GRAFICELE FUNCȚIILOR TRIGONOMETRICE. PROPRIETĂȚI

Trasați graficul și studiați
a) periodicitatea b) mărginirea
c) monotonia d) convexitatea
e) bijectivitatea f) restricția
g) inversa
pentru

1) $f(x) = \cos x$
2) $f(x) = \sin x$
3) $f(x) = tg\, x$
4) $f(x) = ctg\, x$

Comparați:

11) cos 1 cu cos 2
12) sin 1 cu sin 2
13) cos 10 cu cos 11
14) sin 8 cu sin 9
15) tg 1 cu tg 2
16) tg 2 cu tg 3
17) ctg 11 cu ctg 12
18) tg 8 cu ctg 9
19) cos 1, cos 2, cos -
20) sin 1, sin 2, sin 3

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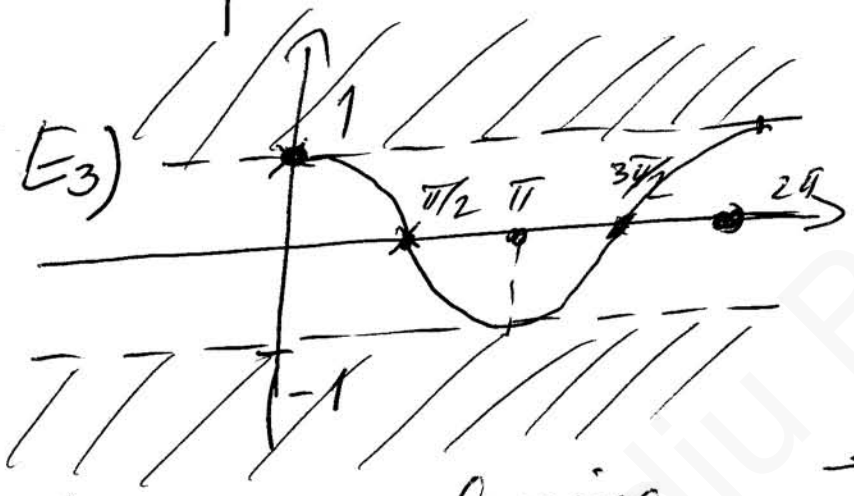
TRASAREA GRAFICULUI LUI cos

E1) $f(x) = \cos x \Rightarrow f: \mathbb{R} \to [-1,1]$, perioadă 2π

E2)

x	0	$\pi/2$	π	$3\pi/2$	2π
$\cos x$	1	0	-1	0	1

E3)



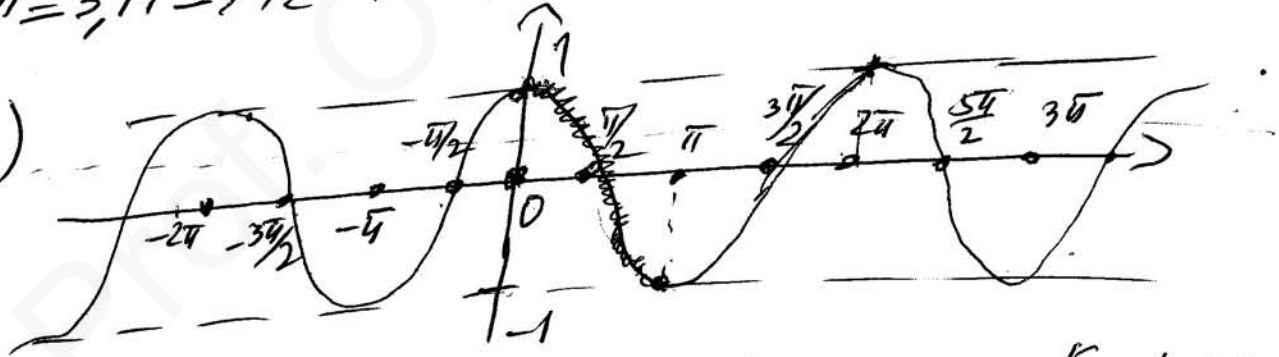
Deci, cos are perioadă 2π

$\Rightarrow$ Graficul se repetă pe porțiuni de lung 2π

Pe grafic, π ca lungime

$\Rightarrow \pi = 3{,}14 \Rightarrow \pi/2 = 1{,}57$

E4)



