ECUAȚII ȘI INECUAȚII

NONSTANDARD

Rezolvați:

1) $3^{\log_3^2 x} + x^{\log_3 x} = 162$

2) $\log_2(2^{x+3}+16) = 2x + \log_2 3$

3) $x \cdot 2^{\log_x 5} = 10$

4) $9^{\log_5(x-1)} = 4 + 5^{\log_9(x+3)}$

5) $10^{\lg^2 x} + x^{\lg x} = 20$

6) $\frac{\sqrt[3]{x+7}-1}{\sqrt[3]{x^2}-1} - \frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x}+1} = 4$

7) $x^3 + x + \sqrt[3]{x^3+x-2} = 12$

8) $x^3 + 3x - 18 + 4\sqrt{x^2+3x-6} = 0$

9) $\left(\frac{1}{3}\right)^{\sqrt{x+2}} < 3^{-x}$

Prof. Ovidiu Bădescu

Rezolvați ecuația:

$$3^{\log_3^2 x} + x^{\log_3 x} = 162$$

E_1) Cond: $x > 0 \Rightarrow x \in (0, \infty)$

E_2) $3^{\log_3^2 x} = 3^{(\log_3 x)^2} = \left(3^{\log_3 x}\right)^{\log_3 x} =$

$= \left(x^{\log_3 3}\right)^{\log_3 x} = x^{\log_3 x}$

E_3) ec. devine:

$$x^{\log_3 x} + x^{\log_3 x} = 162$$

$$2x^{\log_3 x} = 162 \mid :2$$

$$x^{\log_3 x} = 81$$

E_4) $x^{\log_3 x} = 81 \mid \log_3(\cdots)$

$\Rightarrow \log_3\left(x^{\log_3 x}\right) = \log_3 81 \Leftrightarrow$

$(\log_3 x) \cdot \log_3 x = 4 \Leftrightarrow (\log_3 x)^2 = 4$

$\Leftrightarrow \log_3 x = \pm 2$

E_5) $\log_3 x = 2 \Leftrightarrow x = 3^2 \Leftrightarrow x = 9 \in I_C$

$\log_3 x = -2 \Leftrightarrow x = 3^{-2} \Leftrightarrow x = \frac{1}{9} \in I_C$

$$\boxed{S_f: x \in \left\{\frac{1}{9}; 9\right\}}$$

Prof. Ovidiu Bădescu

Rezolvați:

$$\log_2(2^{x+3}+16) = 2x + \log_2 3$$

E_1) Cond: $2^{x+3}+16 > 0$

E_2) folosim $\log_b A = N \Leftrightarrow A = b^N$

E_3) în cazul nostru,

$$2^{x+3}+16 = 2^{2x+\log_2 3}$$

$$\Leftrightarrow 2^x \cdot 8 + 16 = 2^{2x} \cdot 2^{\log_2 3}$$

E_4) dar, $2^{\log_2 3} = 3^{\log_2 2} = 3$

și notând $2^x = t > 0 \Rightarrow 2^{2x} = t^2 \Rightarrow$

E_5) $8t + 16 = t^2 \cdot 3$

$$\Rightarrow 3t^2 - 8t - 16 = 0$$

$$t_{1,2} = \frac{8 \pm \sqrt{64+192}}{2 \cdot 3} = \frac{8 \pm \sqrt{256}}{6} = \frac{8 \pm 16}{6}$$

$$\Rightarrow \begin{cases} t_1 = \frac{24}{6} = 4 \\ t_2 = \frac{-8}{6} = -\frac{4}{3} \end{cases}$$

E_6) $t_1 = 4 \Rightarrow 2^x = 4 \Rightarrow 2^x = 2^2$, cum $f(x) = 2^x$ inj

$\Rightarrow \boxed{x = 2}$ $\quad S_f: \boxed{x = 2}$

$t_2 = -\frac{4}{3} \Rightarrow 2^x = -\frac{4}{3} < 0 \Rightarrow x \in \emptyset$

Prof. Ovidiu Badescu

Rezolvați: $x \cdot 2^{\log_x 5} = 10$

E1) $x \cdot 2^{\log_x 5} = 10 \mid \log_x(\cdots)$

$\log_x\left(x \cdot 2^{\log_x 5}\right) = \log_x 10$

$\log_x x + \log_x 2^{\log_x 5} = \log_x(2 \cdot 5)$

$1 + \log_x 5 \cdot \log_x 2 = \log_x 2 + \log_x 5$

E2) $1 + \log_x 5 \cdot \log_x 2 - \log_x 2 - \log_x 5 = 0$

$\log_x 2\left(\log_x 5 - 1\right) - \left(\log_x 5 - 1\right) = 0$

$\left(\log_x 5 - 1\right)\left(\log_x 2 - 1\right) = 0$

E3) Caz I: $\log_x 5 - 1 = 0 \Rightarrow \log_x 5 = 1 \Rightarrow 5 = x^1$

$\Rightarrow x = 5$ sol posibilă, o verificăm

$\Rightarrow 5 \cdot 2^{\log_5 5} = 10 \Leftrightarrow 5 \cdot 2 = 10$ adev $\Rightarrow \boxed{x = 5 \text{ sol}}$

