ECUAȚII TRIGONOMETRICE FUNDAMENTALE

Rezolvați în $\mathbb{R}$:

1) $\sin x = \frac{1}{2}$ și $\cos x = \frac{\sqrt{2}}{2}$

2) $\sin x = 1$ și $\cos x = \frac{\sqrt{3}}{2}$

3) $\sin x = \frac{\sqrt{3}}{2}$ și $\operatorname{tg} x = \sqrt{3}$

4) $\cos x = \frac{\sqrt{3}}{2}$ și $\operatorname{tg} x = \frac{1}{\sqrt{3}}$

5) $\cos x = \frac{\sqrt{2}}{2}$ și $\operatorname{tg} x = 1$

6) $\sin x = -\frac{1}{2}$ și $\operatorname{ctg} x = \sqrt{3}$

7) $\sin x = -\frac{\sqrt{3}}{2}$ și $\operatorname{tg} x = 1$

8) $\cos x = -\frac{\sqrt{2}}{2}$ și $\operatorname{ctg} x = -\sqrt{3}$

9) $\cos x = -\frac{1}{2}$ și $\sin x = -\frac{\sqrt{2}}{2}$

10) $\operatorname{ctg} x = -\frac{1}{\sqrt{3}}$ și $\cos x = \frac{\sqrt{3}}{2}$

11) $\sin\left(2x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$

12) $\cos\left(3x + \frac{\pi}{3}\right) = -\frac{1}{2}$

13) $\operatorname{tg}\left(x - \frac{\pi}{6}\right) = 1$

14) $\operatorname{ctg}\left(\frac{\pi}{4} - x\right) = -\sqrt{3}$

15) $\cos x = \frac{\sqrt{5}}{2}$ și $\operatorname{tg} x = \sqrt{3}$

16) $\sin x = \sqrt{3}$ și $\operatorname{ctg} x = \frac{1}{\sqrt{3}}$

17) $\sin\left(4x + \frac{\pi}{3}\right) = -1$

18) $\cos\left(5x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

19) $\operatorname{tg}\left(x - \frac{\pi}{3}\right) = -1$

20) $\operatorname{ctg}\left(\frac{\pi}{6} - 3x\right) = -\frac{1}{2}$

Rezolvați în $[0; 2\pi)$

21) $\sin x = -\frac{\sqrt{3}}{2}$

22) $\cos\left(4x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$

23) $\operatorname{tg}\left(5x - \frac{\pi}{4}\right) = -1$

24) $\operatorname{ctg}\left(3x - \frac{\pi}{3}\right) = -1$

25) $\sin\left(3x + \frac{\pi}{3}\right) = -\frac{1}{2}$ în $[-2\pi; 0]$

26) $\cos x = -\frac{\sqrt{3}}{2}$

27) $\operatorname{tg} x = -\sqrt{3}$

28) $\operatorname{ctg} x = -\sqrt{3}$

29) $\sin x = -\frac{1}{2}$

30) $\cos x = \frac{\sqrt{3}}{2}$

31) $\sin 2x = \frac{1}{2}$

32) $\operatorname{tg} x = -1$

33) $\cos 2x = \frac{1}{2}$

Prof. Ovidiu Badescu

Rezolvați în $\mathbb{R}$: a) $\cos x = -\frac{1}{2}$ b) $\sin x = -\frac{\sqrt{2}}{2}$ 9

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a) $\cos x = -\frac{1}{2}$

$E_1) -\frac{1}{2} \in [-1;1] \Rightarrow$ ec are soluții de forma

$E_2) x \in \{\arccos(-\frac{1}{2}) + 2k\pi / k \in \mathbb{Z}\} \cup \{-\arccos(-\frac{1}{2}) + 2k\pi / k \in \mathbb{Z}\}$