E5) $f \searrow$ pe $x \in [0+2k\pi; \pi+2k\pi]$ și $f \nearrow$ pe $x \in [\pi+2k\pi, 2\pi+2k\pi]$

f concavă pe $[-\frac{\pi}{2}+2k\pi, \frac{\pi}{2}+2k\pi]$ și f convexă $[\frac{\pi}{2}+2k\pi; \frac{3\pi}{2}+2k\pi]$

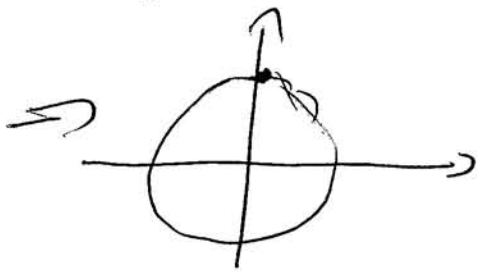
f nu e bijectă alegem restricția $\cos: [0;\pi] \to [-1,1]$

$\Rightarrow \cos$ inversabilă, $\cos^{-1}$ not arccos

$\exists \arccos: [-1,1] \to [0,\pi]$

TRASAREA GRAFICULUI LUI „tg"

$E_1)$ $f(x) = \operatorname{tg} x$, $\operatorname{tg} x = \frac{\sin x}{\cos x}$ deci cond.: $\cos x \neq 0$



din cerc, $\cos x = 0 \Leftrightarrow$

$x \in \{\frac{\pi}{2}; \frac{3\pi}{2}; \frac{5\pi}{2}; \dots\}$

$\Rightarrow f: \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\} \to \mathbb{R}$, periodă π

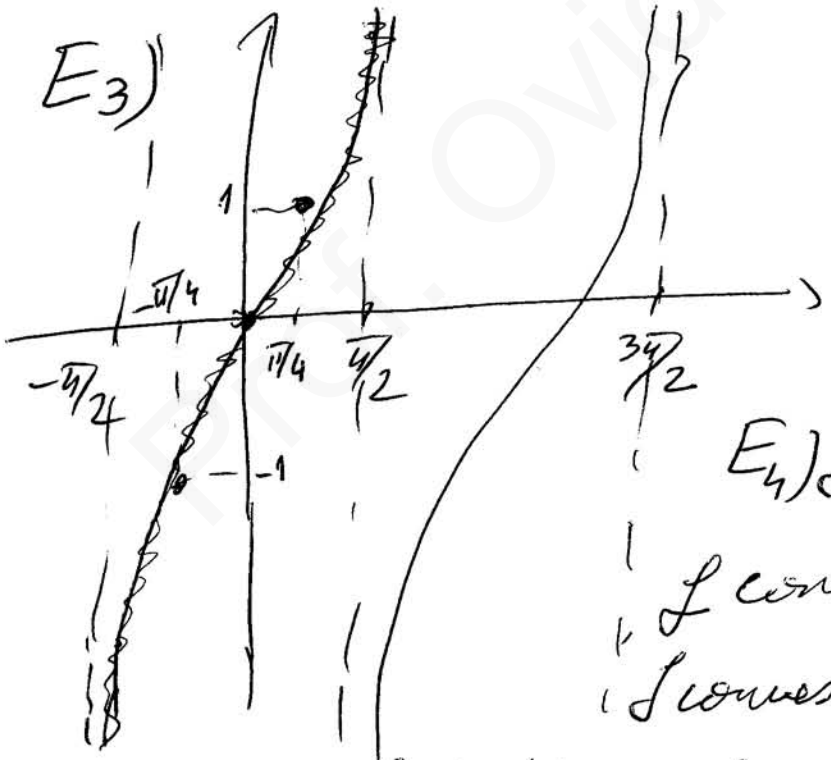
$E_2)$

x		$-\pi/2$		$-\pi/4$		0		$\pi/4$		$\pi/2$		$3\pi/2$
tg	$\nearrow$	$\mid -\infty$	$\nearrow$	-1	$\nearrow$	0	$\nearrow$	1	$\nearrow$	$\infty \mid$	$\nearrow$	

