+2k\pi$$

$$-4x=\frac{-13\pi}{42}+2k\pi \text{ sau } 2x=\frac{\pi}{42}+2k\pi$$

$$\Rightarrow x=\frac{13\pi}{168}-\frac{k\pi}{2} \text{ sau } x=\frac{\pi}{84}+k\pi$$

E_4) $x\in\left\{\frac{13\pi}{168}-\frac{k\pi}{2}\mid k\in\mathbb{Z}\right\}\cup\left\{\frac{\pi}{84}+k\pi\mid k\in\mathbb{Z}\right\}$

Prof. Ovidiu Bădescu

Să se rezolve în $[0, 2\pi)$ ecuația

$$\sin 2x = \sin x$$

$E_1)$ $\sin f(x) = \sin g(x) \Rightarrow$

$\Rightarrow f(x) = g(x) + 2k\pi$ sau $f(x) = \pi - g(x) + 2k\pi$

$E_2)$ $\sin 2x = \sin x \Rightarrow$

$2x = x + 2k\pi$ sau $2x = \pi - x + 2k\pi$

$\Rightarrow x = 2k\pi$ sau $x = \frac{\pi}{3} + \frac{2k\pi}{3}$

Obs.: Dacă cerea soluții din $\mathbb{R} \Rightarrow$ pb. era rezolvată

$E_3)$ Se cer soluții din $[0, 2\pi)$

$k = -1 \Rightarrow x = -2\pi \notin [0, 2\pi)$ sau $x = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3} \notin [0, 2\pi)$

$k = 0 \Rightarrow x = 0 \in [0, 2\pi)$ sau $x = \frac{\pi}{3} + 0 = \frac{\pi}{3} \in [0, 2\pi)$

$k = 1 \Rightarrow x = 2\pi \notin [0, 2\pi)$ sau $x = \frac{\pi}{3} + \frac{2\pi}{3} = \pi \in [0, 2\pi)$

$k = 2 \Rightarrow x = 4\pi \notin [0, 2\pi)$ sau $x = \frac{\pi}{3} + \frac{4\pi}{3} = \frac{5\pi}{3} \in [0, 2\pi)$

$k = 3 \Rightarrow x = 6\pi \notin [0, 2\pi)$ sau $x = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3} \notin [0, 2\pi)$

$E_4)$ $S_f : x \in \left\{0; \frac{\pi}{3}; \pi; \frac{5\pi}{3}\right\}$

Prof. Ovidiu Bădescu

Rezolvați în $[0; 2\pi]$ ecuația:

$$\cos\left(3x + \frac{\pi}{7}\right) = \cos\left(x + \frac{\pi}{3}\right)$$

E1) folosim $\cos f(x) = \cos g(x) \Leftrightarrow$

$f(x) = g(x) + 2k\pi, k \in \mathbb{Z}$ sau $f(x) = -g(x) + 2k\pi / k \in \mathbb{Z}$

E2) $3x + \frac{\pi}{7} = x + \frac{\pi}{3} + 2k\pi$ sau $3x + \frac{\pi}{7} = -\left(x + \frac{\pi}{3}\right) + 2k\pi$

$2x = \frac{7\pi}{3} - \frac{3\pi}{7} + 2k\pi$ sau $4x = -\frac{7\pi}{3} - \frac{3\pi}{7} + 2k\pi$

$2x = \frac{14\pi}{21} + 2k\pi$ sau $4x = -\frac{10\pi}{21} + 2k\pi$

$\Rightarrow x = \frac{2\pi}{21} + k\pi$ sau $x = -\frac{5\pi}{42} + \frac{k\pi}{2}$

E3) $x \in \left\{\frac{2\pi}{21} + k\pi / k \in \mathbb{Z}\right\} \cup \left\{-\frac{5\pi}{42} + \frac{k\pi}{2} / k \in \mathbb{Z}\right\}$

E4) x cer soluții din $[0; 2\pi]$, dăm valori lui k

$k = -1 \rightarrow \left\{\frac{2\pi}{21} - \frac{21\pi}{21}; -\frac{5\pi}{42} - \frac{21\pi}{42}\right\} \cap [0; 2\pi] = \left\{-\frac{19\pi}{21}; -\frac{26\pi}{42}\right\} \cap [0; 2\pi] = \emptyset$

$k = 0 \rightarrow \left\{\frac{2\pi}{21}; -\frac{5\pi}{42}\right\} \cap [0; 2\pi] = \left\{\frac{2\pi}{21}\right\}$

$k = 1 \rightarrow \left\{\frac{2\pi}{21} + \frac{21\pi}{21}; -\frac{5\pi}{42} + \frac{21\pi}{42}\right\} = \left\{\frac{23\pi}{21}; \frac{16\pi}{42}\right\} \cap [0; 2\pi] = \left\{\frac{23\pi}{21}; \frac{16\pi}{42}\right\}$