Caz II: $\log_x 2 - 1 = 0 \Rightarrow \log_x 2 = 1 \Rightarrow 2 = x^1$

$\Rightarrow x = 2$ sol posibilă, o verificăm

$\Rightarrow 2 \cdot 2^{\log_2 5} = 10 \Leftrightarrow 2 \cdot 5^{\log_2 2} = 10 \Rightarrow 10 = 10$

adev $\Rightarrow x = 2$ soluții

$S_f : x \in \{2; 5\}$

Prof. Ovidiu Băgescu

Rezolvați: $9^{\log_5(x-1)} = 4+5^{\log_9(x+3)}$

$E_1)$ $u = \log_5(x-1) \Rightarrow x-1 = 5^u \Rightarrow x = 5^u + 1$

$v = \log_9(x+3) \Rightarrow x+3 = 9^v \Rightarrow x = 9^v - 3$

$E_2)$ din ec $\Rightarrow \boxed{9^u = 4+5^v}$

din relații $\Rightarrow 5^u + 1 = 9^v - 3 \Rightarrow \boxed{5^u - 9^v + 4 = 0}$

$E_3)$ $\begin{cases} 9^u = 4+5^v \\ 9^v = 4+5^u \end{cases}$

Caz I: $u < v \Rightarrow 9^u < 9^v \Rightarrow 4+5^v < 4+5^u$

$\Rightarrow 5^v < 5^u$ $\Big\} \Rightarrow \underline{v < u}$ contradicție

baza $b = 5 > 1$

Caz II: $u > v \Rightarrow 9^u > 9^v \Rightarrow 4+5^v > 4+5^u$

$\Rightarrow 5^v > 5^u$ $\Big\} \Rightarrow \underline{v > u}$ contradicție

baza $b = 5 > 1$

Caz III: $u = v$ (singurul caz rămas)

$E_4)$ $9^u = 4+5^u \mid :5^u \Rightarrow \left(\frac{9}{5}\right)^u = \frac{4}{5^u} + 1$

$\Rightarrow \left(\frac{9}{5}\right)^u - \frac{4}{5^u} = 1$, $f_1(x) = \left(\frac{9}{5}\right)^u \Rightarrow f_1$ ↗

$f_2(x) = \frac{4}{5^u} \Rightarrow f_2$ ↘ $\Rightarrow -f_2$ ↗

$\Rightarrow f_1 + (-f_2)$ ↗ $\Rightarrow$ cel mult o sol, $u = 1 \Rightarrow \log_5(x-1) = 1 \Rightarrow x = 6$ se verifică în ecuație

Prof. Ovidiu Bădescu

Rezolvați: $10^{\lg^2 x} + x^{\lg x} = 20$

E_1) cond $x > 0 \Rightarrow x \in (0; \infty)$

E_2) $10^{\lg^2 x} = 10^{\lg x \cdot \lg x} = \left(10^{\lg x}\right)^{\lg x} =$

$= \left(x^{\lg 10}\right)^{\lg x} = x^{\lg x}$

E_3) ec. devine: $x^{\lg x} + x^{\lg x} = 20$

$\Rightarrow 2x^{\lg x} = 20 \mid :2 \Rightarrow x^{\lg x} = 10 \mid \lg(-)$

$\lg x^{\lg x} = \lg 10 \Rightarrow \lg x \cdot \lg x = 1$

$\Rightarrow (\lg x)^2 = 1 \Rightarrow \lg x = \pm 1$

E_4) $\lg x = 1 \Rightarrow x = 10^1 \Rightarrow x = 10 \in (0; \infty)$ ok.

$\lg x = -1 \Rightarrow x = 10^{-1} \Rightarrow x = \frac{1}{10} \in (0; \infty)$ ok.