$\operatorname{tg}\frac{\pi}{4} = 1$; $\operatorname{tg}(-\frac{\pi}{4}) = -1$; $\operatorname{tg}\frac{\pi}{2} = \frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}} \to \frac{1}{0} \to \infty$

$\operatorname{tg}(-\pi/2) = -\operatorname{tg}\pi/2 \to -\infty$

$E_3)$



am trasat Gf pe $(-\pi/2; \pi/2)$ și am folosit periodicitatea

$E_4)$ $f_\Delta \nearrow$ pe $(-\pi/2 + k\pi; \frac{\pi}{2} + k\pi)$

f concavă $(-\pi/2 + k\pi; 0 + k\pi]$

f convexă pe $[0 + k\pi; \pi/2 + k\pi)$

f nu e bijectivă, alegem

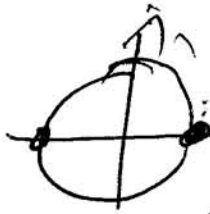
$\operatorname{tg}: (-\pi/2; \pi/2) \to \mathbb{R} \Rightarrow \exists \operatorname{tg}^{-1}: \mathbb{R} \to (-\pi/2; \pi/2)$

tg^{-1} not arctg $: \mathbb{R} \to (-\pi/2; \pi/2)$

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TRASAREA GRAFICULUI LUI „ctg"

E1) $f(x) = ctg\,x$, $ctg\,x = \frac{\cos x}{\sin x}$ deci cond.: $\sin x \neq 0$



$\Rightarrow$ din cerc $\Rightarrow x \neq 0; x \neq \pi; x \neq 2\pi; \ldots$

$f: \mathbb{R} \setminus \{0 + k\pi \mid k \in \mathbb{Z}\} \to \mathbb{R}$, perioadă π

E2)

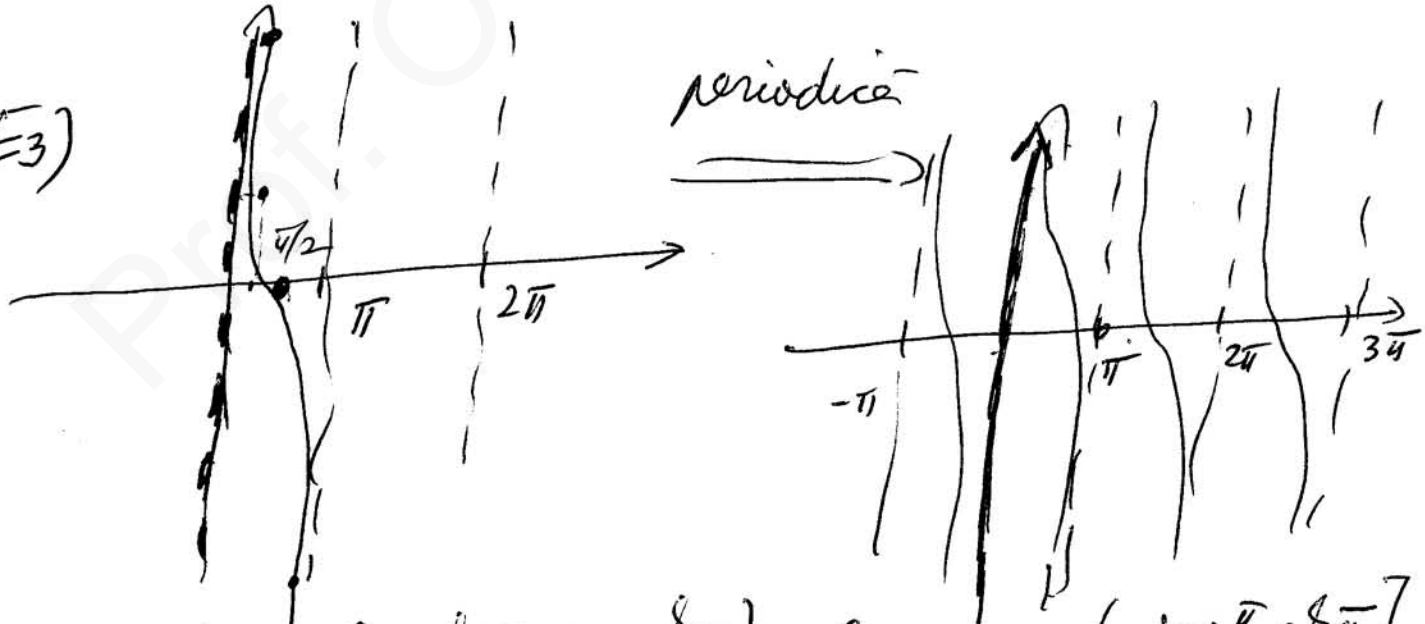
x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$ctg\,x$	∞	1	0	-1	$-\infty$