$k = 2 \rightarrow \left\{\frac{2\pi}{21} + \frac{42\pi}{21}; -\frac{5\pi}{42} + \frac{42\pi}{42}\right\} = \left\{\frac{44\pi}{21}; \frac{37\pi}{42}\right\} \cap [0; 2\pi] = \left\{\frac{37\pi}{42}\right\}$

$k = 3 \rightarrow \left\{\frac{2\pi}{21} + \frac{63\pi}{21}; -\frac{5\pi}{42} + \frac{63\pi}{42}\right\} = \left\{\frac{65\pi}{21}; \frac{58\pi}{42}\right\} \cap [0; 2\pi] = \left\{\frac{58\pi}{42}\right\}$

$k = 4 \rightarrow \left\{\frac{2\pi}{21} + \frac{84\pi}{21}; -\frac{5\pi}{42} + \frac{84\pi}{42}\right\} = \left\{\frac{86\pi}{21}; \frac{79\pi}{42}\right\} \cap [0; 2\pi] = \left\{\frac{79\pi}{42}\right\}$

$k = 5 \rightarrow \left\{\frac{2\pi}{21} + \frac{105\pi}{21}; -\frac{5\pi}{42} + \frac{105\pi}{42}\right\} = \left\{\frac{107\pi}{21}; \frac{100\pi}{42}\right\} \cap [0; 2\pi] = \emptyset$

E5) $S_f: x \in \left\{\frac{2\pi}{21}; \frac{23\pi}{21}; \frac{8\pi}{21}; \frac{37\pi}{42}; \frac{29\pi}{21}; \frac{79\pi}{42}\right\}$

Rezolvați: $\cos\left(4x+\frac{\pi}{3}\right)=\cos\left(3x+\frac{\pi}{5}\right)$ în $[0;2\pi]$

E_1) folosim $\cos f(x)=\cos g(x) \Leftrightarrow$

$f(x)=g(x)+2k\pi$ sau $f(x)=-g(x)+2k\pi$

E_2) în cazul nostru,

$4x+\frac{\pi}{3}=3x+\frac{\pi}{5}+2k\pi$ sau $4x+\frac{\pi}{3}=-\left(3x+\frac{\pi}{5}\right)+2k\pi$

$\Leftrightarrow x=\frac{\pi}{5}-\frac{\pi}{3}+2k\pi$ sau $7x=-\frac{\pi}{5}-\frac{\pi}{3}+2k\pi$

$\Leftrightarrow x=-\frac{2\pi}{15}+2k\pi$ sau $x=\frac{-\frac{8\pi}{15}+2k\pi}{7}$

E_3) $x\in\left\{-\frac{2\pi}{15}+2k\pi \mid k\in\mathbb{Z}\right\}\cup\left\{-\frac{8\pi}{107}+\frac{2k\pi}{7} \mid k\in\mathbb{Z}\right\}$

E_4) soluții din $[0;2\pi]$, dăm valori lui k

$k=-1 \Rightarrow x\in\left\{-\frac{2\pi}{15}-2\pi;-\frac{8\pi}{107}-\frac{2\pi}{7}\right\}\cap[0;2\pi]=\emptyset$

$k=0 \Rightarrow x\in\left\{-\frac{2\pi}{15};-\frac{8\pi}{107}\right\}\cap[0;2\pi]=\emptyset$

$k=1 \Rightarrow x\in\left\{-\frac{2\pi}{15}+2\pi;-\frac{8\pi}{107}+2\pi\right\}=\left\{\frac{28\pi}{15};\frac{306\pi}{107}\right\}\cap[0;2\pi]=$

$=\left\{\frac{28\pi}{15};\frac{306\pi}{107}\right\}$

$k=2 \Rightarrow x\in\left\{-\frac{2\pi}{15}+4\pi;-\frac{8\pi}{107}+4\pi\right\}=\left\{\frac{58\pi}{15};\frac{620\pi}{107}\right\}\cap[0;2\pi]=\emptyset$

E_5) $S_g: x\in\left\{\frac{28\pi}{15};\frac{306\pi}{107}\right\}$

Prof. Viviu Bădescu

Rezolvați în $[-2\pi, 0)$ ecuația:

$$\sin\left(x+\frac{\pi}{6}\right)=\cos\left(2x+\frac{\pi}{3}\right)$$

E_1) Dacă aveam $\sin f(x)=\sin g(x) \Rightarrow$
$f(x)=g(x)+2k\pi$ sau $f(x)=\pi-g(x)+2k\pi$