$\Leftrightarrow x \in \{\pi - \arccos\frac{1}{2} + 2k\pi / k \in \mathbb{Z}\} \cup \{-(\pi - \arccos\frac{1}{2}) + 2k\pi / k \in \mathbb{Z}\}$

$E_3)$ cum $\arccos\frac{1}{2} = \frac{\pi}{3}$ pt că $\cos\frac{\pi}{3} = \frac{1}{2}$

$\Rightarrow x \in \{\pi - \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z}\} \cup \{-(\pi - \frac{\pi}{3}) + 2k\pi / k \in \mathbb{Z}\}$

$$\boxed{x \in \{\frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z}\} \cup \{-\frac{2\pi}{3} + 2k\pi / k \in \mathbb{Z}\}}$$

b) $\sin x = -\frac{\sqrt{2}}{2}$

$E_1) -\frac{\sqrt{2}}{2} \in [-1;1] \Rightarrow$ ec are soluții de forma

$x \in \{\arcsin(-\frac{\sqrt{2}}{2}) + 2k\pi / k \in \mathbb{Z}\} \cup \{\pi - \arcsin(-\frac{\sqrt{2}}{2}) + 2k\pi / k \in \mathbb{Z}\}$

$x \in \{-\arcsin\frac{\sqrt{2}}{2} + 2k\pi / k \in \mathbb{Z}\} \cup \{\pi + \arcsin\frac{\sqrt{2}}{2} + 2k\pi / k \in \mathbb{Z}\}$

$x \in \{-\frac{\pi}{4} + 2k\pi / k \in \mathbb{Z}\} \cup \{\pi + \frac{\pi}{4} + 2k\pi / k \in \mathbb{Z}\}$

$$\boxed{x \in \{-\frac{\pi}{4} + 2k\pi / k \in \mathbb{Z}\} \cup \{\frac{5\pi}{4} + 2k\pi / k \in \mathbb{Z}\}}$$

Prof. Ovidiu Badescu

Rezolvați în $[0; 2\pi]$ ecuația

$$\sin x = -\frac{\sqrt{3}}{2}$$

E_1) $\sin x = -\frac{\sqrt{3}}{2}$, cum $-\frac{\sqrt{3}}{2} \in [-1; 1] \Rightarrow$ ec are soluții

$x \in \{\arcsin\left(-\frac{\sqrt{3}}{2}\right) + 2k\pi / k \in \mathbb{Z}\} \cup \{\pi - \arcsin\left(-\frac{1}{2}\right) + 2k\pi / k \in \mathbb{Z}\}$

$x \in \{-\arcsin\frac{\sqrt{3}}{2} + 2k\pi / k \in \mathbb{Z}\} \cup \{\pi + \arcsin\frac{1}{2} + 2k\pi / k \in \mathbb{Z}\}$

E_2) cum $\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3} \Rightarrow$

$x \in \{-\frac{\pi}{3} + 2k\pi / k \in \mathbb{Z}\} \cup \{\pi + \frac{\pi}{3} + 2k\pi / k \in \mathbb{Z}\}$

$\Rightarrow x \in \{-\frac{\pi}{3} + 2k\pi / k \in \mathbb{Z}\} \cup \{\frac{4\pi}{3} + 2k\pi / k \in \mathbb{Z}\}$

E_3) x cer soluții din $[0; 2\pi] \Rightarrow$ dăm valori lui k

$k = -1 \Rightarrow x \in \{-\frac{\pi}{3} - 2\pi; \frac{4\pi}{3} - 2\pi\} \cap [0; 2\pi] = \{-\frac{7\pi}{3}; -\frac{2\pi}{3}\} \cap [0; 2\pi] = \emptyset$

$k = 0 \Rightarrow x \in \{-\frac{\pi}{3} + 0; \frac{4\pi}{3} + 0\} \cap [0; 2\pi] = \{-\frac{\pi}{3}; \frac{4\pi}{3}\} \cap [0; 2\pi] = \{\frac{4\pi}{3}\}$

