BINOMUL LUI NEWTON. RANGUL CELUI MAI MARE TERMEN AL DEZVOLTĂRII

Găsiți rangul celui mai mare termen al dezvoltării:

1) $(0,1+1)^{500}$

~~2)~~ $(1,3+2,1)^{500}$

~~3)~~ $(5,2+0,3)^{240}$

4) $(4,3+2)^{320}$

5) $(2+0,1)^{300}$

6) $(0,2+3)^{400}$

7) $(0,1+0,2)^{300}$

8) $(5+20)^{400}$

9) $(2+0,5)^{309}$

10) $(3+0,8)^{400}$

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Găsiți rangul celui mai mare termen din $(1,3+2,1)^{500}$

E_1) $\forall$ binom cu termeni pozitivi $\Rightarrow$

$$T_1 \le T_2 \le \dots \le T_k \le T_{k+1} \ge T_{k+2} \ge \dots \ge T_{n+1}$$

$$\Rightarrow \begin{cases} \frac{T_{k+1}}{T_k} \ge 1 \\ \frac{T_{k+2}}{T_{k+1}} \le 1 \end{cases} \Rightarrow \begin{cases} \frac{b}{a} \cdot \frac{n-k+1}{k} \ge 1 \\ \frac{b}{a} \cdot \frac{n-k}{k+1} \le 1 \end{cases}$$

E_2) $\begin{cases} \frac{2,1}{1,3} \cdot \frac{500-k+1}{k} \ge 1 \\ \frac{2,1}{1,3} \cdot \frac{500-k}{k+1} \le 1 \end{cases} \hookrightarrow \begin{cases} \frac{21}{13} \cdot \frac{501-k}{k} \ge 1 \,/\cdot 13k>0 \\ \frac{21}{13} \cdot \frac{500-k}{k+1} \le 1 \,/\cdot 13(k+1)>0 \end{cases}$

E_3) $\begin{cases} 21(501-k) \ge 13k \\ 21(500-k) \le 13(k+1) \end{cases} \hookrightarrow \begin{cases} 10521-21k \ge 13k \\ 10500-21k \le 13k+13 \end{cases}$

$\Rightarrow \begin{cases} 10521 \ge 34k \\ 10500 \le 34k \end{cases} \hookrightarrow \begin{cases} k \le \frac{10521}{34} = 309,\dots \\ k \ge \frac{10500}{34} = 308,\dots \end{cases}$

$10521 \,|\, 34$ (102, 309, 32, 0, 321, 306, 15)

$10500 \,|\, 34$ (102, 308, ==30, 0, 300, 272, 28)

$\Rightarrow k \in [308,\dots ; 309,\dots] \cap \mathbb{N}$

$k = 309$

E_4) $\Rightarrow$ termenul este $T_{k+1} = T_{310}$

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Găsiți rangul celui mai mare termen al dezvoltării $(5,2+0,3)^{240}$

E_1) în orice binom cu termeni pozitivi $\Rightarrow$

$$T_1 \le T_2 \le \dots \le T_k \le T_{k+1} \ge T_{k+2} \ge \dots \ge T_{n+1}$$

$$\Rightarrow \begin{cases} \frac{T_{k+1}}{T_k} \ge 1 \\ \frac{T_{k+2}}{T_{k+1}} \le 1 \end{cases} \Leftrightarrow \begin{cases} \frac{b}{a} \cdot \frac{n-k+1}{k} \ge 1 \\ \frac{b}{a} \cdot \frac{n-k}{k+1} \le 1 \end{cases}$$

$$E_2) \begin{cases} \frac{0,3}{5,2} \cdot \frac{240-k+1}{k} \ge 1 \\ \frac{0,3}{5,2} \cdot \frac{240-k}{k+1} \le 1 \end{cases} \Leftrightarrow \begin{cases} \frac{3}{52} \cdot \frac{241-k}{k} \ge 1 \,/\cdot 52k>0 \\ \frac{3}{52} \cdot \frac{240-k}{k+1} \le 1 \,/\cdot 52(k+1)>0 \end{cases}$$

