INECUAȚII IRAȚIONALE

Rezolvați:

1) $\sqrt{5+x} \geq x-1$

2) $\sqrt{3-x} < x-1$

3) $\sqrt[3]{x-1} < 2x-3$

4) $\sqrt{x-1} < 3-x$

5) $\sqrt{x^2+3x-18} > 2x+3$

6) $\sqrt{x^2-3x+2} > 2-x$

7) $\sqrt[3]{x^3-3x^2+5x-6} > x-2$

8) $\sqrt{2x+1} \leq x-1$

Prof. Ovidiu Bădescu

Să se rezolve:

$$\sqrt{3-x} < x-1$$

E_1) Cond: $\begin{cases} 3-x \geq 0 \Rightarrow x \in (-\infty, 3] = I_1 \\ x-1 \geq 0 \Rightarrow x \in [1, \infty) = I_2 \end{cases}$

$$I_c = I_1 \cap I_2 = [1, 3]$$

E_2) $\sqrt{3-x} < x-1 \mid (\,)^2$

$$3-x < (x-1)^2 \Leftrightarrow 3-x < x^2-2x+1$$

$$x^2 - x - 2 > 0$$

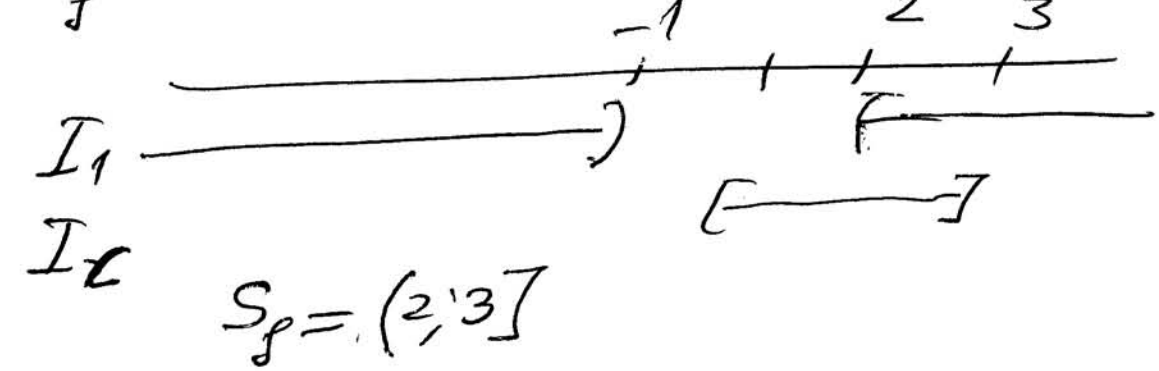
E_3) $x_{1,2} = \dfrac{1 \pm \sqrt{1^2-4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \dfrac{1 \pm \sqrt{9}}{2} < \begin{matrix} 2 \\ -1 \end{matrix}$

E_4)

x		-1		2	
x^2-x-2	$+ + +$	0	$- - -$	0	$+ + +$

$$x^2 - x - 2 > 0 \Rightarrow x \in (-\infty, -1) \cup (2, \infty) = I_3$$

E_5) $S_f = I_3 \cap I_c$



$$S_f = (2, 3]$$

Prof. Ovidiu Bădescu

Să se rezolve: $\sqrt{5+x} \geq x-1$

$E_1)$ Cond: $5+x \geq 0 \Rightarrow x \in [-5, \infty) = I_c$

$E_2)$ Caz I: $x-1 < 0 \Leftrightarrow x \in (-\infty, 1) \Rightarrow$ inec adevărată $\forall x \in I_c \cap (-\infty, 1)$

inec adev. $\forall x \in [-5, \infty) \cap (-\infty, 1) = [-5, 1)$

$\Rightarrow S_I = [-5, 1)$

$E_3)$ Caz II: $x-1 \geq 0 \Rightarrow x \in [1, \infty)$

Din cond $x \in [-5, \infty) \Rightarrow$

$x \in [-5, \infty) \cap [1, \infty) = [1, \infty)$

$\sqrt{5+x} \geq x-1 \mid ()^2 \Rightarrow 5+x \geq x^2 - 2x + 1$

$x^2 - 3x - 4 \leq 0$

$x_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \begin{matrix} 4 \\ -1 \end{matrix}$