$k = 1 \Rightarrow x \in \{-\frac{\pi}{3} + 2\pi; \frac{4\pi}{3} + 2\pi\} \cap [0; 2\pi] = \{\frac{5\pi}{3}; \frac{10\pi}{3}\} \cap [0; 2\pi] = \{\frac{5\pi}{3}\}$

$k = 2 \Rightarrow x \in \{-\frac{\pi}{3} + 4\pi; \frac{4\pi}{3} + 4\pi\} \cap [0; 2\pi] = \{\frac{11\pi}{3}; \frac{16\pi}{3}\} \cap [0; 2\pi] = \emptyset$

$S_f: x \in \{\frac{4\pi}{3}; \frac{5\pi}{3}\}$

Prof. Ovidiu Pădesu

Rezolvați în $[0, 2\pi)$ ecuația:

$$\sin x = -\frac{1}{2}$$

E1) $\sin x = -\frac{1}{2}$, $-\frac{1}{2} \in [-1, 1] \Rightarrow$

$x \in \{\arcsin(-\frac{1}{2}) + 2k\pi / k \in \mathbb{Z}\} \cup \{\pi - \arcsin(-\frac{1}{2}) + 2k\pi / k \in$

$x \in \{-\frac{\pi}{6} + 2k\pi / k \in \mathbb{Z}\} \cup \{\frac{7\pi}{6} + 2k\pi / k \in \mathbb{Z}\}$

E2) x cu soluții din $[0, 2\pi)$

$k = -1 \Rightarrow \{-\frac{\pi}{6} - 2\pi, \frac{7\pi}{6} - 2\pi\} = \{-\frac{13\pi}{6}, -\frac{5\pi}{6}\}$

$\hat{\in} [0, 2\pi) \Rightarrow x \in \emptyset$

$k = 0 \Rightarrow \{-\frac{\pi}{6}, \frac{7\pi}{6}\} \hat{\in} [0, 2\pi) \Rightarrow \underline{x = \frac{7\pi}{6}}$

$k = 1 \Rightarrow \{-\frac{\pi}{6} + 2\pi, \frac{7\pi}{6} + 2\pi\} = \{\frac{11\pi}{6}, \frac{19\pi}{6}\}$

$\hat{\in} [0, 2\pi) \Rightarrow \underline{x = \frac{11\pi}{6}}$

$k = 2 \Rightarrow \{-\frac{\pi}{6} + 4\pi, \frac{7\pi}{6} + 4\pi\} = \{\frac{23\pi}{6}, \frac{31\pi}{6}\}$

$\hat{\in} [0, 2\pi) \Rightarrow x \in \emptyset$

E3) $S_g: x \in \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$

Prof. Ovidiu Badescu

Rezolvați în $[0;2\pi)$ ecuația $\cos x = \frac{\sqrt{3}}{2}$

E_1) $\frac{\sqrt{3}}{2} \in [-1;1] \Rightarrow x \in \{\pm \arccos\frac{\sqrt{3}}{2} + 2k\pi / k \in \mathbb{Z}\}$

$x \in \{\pm \frac{\pi}{6} + 2k\pi / k \in \mathbb{Z}\}$

E_2) Dar, x cu soluțiile din $[0;2\pi)$

$k=0 \Rightarrow x = \{\pm \frac{\pi}{6}\} \rightarrow \in [0;2\pi) \Rightarrow \frac{\pi}{6} \in [0;2\pi)$

$k=-1 \Rightarrow x = \{\frac{\pi}{6} + 2k\pi; -\frac{\pi}{6} + 2k\pi\} \rightarrow \emptyset$

$k=1 \Rightarrow x = \{\frac{\pi}{6} + 2\pi; -\frac{\pi}{6} + 2\pi\} \rightarrow \frac{5\pi}{6} \in [0;2\pi)$