$$E_3) \begin{cases} 3(241-k) \ge 52k \\ 3(240-k) \le 52(k+1) \end{cases} \Leftrightarrow \begin{cases} 723-3k \ge 52k \\ 720-3k \le 52k+52 \end{cases}$$

$$\Leftrightarrow \begin{cases} 55k \le 723 \\ 55k \ge 668 \end{cases} \Leftrightarrow \begin{cases} k \le \frac{723}{55} \Rightarrow k \le 13,14\dots \\ k \ge \frac{668}{55} \Rightarrow k \ge 12,14\dots \end{cases}$$

$$\Rightarrow k \in [12,14\dots ; 13,14\dots] \cap \mathbb{N} \Rightarrow k = 13$$

E_4) Term cerut este $T_{k+1} = T_{14}$

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Găsiți rangul celui mai mare termen din $\left(\frac{1}{2}+\frac{2}{3}\right)^{100}$

E_1) Există: $T_1 \le T_2 \le \dots \le T_k \le \underbrace{T_{k+1}}_{\text{cel mai mare}} \ge T_{k+2} \ge \dots \ge T_{n+1}$

E_2) $\begin{cases} \frac{T_{k+1}}{T_k} \ge 1 \\ \frac{T_{k+1}}{T_{k+2}} \ge 1 \end{cases} \Leftrightarrow \begin{cases} \frac{T_{k+1}}{T_k} \ge 1 \\ \frac{T_{k+2}}{T_{k+1}} \le 1 \end{cases}$

E_3) Folosim formula $\boxed{\frac{T_{k+1}}{T_k} = \frac{b}{a} \cdot \frac{n-k+1}{k}}$

$\Rightarrow \begin{cases} \frac{b}{a} \cdot \frac{n-k+1}{k} \ge 1 \\ \frac{b}{a} \cdot \frac{n-(k+1)+1}{k+1} \le 1 \end{cases} \Leftrightarrow \begin{cases} \frac{\frac{2}{3}}{\frac{1}{2}} \cdot \frac{100-k+1}{k} \ge 1 \\ \frac{\frac{2}{3}}{\frac{1}{2}} \cdot \frac{100-k}{k+1} \le 1 \end{cases}$

$\Leftrightarrow \begin{cases} 4(100-k+1) \ge 3k \\ 4(100-k) \le 3(k+1) \end{cases} \Leftrightarrow \begin{cases} 404-4k \ge 3k \\ 400-4k \le 3k+3 \end{cases}$

$\Leftrightarrow \begin{cases} 7k \le 404 \\ 7k \ge 397 \end{cases} \Leftrightarrow \left\{ \begin{array}{l} k \le \frac{404}{7} = 57,7\dots \\ k \ge \frac{397}{7} = 56,8\dots \\ \text{Dar, } k \in \mathbb{Z} \end{array} \right| \Rightarrow$

$\Rightarrow k = 57 \Rightarrow$ rangul este $k+1 = 58$

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Găsiți o relație de recurență de forma

$$\frac{T_{k+1}}{T_k}$$ pentru $(a+b)^n$

E1) $T_{k+1} = C_n^k a^{n-k} b^k \Rightarrow$

$\Rightarrow T_k = C_n^{k-1} a^{n-(k-1)} b^{k-1} \Rightarrow$

$\Rightarrow T_k = C_n^{k-1} a^{n-k+1} b^{k-1}$

E2) $\frac{T_{k+1}}{T_k} = \frac{C_n^k a^{n-k} b^k}{C_n^{k-1} a^{n-k+1} b^{k-1}} = \frac{b}{a} \cdot \frac{\frac{n!}{k!(n-k)!}}{\frac{n!}{(k-1)!(n-k+1)!}}$

$= \frac{b}{a} \cdot \frac{n!}{k!(n-k)!} \cdot \frac{(k-1)!(n-k+1)!}{n!} =$

$= \frac{b}{a} \cdot \frac{(k-1)!}{(k-1)! \cdot k} \cdot \frac{(n-k)!(n-k+1)}{(n-k)!}$

E3) $\Rightarrow \boxed{\frac{T_{k+1}}{T_k} = \frac{b}{a} \cdot \frac{n-k+1}{k}}$ se memorează (sau se deduce)

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