$ctg\frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0$; $ctg\frac{\pi}{4} = \frac{\cos\pi/4}{\sin\pi/4} = 1$

$x \to 0,\ x > 0 \Rightarrow \cos x \to \cos 0 = 1,\ \sin x \to \sin 0 = 0 \Rightarrow \frac{\cos x}{\sin x} \to \frac{1}{0} \to \infty$

$x \to \pi,\ x < \pi \Rightarrow \cos x \to \cos\pi = -1,\ \sin x \to \sin\pi = 0 \Rightarrow \frac{\cos x}{\sin x} \to \frac{-1}{0} \to -\infty$

E3)



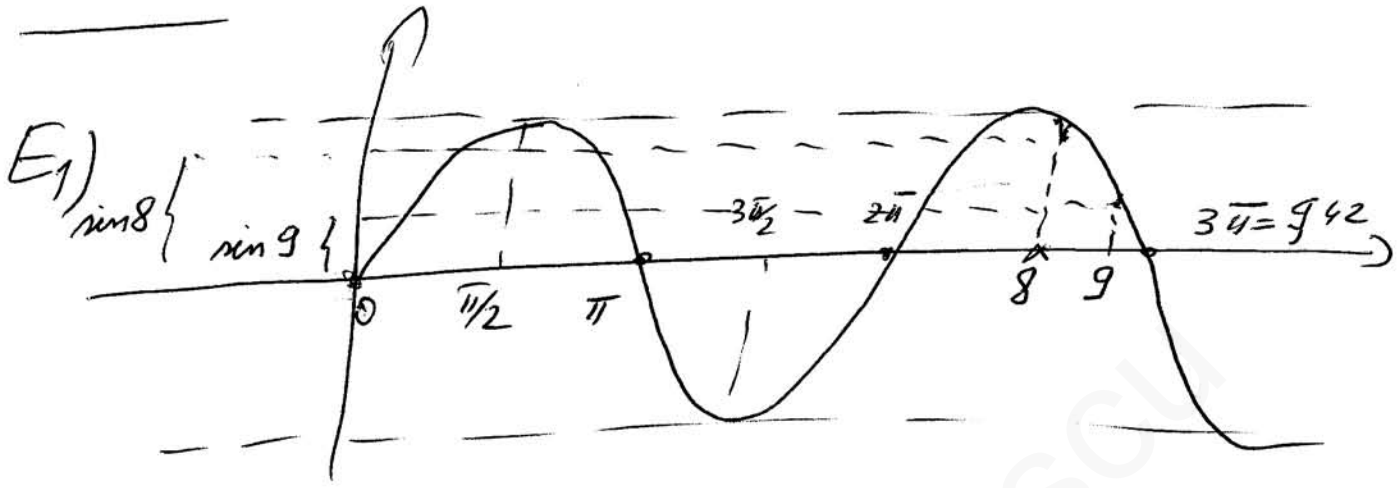
E4) f ↓ pe $(0 + k\pi; \pi + k\pi)$, f convexă $(0 + k\pi; \frac{\pi}{2} + k\pi]$