$S_f: x \in \{\frac{\pi}{6}; \frac{5\pi}{6}\}$

Prof. Ovidiu Bădescu

Rezolvați în $[0;2\pi)$ ecuația

$$\sin 2x = \frac{1}{2}$$

$E_1)$ $2x \in \{\arcsin\frac{1}{2} + 2k\pi \mid k \in \mathbb{Z}\} \cup$

$\cup \{\pi - \arcsin\frac{1}{2} + 2k\pi \mid k \in \mathbb{Z}\}$

$2x \in \{\frac{\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\} \cup \{\frac{5\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\}$

$E_2)$ $x \in \{\frac{\pi}{12} + k\pi \mid k \in \mathbb{Z}\} \cup \{\frac{5\pi}{12} + k\pi \mid k \in \mathbb{Z}\}$

$E_3)$ Dar, se cer soluțiile din $[0;2\pi) = I$

dăm valori lui k

$k = -1 \Rightarrow \frac{\pi}{12} - \pi \notin I$, $\frac{5\pi}{12} - \pi \notin I$

$k = 0 \Rightarrow \frac{\pi}{12} \in I$; $\frac{5\pi}{12} \in I$

$k = 1 \Rightarrow \frac{\pi}{12} + \pi \in I$; $\frac{5\pi}{12} + \pi \in I$

$k = 2 \Rightarrow \frac{\pi}{12} + 2\pi \notin I$; $\frac{5\pi}{12} + 2\pi \notin I$

$E_4)$ $S_f : x \in \{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\}$

Prof. Ovidiu Bădescu

Rezolvați în $[0;2\pi)$ ecuația

$$\operatorname{tg} x = -1$$

E_1) Folosim: $\operatorname{tg} x = a \Rightarrow$

$x \in \{\operatorname{arctg} a + k\pi \mid k \in \mathbb{Z}\}$

Dacă se cere soluție dintr-un interval, se dau lui k valori întregi astfel încât să avem doar soluții din interval, de obicei $k=-1, k=0, \dots$

Se mai folosește $\operatorname{arctg}(-a) = -\operatorname{arctg} a$

E_2) $\operatorname{tg} x = -1 \Rightarrow x \in \{\operatorname{arctg}(-1) + k\pi \mid k \in \mathbb{Z}\}$

$\Rightarrow x \in \{-\operatorname{arctg} 1 + k\pi \mid k \in \mathbb{Z}\} \Rightarrow x \in \{-\frac{\pi}{4} + k\pi \mid k \in \mathbb{Z}\}$

E_3) se cer soluții din $[0;2\pi)$,

$k=-1 \Rightarrow x \in \{-\frac{\pi}{4} - \pi\} \cap [0;2\pi) = \emptyset$

$k=0 \Rightarrow x \in \{-\frac{\pi}{4} + 0\} \cap [0;2\pi) = \emptyset$

$k=1 \rightarrow x \in \{-\frac{\pi}{4} + \frac{4}{4}\} \cap [0;2\pi) = \{\frac{3\pi}{4}\}$

$k=2 \Rightarrow x \in \{-\frac{\pi}{4} + 2\pi\} \cap [0;2\pi) = \{\frac{7\pi}{4}\}$

$k=3 \rightarrow x \in \{-\frac{\pi}{4} + 3\pi\} \cap [0;2\pi) = \emptyset$

E_4) S_f: $x \in \{\frac{3\pi}{4}, \frac{7\pi}{4}\}$

Prof. Ovidiu Dăescu

Rezolvați în $[0, 2\pi)$ ecuația:

$$\cos 2x = \frac{1}{2}$$

E_1) $2x \in \{\arccos\frac{1}{2} + 2k\pi \mid k \in \mathbb{Z}\} \cup$

$\{-\arccos\frac{1}{2} + 2k\pi \mid k \in \mathbb{Z}\}$

$2x \in \{\frac{\pi}{3} + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\frac{\pi}{3} + 2k\pi \mid k \in \mathbb{Z}\}$

