ECUAȚII IRAȚIONALE DE TIPUL

$$\sqrt[3]{f^2(x)} \pm \sqrt[3]{f(x)g(x)} + \sqrt[3]{g^2(x)} = \alpha$$

Rezolvați:

1) $\sqrt[3]{(x-1)^2} - \sqrt[3]{(x-1)(3-x)} + \sqrt[3]{(3-x)^2} = 1$

2) $\sqrt[3]{(3-x)^2} - \sqrt[3]{(3-x)(6+x)} + \sqrt[3]{(6+x)^2} = 3$

3) $\sqrt[3]{(3-x)^2} + \sqrt[3]{(3-x)(x-1)} + \sqrt[3]{(x-1)^2} = 3$

4) $\sqrt[3]{(3-x)^2} + \sqrt[3]{(3-x)(6+x)} + \sqrt[3]{(6+x)^2} = 7$

5) $\sqrt[3]{(2x-1)^2} - \sqrt[3]{(2x-1)(3-2x)} + \sqrt[3]{(3-2x)^2} = 1$

6) $\sqrt[3]{(3-5x)^2} + \sqrt[3]{(3-5x)(5x-1)} + \sqrt[3]{(5x-1)^2} = 3$

7) $\sqrt[3]{(2x+1)^2} + \sqrt[3]{(2x+1)(x-1)} + \sqrt[3]{(x-1)^2} = 1$

8) $\sqrt[3]{(x-7)^2} + \sqrt[3]{(x-7)x} + \sqrt[3]{x^2} = 7$

9) $\sqrt[3]{(x-5)^2} + \sqrt[5]{(x-5)(x-24)} + \sqrt[3]{(x-24)^2} = 19$

10) $\sqrt[3]{(x-6)^2} - \sqrt[3]{(x-6)(x-25)} + \sqrt[3]{(x-25)^2} = 7$

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Rezolvați:

$$\sqrt[3]{(x-1)^2} - \sqrt[3]{(x-1)(3-x)} + \sqrt[3]{(3-x)^2} = 1$$

E_1) obs. că avem $a^2 - ab + b^2$ și folosim $(a+b)(a^2-ab+b^2) = a^3 + b^3$

E_2) $\sqrt[3]{(x-1)^2} - \sqrt[3]{(x-1)(3-x)} + \sqrt[3]{(3-x)^2} = 1 \mid \cdot (\sqrt[3]{x-1} + \sqrt[3]{3-x})$

E_3) $\left(\sqrt[3]{x-1}\right)^3 + \left(\sqrt[3]{3-x}\right)^3 = \sqrt[3]{x-1} + \sqrt[3]{3-x}$

$\Rightarrow x - 1 + 3 - x = \sqrt[3]{x-1} + \sqrt[3]{3-x}$

$\sqrt[3]{x-1} + \sqrt[3]{3-x} = 2 \mid ()^3$

E_4) $x - 1 + 3 - x + 3\sqrt[3]{(x-1)(3-x)} \cdot (\sqrt[3]{x-1} + \sqrt[3]{3-x}) = 8$

$6\sqrt[3]{(x-1)(3-x)} = 6 \mid : 6 \Rightarrow \sqrt[3]{(x-1)(3-x)} = 1 \mid ()^3$

E_5) $(x-1)(3-x) = 1 \Rightarrow 3x - x^2 - 3 + x = 1$

$-x^2 + 4x - 4 = 0 \mid \cdot (-1)$

$x^2 - 4x + 4 = 0 \Rightarrow x_{1,2} = \frac{4 \pm \sqrt{16-16}}{2 \cdot 1}$

$x_{1,2} = \frac{4 \pm 0}{2} = 2$ sol posibilă

E_6) o verifică în ec.: $\sqrt[3]{1^2} - \sqrt[3]{1 \cdot 1} + \sqrt[3]{1^2} = 1$ adev.

$\Rightarrow S_g : \boxed{x = 2}$

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