x	-1		4	
x^2-3x-4	$+$	0	$-$	0 $+$

$x^2 - 3x - 4 \leq 0 \Rightarrow x \in [-1, 4]$

Deci, $x \in [1, \infty) \cap [-1, 4] = [1, 4] = S_{II}$

$E_4)$ $S_f = S_I \cup S_{II} = [-5, 1) \cup [1, 4] = [-5, 4]$

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Să se rezolve:

$$\sqrt[3]{x-1} < 2x-3$$

E_1) Cond.: $x \in \mathbb{R}$

E_2) $\sqrt[3]{x-1} < 2x-3 \mid (\,)^3$

$\Rightarrow x-1 < (2x-3)^3$

E_3) Folosim $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$\Rightarrow (x-1) < (2x)^3 - 3\cdot(2x)^2 3 + 3\cdot(2x)9 - 27$

$x-1 < 8x^3 - 36x^2 + 54x - 27$

$8x^3 - 36x^2 + 53x - 26 > 0$

E_4) D_{-26}

	x^3	x^2	x	x^0
	8	−36	53	−26
1	8	−28	25	$-1 \neq 0$
−1	8	−44	97	$-123 \neq 0$
2	8	−20	13	0

E_5) $(x-2)(8x^2 - 20x + 13) \geq 0$

$\Rightarrow$ $x-2=0 \Rightarrow x=2$ sau $8x^2 - 20x + 13 = 0$

x		2	
$x-2$	− − − −	0	+ + +
$8x^2-20x+13$	+ + + +	+	+ + +
$E(x)$	− − − −	0	+ + +

$x \in [2, \infty)$

Rezolvați: $\sqrt{x-1} < 3-x$

$E_1)$ cond: $\begin{cases} x-1 \geq 0 \Rightarrow x \in [1;\infty) = I_1 \\ 3-x \geq 0 \Rightarrow x \in (-\infty;3] = I_2 \end{cases}$

$I_C = I_1 \cap I_2 = [1;3]$

$E_2)$ $\sqrt{x-1} < 3-x \mid ()^2 \Leftrightarrow x-1 < (3-x)^2$

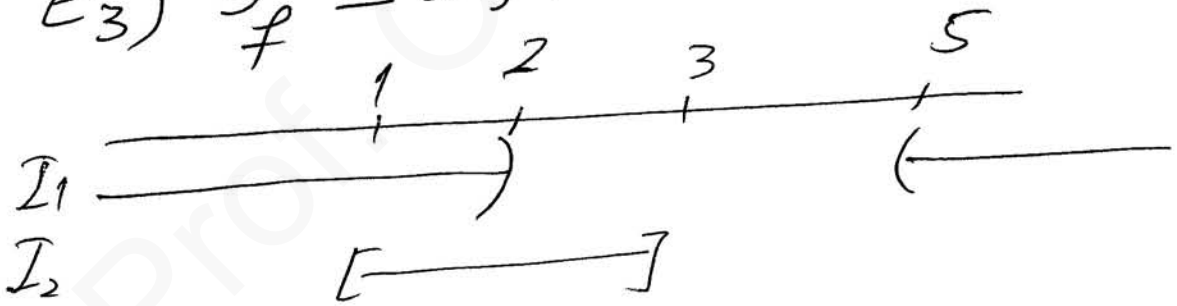
$\Leftrightarrow x-1 < 9-6x+x^2 \Rightarrow x^2-7x+10>0$

$x_{1,2} = \frac{7 \pm \sqrt{49-40}}{2 \cdot 1} = \frac{7 \pm 3}{2} \begin{matrix} 5 \\ < \\ 2 \end{matrix}$

x		2		5	
$x^2-7x+10$	$+$	0	$-$	0	$+$

$x^2-7x+10>0 \Leftrightarrow x \in (-\infty;2) \cup (5;\infty) = I_3$

$E_3)$ $S_f = I_3 \cap I_C$



$\Rightarrow S_f = [1;2)$

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Rezolvați: $\sqrt{x^2+3x-18} > 2x+3$