E_2) $x \in \{\frac{\pi}{6} + k\pi \mid k \in \mathbb{Z}\} \cup \{-\frac{\pi}{6} + k\pi \mid k \in \mathbb{Z}\}$

E_3) Aceste soluții din $[0, 2\pi)$

$k = -1$

Prof. Ovidiu Băgosu

Rezolvați în $[-2\pi, 0]$ ecuația

$$\sin\left(3x+\frac{\pi}{3}\right)=-\frac{1}{2}$$

E1) $3x+\frac{\pi}{3}\in\{\arcsin\left(-\frac{1}{2}\right)+2k\pi \mid k\in\mathbb{Z}\}\cup\{\pi-\arcsin\left(-\frac{1}{2}\right)+2k\pi \mid k\in\mathbb{Z}\}$

$3x+\frac{\pi}{3}\in\{-\arcsin\frac{1}{2}+2k\pi \mid k\in\mathbb{Z}\}\cup\{\pi+\arcsin\frac{1}{2}+2k\pi \mid k\in\mathbb{Z}\}$

$3x+\frac{\pi}{3}\in\{-\frac{\pi}{6}+2k\pi \mid k\in\mathbb{Z}\}\cup\{\frac{7\pi}{6}+2k\pi \mid k\in\mathbb{Z}\}$

E2) $3x\in\{-\frac{\pi}{6}-\frac{\pi}{3}+2k\pi \mid k\in\mathbb{Z}\}\cup\{\frac{7\pi}{6}-\frac{\pi}{3}+2k\pi \mid k\in\mathbb{Z}\}$

$3x\in\{-\frac{\pi}{2}+2k\pi \mid k\in\mathbb{Z}\}\cup\{\frac{5\pi}{6}+2k\pi \mid k\in\mathbb{Z}\}$

E3) $x\in\{-\frac{\pi}{6}+\frac{2k\pi}{3} \mid k\in\mathbb{Z}\}\cup\{\frac{5\pi}{18}+\frac{2k\pi}{3} \mid k\in\mathbb{Z}\}$

E4) Dacă se cerea soluție din $\mathbb{R}$, pb era rezolvată.
Dacă se cer soluții din interval, dăm valori lui k

E5) $k=-1\Rightarrow x\in\{-\frac{\pi}{6}-\frac{2\pi}{3}\}\cup\{\frac{5\pi}{18}-\frac{2\pi}{3}\}=\{-\frac{5\pi}{6};\frac{\pi}{18}\}\cap[-2\pi,0]=\{-\frac{5\pi}{6}\}$

$k=-2\Rightarrow x\in\{-\frac{\pi}{6}-\frac{4\pi}{3};\frac{5\pi}{18}-\frac{4\pi}{3}\}=\{-\frac{9\pi}{6};-\frac{19\pi}{18}\}\cap[-2\pi,0]=\{-\frac{3\pi}{2};-\frac{19\pi}{18}\}$

$k=-3\Rightarrow x\in\{-\frac{\pi}{6}-\frac{6\pi}{3};\frac{5\pi}{18}-\frac{6\pi}{3}\}=\{-\frac{13\pi}{6};-\frac{31\pi}{18}\}\cap[-2\pi,0]=\{-\frac{31\pi}{18}\}$

$k=-4\Rightarrow x\in\{-\frac{\pi}{6}-\frac{8\pi}{3};\frac{5\pi}{18}-\frac{8\pi}{3}\}=\{-\frac{17\pi}{6};-\frac{43\pi}{18}\}\cap[-2\pi,0]=\emptyset$

$k=0\Rightarrow x\in\{\frac{\pi}{18};\frac{5\pi}{6}\}\cap[-2\pi,0]=\emptyset$

E6) $x\in\{-\frac{5\pi}{6};-\frac{3\pi}{2};-\frac{19\pi}{18};-\frac{31\pi}{18}\}$

Profmidii Bodescu