INECUAȚII LOGARITMICE

Rezolvați:

1) $\log_{\frac{1}{2}}(x-1) \leq 1$

2) $\log_2(3-x) > 2$

3) $\log_{\frac{1}{5}}(5+x) < 0$

4) $\log_2(\log_5 x) > 0$

5) $\log_3(\log_{\frac{1}{2}}(3-x)) \leq 1$

6) $\log_2(\log_{\frac{1}{4}}(5+2x)) \geq 2$

7) $2 \leq \log_{\frac{1}{2}}(5-x)$

8) $-1 \geq \log_{\frac{1}{3}}(4+x)$

9) $-4 > \log_2(\log_{\frac{1}{4}}(5-x))$

10) $\log_2 x + \log_4 x > 3$

11) $\log_{\frac{1}{3}}(x^2-4) > 0$

12) $\log_3 x + \log_9 x > 3$

13) $\frac{\lg(1-x)}{4-\lg^2 x} \geq 0$

14) $\log_x(x+2) < 2$

15) $\log_{x+1}(x+3) \geq 2$

16) $x^{\log_4 x+1} > 256x$

17) $\frac{1}{1+\lg x} + \frac{1}{1-\lg x} > 2$

18) $\log_x(2x+1) > \log_x(x-3)$

19) $\log_x 3 > \log_x 2$

20) $\log_{\frac{1}{5}}(2x+5) - \log_{\frac{1}{5}}(16-x^2) \leq 1$

21) $\log_3(\log_2 \frac{1+2x}{1+x}) > 0$

22) $(x-1)\lg 2 + \lg(2^{x+1}+1) < \lg(7\cdot 2^x+12)$

23) $\log_{\frac{1}{2}} x < \log_{\frac{1}{2}}(4x-1) - 1$

24) $\frac{4}{\lg(10x)} - \frac{5}{\lg(100x)} \geq 0$

25) $\log_x(x^2-2x+3) \leq 2\log_x(x+1)$

26) $\log_{\frac{1}{2}}(3x^2-5x-3) < \log_{\frac{1}{2}}(4x-3)$

Profesor Gridiu Eduard

10.2.15

Rezolvați: $\log_{\frac{1}{2}}(x-1) \leq 1$

$E_1)$ cond: $x-1>0 \Rightarrow x>1 \Rightarrow x \in (1;\infty) = I_c$

$E_2)$ $\log_{\frac{1}{2}}(x-1) \leq \log_{\frac{1}{2}} \frac{1}{2}$

$E_3)$ $b = \frac{1}{2} \in (0;1)$

$\Rightarrow x-1 \geq \frac{1}{2} \Rightarrow x \geq 1+\frac{1}{2}$

$\Rightarrow x \geq \frac{3}{2} \Rightarrow x \in \left[\frac{3}{2};\infty\right) = I_1$

$E_4)$ $x \in I_1 \cap I_c = \left[\frac{3}{2};\infty\right) \cap (1;\infty) =$

$= \left[\frac{3}{2};\infty\right)$

Prof. Ovidiu Bădescu

Rezolvați: $\log_2(3-x) > 2$

E_1) cond.: $3-x>0 \Leftrightarrow x<3 \Rightarrow x \in (-\infty, 3) = I_c$

E_2) $2 = \log_2 2^2 \Rightarrow 2 = \log_2 4$

E_3) inec $\Leftrightarrow \log_2(3-x) > \log_2 4$

$b = 2 > 1 \Rightarrow 3-x > 4 \Leftrightarrow -x > 1$

$\Leftrightarrow x < -1 \Rightarrow x \in (-\infty, -1) = I_1$

E_4) $S_f = I_1 \cap I_c = (-\infty, -1)$

Prof. Ovidiu Badescu

Rezolvați $\log_{\frac{1}{5}}(5+x) < 0$

E_1) Cond.: $5+x>0 \Rightarrow x > -5$

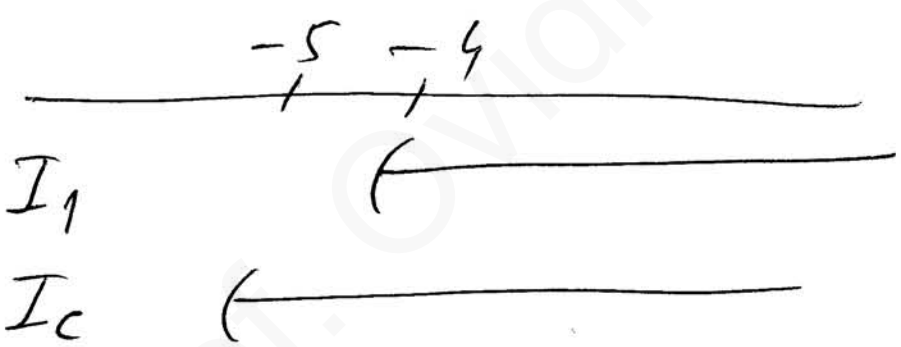
$\Rightarrow x \in (-5; \infty) = I_C$

E_2) inec devine: $\log_{\frac{1}{5}}(5+x) < \log_{\frac{1}{5}} 1$

baza $b = \frac{1}{5} \in (0;1) \Rightarrow 5+x > 1$.