$E_1)$ $\sqrt{f(x)} > g(x) \Rightarrow$ 2 cazuri

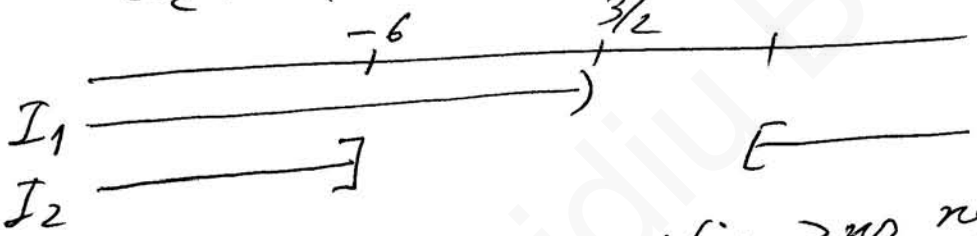
$E_2)$ Caz I: $\begin{cases} x^2+3x-18 \geq 0 \\ 2x-3 < 0 \Rightarrow x < \frac{3}{2} \Rightarrow x \in (-\infty; \frac{3}{2}) = I_1 \end{cases}$

$x^2+3x-18 \geq 0$, $x_{1,2} = \frac{-3 \pm \sqrt{9+72}}{2 \cdot 1} = \frac{-3 \pm 9}{2} \begin{matrix} 3 \\ -6 \end{matrix}$

x	-6		3	
$x^2+3x-18$	$+$	0 $-$	$-$	0 $+$

$\Rightarrow x^2+3x-18 \geq 0 \Leftrightarrow x \in (-\infty; -6] \cup [3; \infty) = I_2$

$I_C = I_1 \cap I_2 =$



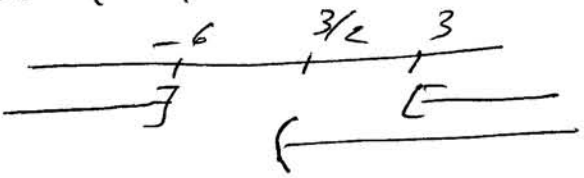
$\Rightarrow I_C = (-\infty; -6]$

$E_3)$ Deoarece un nr. pozitiv > nr negativ, $\forall x \in \mathbb{R}$

$S_I = \mathbb{R} \cap (-\infty; -6] = (-\infty; -6]$

$E_4)$ Caz II: $\begin{cases} x^2+3x-18 \geq 0 \Rightarrow x \in (-\infty; -6] \cup [3; \infty) = I_2 \\ 2x-3 > 0 \Rightarrow x \in (\frac{3}{2}; \infty) = I_3 \end{cases}$

$I_C = I_2 \cap I_3 = [3; \infty)$



$E_5)$ $\sqrt{x^2+3x-18} > 2x+3 \mid ()^2 \Rightarrow$

$x^2+3x-18 > 4x^2+12x+9 \Rightarrow 3x^2+9x+27 < 0 \mid :3$

$x^2+3x+9 < 0$, $x_{1,2} = \frac{-3 \pm \sqrt{9-36}}{2 \cdot 1}$, $\Delta < 0$

x	
x^2+3x+9	$+$ $+$ $+$ $+$ $+$

$\Rightarrow x^2+3x+9 < 0 \Rightarrow x \in \emptyset$

$\Rightarrow I_5 = \emptyset$

$S_{II} = \emptyset \cap [3; \infty) = \emptyset$

$E_6)$ $S_f = S_I \cup S_{II} = (-\infty; -6] \cup \emptyset = (-\infty; -6]$

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Rezolvați: $\sqrt{x^2-3x+2} > 2-x$