$S_f : x \in \left\{\frac{1}{10}; 10\right\}$

Prof. Ovidiu Badescu

Rezolvați: $\frac{\sqrt[3]{x^4}-1}{\sqrt[3]{x^2}-1}-\frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x}+1}=4$

E1) observăm că $\left(\sqrt[3]{x^2}\right)^2=\sqrt[3]{x^4}$ și $\left(\sqrt[3]{x}\right)^2=\sqrt[3]{x^2}$

E2) ec: $\frac{\left(\sqrt[3]{x^2}\right)^2-1}{\sqrt[3]{x^2}-1}-\frac{\left(\sqrt[3]{x}\right)^2-1}{\sqrt[3]{x}+1}=4$

$\Rightarrow \frac{(\sqrt[3]{x^2}-1)(\sqrt[3]{x^2}+1)}{\sqrt[3]{x^2}-1}-\frac{(\sqrt[3]{x}-1)(\sqrt[3]{x}+1)}{\sqrt[3]{x}+1}=4$

$\Rightarrow \sqrt[3]{x^2}+1-\sqrt[3]{x}+1=4$

$\sqrt[3]{x^2}-\sqrt[3]{x}-2=0$

E3) not $\sqrt[3]{x}=t \Rightarrow \sqrt[3]{x^2}=t^2$

$\Rightarrow t^2-t-2=0 \Rightarrow t_{1,2}=\frac{1\pm\sqrt{1+8}}{2\cdot 1}=\frac{1\pm 3}{2}$

$\begin{cases} t_1=2 \\ t_2=-1 \end{cases}$

E4) Cz I: $t_1=2 \Rightarrow \sqrt[3]{x}=2 \mid ()^3 \Rightarrow x=8$

Cz II: $t_2=-1 \Rightarrow \sqrt[3]{x}=-1 \mid ()^3 \Rightarrow x=-1$

$S_8: x\in\{-1,8\}$

Prof. Ovidiu Băleșcu

Rezolvați: $x^3+x+\sqrt[3]{x^3+x-2}=12$

E1) not $t=x^3+x \Rightarrow t+\sqrt[3]{t-2}=12$

$\sqrt[3]{t-2}=12-t \mid ()^3 \Rightarrow$

$\Rightarrow t-2=12^3-3\cdot 12^2\cdot t+3\cdot 12\cdot t^2-t^3$

$t^3-36t^2+\ldots 32 = 0$

Prof. Ovidiu Bădescu

Rezolvați:

$$x^2+3x-18+4\sqrt{x^2+3x-6}=0$$

E_1) not $t=\sqrt{x^2+3x-6}$

$\Rightarrow$ ec.: $\underbrace{x^2+3x-6}-12+4\sqrt{x^2+3x-6}=0$

$$t^2-12+4t=0$$

$$t^2+4t-12=0 \Rightarrow t_{1,2}=\frac{-4\pm\sqrt{16+48}}{2\cdot 1}=\frac{-4\pm 8}{2}$$

$$\begin{cases} t_1=2 \\ t_2=-6 \end{cases}$$

E_2) $t_1=2 \Rightarrow \sqrt{x^2+3x-6}=2 \mid (\,)^2$

$$x^2+3x-6=4 \Rightarrow x^2+3x-10=0$$

$$x_{1,2}=\frac{-3\pm\sqrt{9+40}}{2}=\frac{-3\pm 7}{2}\begin{matrix}\nearrow 2 \\ \searrow -5\end{matrix}$$

$t_2=-6 \Rightarrow \sqrt{x^2+3x-6}=-6$ fals pt că

$\sqrt{\cdots} \geq 0$

Prof. Ovidiu Băcescu

Rezolvați: $\left(\frac{1}{3}\right)^{\sqrt{x+2}} < 3^{-x}$ *

E_1) Cond: $x+2 \geq 0 \Rightarrow x \geq -2 \Rightarrow x \in [-2, \infty) = I_C$

E_2) $\left(\frac{1}{3}\right)^{\sqrt{x+2}} < \left(\frac{1}{3}\right)^{x}$ $\Big\} \Rightarrow \sqrt{x+2} > x$.
$b = \frac{1}{3} \in (0,1)$

E_3) Caz I: $x < 0 \xrightarrow{I_C} x \in [-2, 0) \Rightarrow$ inec. adev.

$\Rightarrow S_I = [-2, 0)$

E_4) Caz II: $x \geq 0 \xrightarrow{I_C} x \in [0, \infty) \Rightarrow \sqrt{x+2} > x \Rightarrow$

$\xrightarrow{(\,)^2} x+2 > x^2 \Rightarrow x^2 - x - 2 < 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{matrix} 2 \\ -1 \end{matrix}$

x	-1		2
x^2-x-2	$+ \quad 0 \quad - \quad - \quad - \quad 0 \quad +$		

$\Rightarrow x^2 - x - 2 < 0$ pt $x \in (-1, 2) \Rightarrow x \in (-1, 2) \cap [0, \infty)$

$\Rightarrow S_{II} = [0, 2)$

E_5) $S_f = S_I \cup S_{II} = [-2, 2)$

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