f concavă pe $[\frac{\pi}{2} + k\pi; \pi + k\pi)$, nu e bijectivă

$\exists\ f^{-1}: (0;\pi) \to D \Rightarrow arcctg = ctg^{-1}: \mathbb{R} \to (0;\pi)$

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Comparați: $\sin 8$ cu $\sin 9$



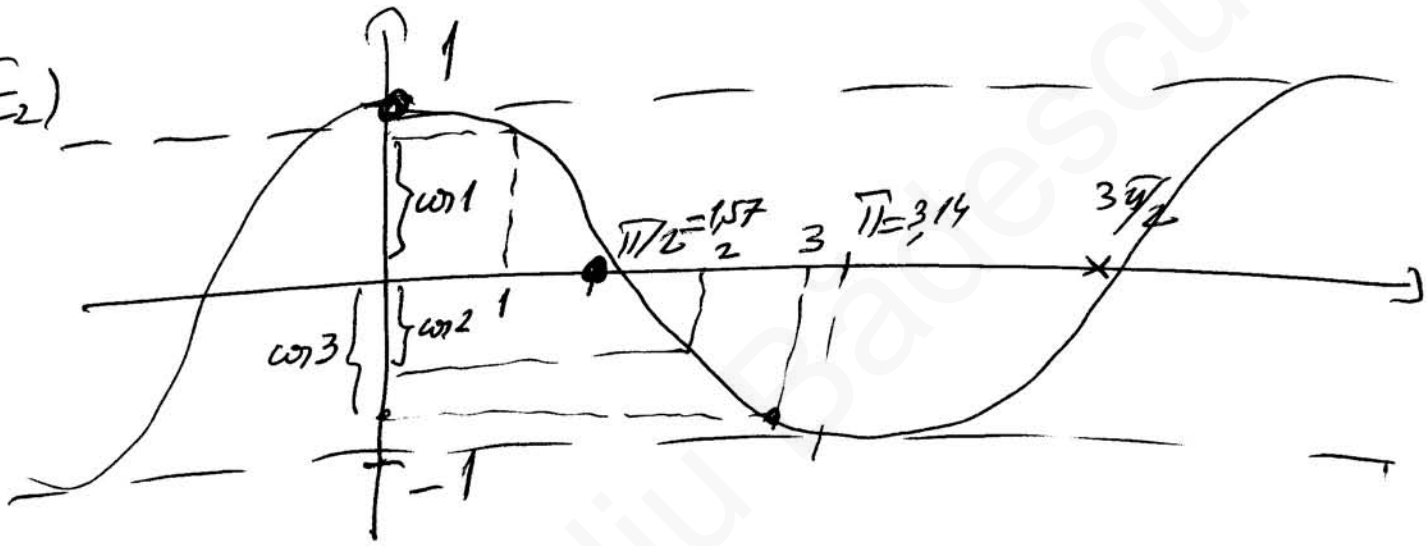
E_2) din grafic $\Rightarrow \sin 8 > \sin 9$

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Comparați: $\cos 1$, $\cos 2$, $\cos 3$

E_1) atunci când se cere compararea fct. trigonometrice se face graficul funcției însă π este 3,14, nu se consideră $\pi = 180°$

E_2)



E_3) Din grafic $\Rightarrow \cos 1 > \cos 2 > \cos 3$

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