E_2) Folosim $\cos x=\sin\left(\frac{\pi}{2}-x\right)$
$\Rightarrow$ ecuația: $\sin\left(x+\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{2}-\left(2x+\frac{\pi}{3}\right)\right)$

$$\sin\left(x+\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{6}-2x\right)$$

E_3) $x+\frac{\pi}{6}=\frac{\pi}{6}-2x+2k\pi$ sau
$x+\frac{\pi}{6}=\pi-\left(\frac{\pi}{6}-2x\right)+2k\pi$

E_4) $3x=2k\pi$ sau $-x=\frac{6}{\pi}-\frac{\pi}{6}-\frac{\pi}{6}+2k\pi$

$x=\frac{2k\pi}{3}$ sau $x=-\frac{2\pi}{3}-2k\pi$

E_5) Se cer soluții din $[-2\pi, 0)=I$

$k=-1 \Rightarrow \boxed{x=-\frac{2\pi}{3}\in I}$; $x=-\frac{2\pi}{3}+2\pi=\frac{4\pi}{3}\notin I$

$k=-2 \Rightarrow \boxed{x=-\frac{4\pi}{3}\in I}$; $x=-\frac{2\pi}{3}+4\pi=\frac{10\pi}{3}\notin I$

$k=-3 \Rightarrow \boxed{x=-2\pi\in I}$; $x=-\frac{2\pi}{3}+6\pi=\frac{16\pi}{3}\notin I$

$k=0 \Rightarrow x=0\notin I$, $\boxed{x=-\frac{2\pi}{3}\in I}$

$\frac{16}{3}\pi\notin I$

Prof. Ovidiu Bădescu

Rezolvați în $(0;\pi)$ ecuația:

$$\cos 5x = \cos 2x$$

E_1) Folosim $\cos f(x) = \cos g(x) \Rightarrow$

$f(x) = g(x) + 2k\pi$ sau $f(x) = -g(x) + 2k\pi$

E_2) $5x = 2x + 2k\pi$ sau $5x = -2x + 2k\pi$

$\Rightarrow 3x = 2k\pi$ sau $7x = 2k\pi$

$x = \frac{2k\pi}{3}$ sau $x = \frac{2k\pi}{7}$

E_3) $x \in \left\{ \frac{2k\pi}{3} \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{2k\pi}{7} \mid k \in \mathbb{Z} \right\}$

E_4) Dacă x cere rezolvare în $\mathbb{R} \Rightarrow$ soluțiile erau OK, x cere rezolvare din $(0;\pi) \Rightarrow$ dăm valori lui k

$k = -1 \Rightarrow x \in \left\{ -\frac{2\pi}{3}; -\frac{2\pi}{7} \right\} \cap (0;\pi) = \emptyset$

$k = 0 \Rightarrow x \in \left\{ 0 \right\} = \emptyset$

$k = 1 \Rightarrow x \in \left\{ \frac{2\pi}{3}; \frac{2\pi}{7} \right\} \cap (0;\pi) = \left\{ \frac{2\pi}{3}; \frac{2\pi}{7} \right\}$

$k = 2 \Rightarrow x \in \left\{ \frac{4\pi}{3}; \frac{4\pi}{7} \right\} \cap (0;\pi) = \left\{ \frac{4\pi}{7} \right\}$

$k = 3 \Rightarrow x \in \left\{ 2\pi; \frac{6\pi}{7} \right\} \cap (0;\pi) = \left\{ \frac{6\pi}{7} \right\}$

$k = 4 \Rightarrow x \in \left\{ \frac{8\pi}{3}; \frac{8\pi}{7} \right\} \cap (0;\pi) = \emptyset$

E_5) $S_f = \left\{ \frac{2\pi}{3}; \frac{2\pi}{7}; \frac{4\pi}{7}; \frac{6\pi}{7} \right\}$

Prof Ovidiu Bârcsu

Rezolvați în $[-4\pi, -2\pi]$

$$\sin\left(3x+\frac{\pi}{4}\right)=\sin\left(2x+\frac{\pi}{6}\right)$$

E_1) Folosim $\sin f(x)=\sin g(x) \Leftrightarrow$

$f(x)=g(x)+2k\pi$ sau $f(x)=\pi-g(x)+2k\pi$

E_2) $\left(3x+\frac{\pi}{4}=2x+\frac{\pi}{6}+2k\pi\right)$ sau $\left(3x+\frac{\pi}{4}=\pi-\left(2x+\frac{\pi}{6}\right)+2k\pi\right)$

$\Leftrightarrow x=\frac{2\pi}{6}-\frac{3\pi}{4}+2k\pi$ sau $5x=\frac{12\pi}{12}-\frac{2\pi}{6}-\frac{3\pi}{4}+2k\pi$

$\Leftrightarrow x=-\frac{\pi}{12}+2k\pi$ sau $x=\frac{\frac{7\pi}{12}+2k\pi}{5}$

