CALCULAREA UNOR SUME ÎN COMBINATORICĂ

Calculați (respectiv arătați că):

1) $S=\sum_{k=1}^{n} k!(k^2+k+1)$

2) $S=\sum_{k=1}^{n} \frac{k+2}{k!+(k+1)!+(k+2)!}$

3) $S=\sum_{k=1}^{n} \frac{(k-1)!}{(k+1)!}$

4) $\sum_{k=1}^{n} A_{n+k}^{n}=\frac{1}{n+1}\left(A_{n+p+1}^{n+1}-P_{n+1}\right)$

5) $S=\sum_{k=0}^{n-1} \frac{A_n^{k+1}}{A_n^k}$

6) $S=\sum_{k=1}^{n} \frac{A_n^k}{A_{n-1}^{k-1}}$

7) $S=\sum_{k=1}^{n} (n-k)A_n^{k-1}=A_n^p-1$

8) $S=\sum_{k=0}^{n} (2k+1)C_n^k$

9) $S=\sum_{k=1}^{n} k C_n^k$

10) $S=\sum_{k=0}^{n} \frac{1}{k+1}\cdot C_n^k$

11) $S=\sum_{k=1}^{n} \frac{3k+4}{k+2} C_{2k+1}^{k+1}$

12) $\sum_{k=1}^{n} \frac{3k+1}{k} C_{2k}^{k+1}=C_{2n+2}^{n+1}-2$

13) $\sum_{k=1}^{n} \frac{k C_n^k}{C_n^{k-1}}=\frac{2n+1}{n+1}$

14) $\sum_{k=0}^{n} C_{n+k}^{n}=C_{n+p+1}^{n+1}$

15) $\sum_{k=0}^{n} \frac{(n+k-1)!\,n}{k!}=\frac{(2n)!}{n!}$

16) $\sum_{p=0}^{n} \frac{(k+n)!}{n!}=\frac{(k+n+1)!}{n!(k+1)}$

17) $\sum_{k=0}^{n} \frac{1}{(k+1)(k+2)} C_n^k=\frac{2^{n+2}-(n+3)}{(n+1)(n+2)}$

18) $\sum_{k=1}^{n}\left(k\cdot\sum_{p=1}^{k} C_k^p\right)=$
$=(n-1)2^{n+1}+2-\frac{n(n+1)}{2}$

Prof. Ovidiu Bădescu

Calculați $S = C_n^0 + 3C_n^1 + 5C_n^2 + \dots + (2n+1)C_n^n$

E1) $S = \sum_{k=0}^{n} (2k+1) C_n^k = \sum_{k=0}^{n} 2k C_n^k + \sum_{k=0}^{n} C_n^k$

$= 2 \sum_{k=0}^{n} k \cdot C_n^k + (C_n^0 + C_n^1 + C_n^2 + \dots + C_n^n)$

$\Rightarrow \boxed{S = 2 \sum_{k=0}^{n} k \cdot C_n^k + 2^n}$

E2) $S_1 = \sum_{k=0}^{n} k \cdot C_n^k$

$\exists$ formula $\boxed{k \cdot C_n^k = n \cdot C_{n-1}^{k-1}}$ (*)

E3) $S_1 = \sum_{k=0}^{n} k C_n^k = \underbrace{0 \cdot C_n^0}_{0} + 1 C_n^1 + 2 C_n^2 + \dots + n C_n^n \Rightarrow$

$\Rightarrow S_1 = \sum_{k=1}^{n} k C_n^k \overset{(*)}{=\!=} \sum_{k=1}^{n} n \cdot C_{n-1}^{k-1} =$

$= n \left(C_{n-1}^0 + C_{n-1}^1 + \dots + C_{n-1}^{n-1} \right) = n \cdot 2^{n-1}$

E4) $S = 2 \cdot n \cdot 2^{n-1} + 2^n = 2^n \cdot n + 2^n =$

$= 2^n (n+1)$

Prof. Ovidiu Badescu

Arătați că $k C_n^k = n C_{n-1}^{k-1}$ și calculați

$$C_n^0 + \frac{1}{2} C_n^1 + \frac{1}{3} C_n^2 + \dots + \frac{1}{n} C_n^{n-1}$$

$E_1)$ $k \cdot C_n^k = k \cdot \frac{n!}{k!(n-k)!} = \frac{n!}{(k-1)!(n-k)!}$ (1)

$n C_{n-1}^{k-1} = n \cdot \frac{(n-1)!}{(k-1)!(n-1-k+1)!} = \frac{n!}{(k-1)!(n-k)!}$ (2)

Din (1) + (2) $\Rightarrow$ egalitate

$E_2)$ $k C_n^k = n C_{n-1}^{k-1} \Rightarrow \frac{1}{n} C_n^k = \frac{1}{k} C_{n-1}^{k-1}$

