# PROBLEME NONSTANDARD ÎN COMBINATORICĂ

1) Rezolvați în $\mathbb{N}$:

$$C_{3n+2}^{n^2} \geq 8$$

2) Găsiți $n \in \mathbb{N}$, $n \geq 3$ dacă

$C_n^3$ divide $C_{n+1}^3$

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Se care $n \in \mathbb{N}$, $n \geq 3$ pt. care

$C_n^3$ divide pe $C_{n+1}^3$

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$E_1)$ $C_n^3 = \frac{n!}{3!(n-3)!}$

$C_{n+1}^3 = \frac{(n+1)!}{3!(n-2)!}$

$E_2)$ $a \mid b \Leftrightarrow \frac{b}{a} \in \mathbb{Z}$

$E_3)$ $\frac{C_{n+1}^3}{C_n^3} \in \mathbb{Z} \Rightarrow$

$\Rightarrow \frac{(n+1)!}{3!(n-2)!} \cdot \frac{3!(n-3)!}{n!} \in \mathbb{Z}$

$\Rightarrow \frac{n+1}{n-2} \in \mathbb{Z}$

$E_4)$ $n-2 \mid n+1$ $\Big\}$ $\Rightarrow n-2 \mid (n+1)-(n-2)$

dar, $n-2 \mid n-2$ $\Big\}$ $\Rightarrow n-2 \mid 3$

$\Rightarrow n-2 \in D_3 \Rightarrow n-2 \in \{\pm 1; \pm 3\} \Rightarrow$

$\Rightarrow -1; 1; 3; 5$

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Rezolvați în $\mathbb{N}$:

$$C_{3n+2}^{n^2} \geq 8$$

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$E_1$) Cond.: $\begin{cases} n^2 \in \mathbb{N} \text{ adev} \\ 3n+2 \in \mathbb{N} \text{ adev.} \\ 3n+2 \geq n^2 \end{cases}$

$E_2$) $3n+2 \geq n^2 \Rightarrow n^2 - 3n - 2 \leq 0$

$$n_{1,2} = \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

| $n$ | | $\frac{3-\sqrt{17}}{2}$ | | $\frac{3+\sqrt{17}}{2}$ | |
|---|---|---|---|---|---|
| $n^2-3n-2$ | $+ +$ | $0$ | $- - -$ | $0$ | $+ + +$ |

$$n \in \left[\frac{3-\sqrt{17}}{2}, \frac{3+\sqrt{17}}{2}\right] \cap \mathbb{N} = \{0, 1, 2, 3\}$$

$E_3$) valorile posibile sunt $n \in \{0, 1, 2, 3\}$

le verificăm:

$n = 0 \Rightarrow C_2^0 \geq 8 \Leftrightarrow \frac{2!}{0! \cdot 2!} \geq 8 \Leftrightarrow 1 \geq 8$ fals

$n = 1$

$n = 2$

$n = 3 \Rightarrow C_{11}^9 \geq 8 \Leftrightarrow \frac{11!}{9! \cdot 2!} \geq 8 \Leftrightarrow \frac{10 \cdot 11}{2} \geq 8$ adev.

$E_4$) $S_f$: $n \in \{2, 3\}$

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