ECUAȚII IRAȚIONALE DE TIPUL $\sqrt[3]{f(x)} \pm \sqrt[3]{g(x)} = \alpha$

Rezolvați:

1) $\sqrt[3]{8-x} + \sqrt[3]{1+x} = 3$

2) $\sqrt[3]{5x+7} - \sqrt[3]{5x-7} = 1$

3) $\sqrt[3]{3x-1} + \sqrt[3]{9+x} = 4$

4) $\sqrt[3]{4x+3} - \sqrt[3]{x-5} = 2$

5) $\sqrt[3]{2x+1} + \sqrt[3]{x+9} = 1$

6) $\sqrt[3]{2-x} + \sqrt[3]{3x-2} = 2$

7) $\sqrt[3]{2x-6} + \sqrt[3]{5x-8} = 5$

Prof. Ovidiu Bădescu

Rezolvați:

$$\sqrt[3]{8-x}+\sqrt[3]{1+x}=3$$

not I:

E_1) folosim $(a+b)^3=a^3+b^3+3ab(a+b)$ și înlocuim $a+b$

E_2) $\sqrt[3]{8-x}+\sqrt[3]{1+x}=3\ |(\ )^3$

$\Rightarrow \left(\sqrt[3]{8-x}+\sqrt[3]{1+x}\right)^3=27 \Rightarrow$ (3)

$\Rightarrow (8-x)+(1+x)+3\sqrt[3]{(8-x)(1+x)}\cdot\left(\sqrt[3]{8-x}+\sqrt[3]{1+x}\right)=27$

$9\sqrt[3]{(8-x)(1+x)}=18\ |:9 \Rightarrow \sqrt[3]{(8-x)(1+x)}=2\ |(\ )^3$

$\Rightarrow (8-x)(1+x)=8 \Rightarrow 8+8x-x-x^2=8$

$-x^2+7x=0 \Rightarrow x(-x+7)=0$

$\Rightarrow \begin{cases} x_1=0 \\ \text{sau} \\ x_2=7 \end{cases} \Rightarrow S_p=\{0;7\}$ sol posibile

E_3) o verificăm: $x_1=0 \Rightarrow \sqrt[3]{8}+\sqrt[3]{1}=3$ ok.

$x_2=7 \Rightarrow \sqrt[3]{1}+\sqrt[3]{8}=3$ ok.

$S_p=\{0;7\}$

Prof. Ovidiu Bădescu

Rezolvați:

$$\sqrt[3]{2-x}+\sqrt[3]{3x-2}=2$$

$E_1)$ $\sqrt[3]{2-x}+\sqrt[3]{3x-2}=2 \mid ()^3$

$\Leftrightarrow 2-x+3x-2+3\sqrt[3]{(2-x)(3x-2)}\underbrace{\left(\sqrt[3]{2-x}+\sqrt[3]{3x-2}\right)}_{2}=8$

$$2x+6\sqrt[3]{(2-x)(3x-2)}=8$$

$$6\sqrt[3]{(2-x)(3x-2)}=8-2x \mid :2$$

$E_2)$ $3\sqrt[3]{(2-x)(3x-2)}=4-x \mid ()^3$

$$27(2-x)(3x-2)=64-3\cdot 16x+3\cdot 4\cdot x^2-x^3$$

$$27(6x-4-3x^2+2x)=-x^3+12x^2-48x+64$$

$$-81x^2+216x-108=-x^3+12x^2-48x+64$$

$$x^3-93x^2+264x-172=0$$

$E_3)$

	x^3	x^2	x	x^0
	1	-93	264	-172
1	1	-92	172	0

$\Rightarrow (x-1)(1\cdot x^2-92x+172)=0$

$\Rightarrow$ $\begin{cases} x=1 \\ \text{sau} \\ x^2-92x+172=0 \Rightarrow x_{1,2}=\dfrac{92\pm\sqrt{8464-688}}{2\cdot 1} \end{cases}$

$$=\frac{92\pm\sqrt{7776}}{2}=\frac{92\pm 8\sqrt{106}}{2}=46\pm 4\sqrt{106}$$

Prof. Ovidiu Bădescu