$E_1)\ \sqrt{f(x)} > g(x) \Rightarrow$ 2 cazuri

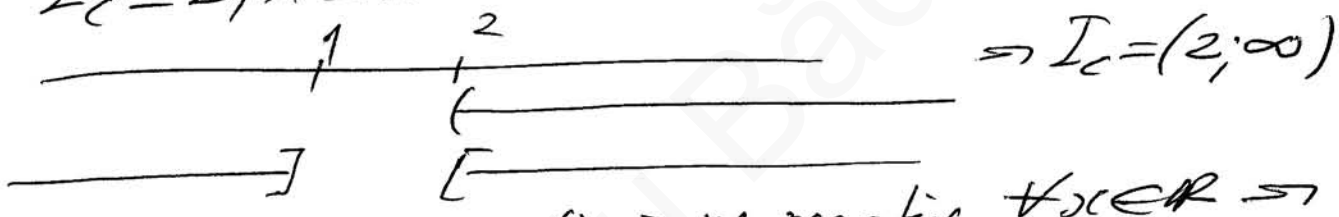
$E_2)$ caz I: $\begin{cases} x^2-3x+2 \geq 0 \\ 2-x < 0 \Rightarrow x > 2 \Rightarrow x \in (2;\infty) = I_1 \end{cases}$

$x^2-3x+2 \geq 0,\ x_{1,2} = \frac{3 \pm \sqrt{9-8}}{2 \cdot 1} = \frac{3 \pm 1}{2} \begin{matrix} \nearrow 2 \\ \searrow 1 \end{matrix}$

x		1		2	
x^2-3x+2	$+$	0	$-$	0	$+$

$\Rightarrow x \in (-\infty;1] \cup [2;\infty) = I_2$

$I_C = I_1 \cap I_2$



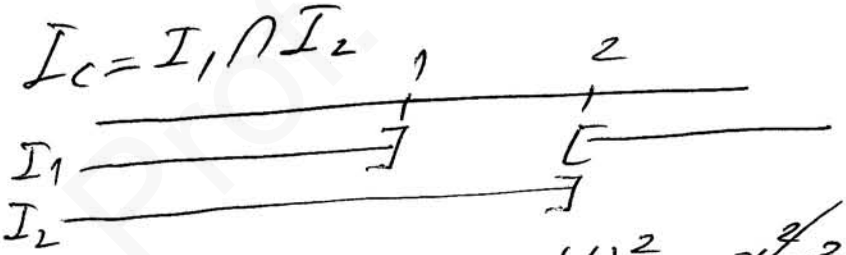
$\Rightarrow I_C = (2;\infty)$

$E_3)$ deoarece $\sqrt{\ }$ pozitiv > 0 negativ $\forall x \in \mathbb{R} \Rightarrow$

$\Rightarrow S_I = \mathbb{R} \cap (2;\infty) \Rightarrow S_I = (2;\infty)$

$E_4)$ caz II cond: $\begin{cases} x^2-3x+2 \geq 0 \Rightarrow x \in (-\infty;1] \cup [2;\infty) = I_1 \\ 2-x \geq 0 \Rightarrow x \in (-\infty;2] = I_2 \end{cases}$

$I_C = I_1 \cap I_2$

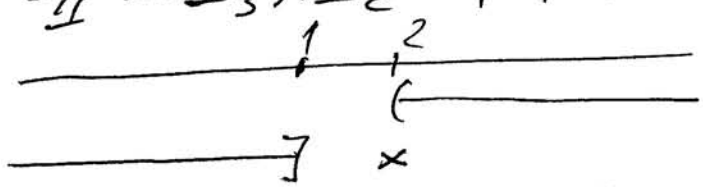


$\Rightarrow I_C = (-\infty;1] \cup \{2\}$

$E_5)\ \sqrt{x^2-3x+2} > 2-x\ /()^2 \Rightarrow x^2-3x+2 > 4-4x+x^2$

$x > 2 \Rightarrow x \in (2;\infty) = I_3$

$S_{II} = I_3 \cap I_C = (2;\infty) \cap ((-\infty;1] \cup \{2\})$



$\Rightarrow S_{II} = \emptyset$

$E_6)\ S_f = S_I \cup S_{II} = (2;\infty) \cup \emptyset = (2;\infty)$

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