$E_3)$ în $\boxed{\frac{1}{k} C_{n-1}^{k-1} = \frac{1}{n} C_n^k}$

$k=1 \Rightarrow 1 \cdot C_{n-1}^0 = \frac{1}{n} C_n^1$

$k=2 \Rightarrow \frac{1}{2} C_{n-1}^1 = \frac{1}{n} C_n^2$

$k=3 \Rightarrow \frac{1}{3} C_{n-1}^2 = \frac{1}{n} C_n^3$

$\dots\dots\dots$

$k=n \Rightarrow \frac{1}{n} C_{n-1}^{n-1} = \frac{1}{n} C_n^n$

„+" $S = \frac{1}{n}\left(C_n^1 + C_n^2 + \dots + C_n^n\right) = \frac{2^n - C_n^0}{n}$

$\Rightarrow S = \frac{2^n - 1}{n}$

Prof. Ovidiu Radescu

Calculați:

$$\frac{C_n^0}{1} + \frac{C_n^1}{2} + \frac{C_n^2}{3} + \dots + \frac{C_n^n}{n+1} = \frac{2^{n+1}-1}{n+1}$$

$$E_1)\ M_S = \sum_{k=0}^{n} \frac{C_n^k}{k+1}$$

$$E_2)\ \frac{C_n^k}{k+1} = \frac{1}{k+1} \cdot C_n^k = \frac{1}{k+1} \cdot \frac{n!}{k!(n-k)!} =$$

$$= \frac{(n+1)!}{(k+1)!\,(n+1-(k+1))} \cdot \frac{1}{n+1} =$$

$$= C_{n+1}^{k+1} \cdot \frac{1}{n+1}$$

$$E_3)\ M_S = \sum_{k=0}^{n} C_{n+1}^{k+1} \cdot \frac{1}{n+1} = \frac{1}{n+1} \sum_{k=0}^{n} C_{n+1}^{k+1} =$$

$$= \frac{1}{n+1}\left(C_{n+1}^1 + C_{n+1}^2 + \dots + C_{n+1}^{n+1}\right) =$$

$$= \frac{1}{n+1}\left(\underbrace{C_{n+1}^0 + C_{n+1}^1 + C_{n+1}^2 + \dots + C_{n+1}^{n+1}}_{2^{n+1}} - C_{n+1}^0\right)$$

$$= \frac{1}{n+1}\left(2^{n+1} - 1\right) = \frac{2^{n+1}-1}{n+1}$$

Obs.: S-a folosit: $C_n^0 + C_n^1 + \dots + C_n^n = 2^n$

Prof. Ovidiu Bădescu

Arătați că: a) $k \cdot C_n^k = n C_{n-1}^{k-1}$

b) Calculați: $S = C_n^0 + 2C_n^1 + 3C_n^2 + \dots + (n+1)C_n^n$

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a) $E_1)$ $k \cdot C_n^k = n C_{n-1}^{k-1} \Leftrightarrow$

$\Leftrightarrow k \cdot \frac{n!}{k!(n-k)!} = n \cdot \frac{(n-1)!}{(k-1)!(n-1-k+1)!}$

$\Leftrightarrow \cancel{k} \cdot \frac{n!}{\cancel{(k-1)!} \cdot \cancel{k}(n-k)!} = \frac{(n-1)! \cdot n}{\cancel{(k-1)!}(n-k)!}$

$\Leftrightarrow n! = (n-1)! \cdot n$ adevărată

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b) $S = C_n^0 + 2C_n^1 + 3C_n^2 + \dots + (n+1)C_n^n$

$E_1) S = (C_n^0 + C_n^1 + \dots + C_n^n) + (C_n^1 + 2C_n^2 + \dots + nC_n^n)$

$\Rightarrow S = 2^n + \underbrace{(C_n^1 + 2C_n^2 + \dots + nC_n^n)}_{S_1}$

$E_2)$ Pt S_1, $1 \cdot C_n^1 = n C_{n-1}^{1-1} = n C_{n-1}^0$

$2 \cdot C_n^2 = n C_{n-1}^{2-1} = n C_{n-1}^1$

$3 C_n^3 = n C_{n-1}^{3-1} = n C_{n-1}^2$

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$n C_n^n = n C_{n-1}^{n-1} = n C_{n-1}^{n-1}$

"+" $\Rightarrow S_1 = n(C_{n-1}^0 + C_{n-1}^1 + \dots + C_{n-1}^{n-1}) = n \cdot 2^{n-1}$

$E_3)$ $\boxed{S = 2^n + n \cdot 2^{n-1} = 2^{n-1}(2+n)}$

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