E_3) $x\in\left\{-\frac{\pi}{12}+2k\pi \mid k\in\mathbb{Z}\right\}\cup\left\{\frac{7\pi}{60}+\frac{2k\pi}{5} \mid k\in\mathbb{Z}\right\}$

E_4) x cer soluții din $[-4\pi;-2\pi] \Rightarrow$ dăm valori

$k=0 \Rightarrow x\in\left\{-\frac{\pi}{12};\frac{7\pi}{6}\right\}\cap[-4\pi;-2\pi]=\emptyset$

$k=-1 \Rightarrow x\in\left\{-\frac{\pi}{12}-2\pi;\frac{7\pi}{60}-\frac{2\pi}{5}\right\}\cap[-4\pi;-2\pi]=\left\{-\frac{25\pi}{12};-\frac{17\pi}{60}\right\}\cap[-4\pi;-2\pi]$

$\Rightarrow \boxed{x\in\left\{-\frac{25\pi}{12}\right\}}$

$k=-2 \Rightarrow x\in\left\{-\frac{\pi}{12}-4\pi;\frac{7\pi}{60}-\frac{4\pi}{5}\right\}\cap[-4\pi;-2\pi]=\left\{-\frac{49\pi}{12};-\frac{41\pi}{60}\right\}\cap[-4\pi;-2\pi]=\emptyset$

$k=-3 \Rightarrow x\in\left\{-\frac{\pi}{12}-6\pi;\frac{7\pi}{60}-\frac{6\pi}{5}\right\}\cap[-4\pi;-2\pi]=\left\{-\frac{73\pi}{12};-\frac{65\pi}{60}\right\}\cap[-4\pi;-2\pi]=\emptyset$

$k=-4 \Rightarrow x\in\left\{-\frac{\pi}{12}-8\pi;\frac{7\pi}{60}-\frac{8\pi}{5}\right\}\cap[-4\pi;-2\pi]=\left\{-\frac{97\pi}{12};-\frac{89\pi}{60}\right\}\cap[-4\pi;-2\pi]=\emptyset$

$k=-5 \Rightarrow x\in\left\{-\frac{\pi}{12}-10\pi;\frac{7\pi}{60}-2\pi\right\}\cap[-4\pi;-2\pi]=\emptyset$

$k=-6 \Rightarrow x\in\left\{-\frac{\pi}{12}-12\pi;\frac{7\pi}{60}-\frac{12\pi}{5}\right\}\cap[-4\pi;-2\pi]=\boxed{\left\{-\frac{137\pi}{60}\right\}}$

$k=-7 \Rightarrow x\in\left\{-\frac{\pi}{12}-14\pi;\frac{7\pi}{60}-\frac{14\pi}{5}\right\}\cap[-4\pi;-2\pi]=\boxed{\left\{-\frac{161\pi}{60}\right\}}$

$k=-8 \Rightarrow \boxed{x\in\left\{-\frac{185\pi}{60}\right\}}$, $k=-9 \Rightarrow \boxed{x\in\left\{-\frac{209\pi}{60}\right\}}$; $k=-10 \Rightarrow \boxed{x\in\left\{-\frac{233\pi}{60}\right\}}$,

$k=-11 \Rightarrow x\in\left\{-\frac{257\pi}{60}\right\}\cap[-4\pi;-2\pi]=\emptyset \Rightarrow S_f=$ este reuniunea celor de sus

Rezolvați:

$$\sin\left(5x+\frac{\pi}{6}\right)=\sin\left(x-\frac{\pi}{3}\right)$$

E1/ Soluție $\sin f(x)=\sin g(x)$

$\Rightarrow f(x)=g(x)+2k\pi$ sau $f(x)=\pi-g(x)+2k\pi$

E2) $5x+\frac{\pi}{6}=x-\frac{\pi}{3}+2k\pi$ sau $5x+\frac{\pi}{6}=\pi-\left(x-\frac{\pi}{3}\right)+2k\pi$

$\Rightarrow 4x=-\frac{2}{3}\pi-\frac{\pi}{6}+2k\pi$ sau $6x=\frac{6}{\pi}+\frac{2}{3}\pi-\frac{\pi}{6}+2k\pi$

$\Rightarrow 4x=-\frac{3\pi}{6}+2k\pi$ sau $6x=\frac{7\pi}{6}+2k\pi$

E3) $x\in\left\{\frac{-\frac{\pi}{2}+2k\pi}{4}\,\middle|\,k\in\mathbb{Z}\right\}\cup\left\{\frac{\frac{7\pi}{6}+2k\pi}{6}\,\middle|\,k\in\mathbb{Z}\right.$

Prof. Ovidiu Bădescu