$\Rightarrow x > -4 \Rightarrow x \in (-4; \infty) = I_1$

E_3) $x \in I_1 \cap I_C = (-4; \infty)$



Prof. Ovidiu Badescu

Rezolvați: $\log_2(\log_5 x) > 0$

$E_1)$ cond.: $\begin{cases} x > 0 \Rightarrow x \in (0;\infty) = I_1 \\ \log_5 x > 0 \Leftrightarrow \log_5 x > \log_5 1, \ b = 5 > 1 \\ \Rightarrow x > 1 \Rightarrow x \in (1;\infty) = I_2 \end{cases}$

$I_c = I_1 \cap I_2 = (1;\infty)$

$E_2)$ inec.: $\log_2(\log_5 x) > 0 \Leftrightarrow$

$\log_2(\log_5 x) > \log_2 1$, baza $b = 2 > 1$

$\Rightarrow \log_5 x > 1 \Leftrightarrow \log_5 x > \log_5 5$

$E_3)$ deoarece $b = 5 > 1 \Rightarrow x > 5 \Rightarrow x \in (5;\infty) = I_1$

$E_4)$ $x \in I_1 \cap I_c = (5;\infty) \cap (1;\infty) = (5;\infty)$

Prof. Ovidiu Băescu

Rezolvați: $\log_3(\log_{\frac{1}{2}}(3-x)) \leq 1$

E_1) cond.: $\begin{cases} 3-x>0 \\ \log_{\frac{1}{2}}(3-x)>0 \end{cases}$

$3-x>0 \Rightarrow x<3 \Rightarrow x\in(-\infty;3)=I_1$

$\log_{\frac{1}{2}}(3-x)>0 \Leftrightarrow \log_{\frac{1}{2}}(3-x)>\log_{\frac{1}{2}}1$, $b=\frac{1}{2}\in(0;1)$

$\Rightarrow 3-x<1 \Leftrightarrow x>2 \Rightarrow x\in(2;\infty)=I_2$

$I_c=I_1\cap I_2=(2;3)$

E_2) folosim $a=\log_b b^a \Rightarrow$ ineq $\Leftrightarrow$

$\log_3(\log_{\frac{1}{2}}(3-x)) \leq \log_3 3$, $b=3>1$

$\Rightarrow \log_{\frac{1}{2}}(3-x)\leq 3$.

E_3) dar, $3=\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^3 \Rightarrow$ ineq $\Leftrightarrow$

$\log_{\frac{1}{2}}(3-x)\leq \log_{\frac{1}{2}}\frac{1}{8}$, $b=\frac{1}{2}\in(0;1)$

$\Rightarrow 3-x\geq\frac{1}{8} \Rightarrow x\leq 3-\frac{1}{8} \Rightarrow x\leq\frac{23}{8}$

$\Rightarrow x\in(-\infty;\frac{23}{8}]=I_3$

E_4) $S_p=I_3\cap I_c=(2;\frac{23}{8}]$

Prof. Ovidiu Bădescu

Rezolvați:

$$\log_2\left(\log_{\frac{1}{4}}(5+2x)\right) \geq 2$$

E_1) cond. $\begin{cases} 5+2x>0 \Rightarrow x>-\frac{5}{2} \Rightarrow x\in(-\frac{5}{2},\infty)=I_1 \\ \log_{\frac{1}{4}}(5+2x)>0 \Rightarrow \log_{\frac{1}{4}}(5+2x)>\log_{\frac{1}{4}}1 \end{cases}$

$b=\frac{1}{4}\in(0;1) \Rightarrow 5+2x<1 \Rightarrow 2x<-4 \Rightarrow x<-2$

$\Rightarrow x\in(-\infty,-2)=I_2$

$I_C=I_1\cap I_2=(-\frac{5}{2},-2)$

E_2) inec: $\log_2\left(\log_{\frac{1}{4}}(5+2x)\right)\geq\log_2 4$

$b=2>1 \Rightarrow \log_{\frac{1}{4}}(5+2x)\geq 4$

$\Leftrightarrow \log_{\frac{1}{4}}(5+2x)\geq\log_{\frac{1}{4}}\left(\frac{1}{4}\right)^4$, $b=\frac{1}{4}\in(0;1)$

$\Leftrightarrow 5+2x\leq\left(\frac{1}{4}\right)^4$

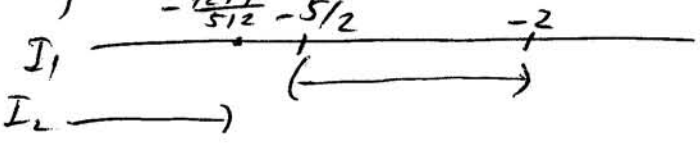
$5+2x\leq\frac{1}{256}\ /\cdot 256$

$1280+512x\leq 1$

$512x\leq -1279 \Rightarrow x\leq -\frac{1279}{512}=-2,4\ldots$

$\Rightarrow x\in(-\infty,-\frac{1279}{512}]=I_1$

E_3) $S_f=I_1\cap I_C=\emptyset$



Prof Ovidiu Badescu

Rezolvați: $-4 > \log_2(\log_{\frac{1}{4}}(5-x))$

$E_1)$ $\log_2(\log_{\frac{1}{4}}(5-x)) < -4$

$E_2)$ cond.: $\begin{cases} 5-x>0 \Rightarrow x<5 \Rightarrow x\in(-\infty;5)=I_1 \\ \log_{\frac{1}{4}}(5-x)>0 \end{cases}$

La toate inecuațiile trebuie găsit intervalul de condiții $\Rightarrow \log_{\frac{1}{4}}(5-x)>0 \Leftrightarrow \log_{\frac{1}{4}}(5-x)>\log_{\frac{1}{4}}1$

$b=\frac{1}{4}\in(0;1) \Rightarrow 5-x<1 \Rightarrow x>4$

$\Rightarrow x\in(4;\infty)=I_2$

$I_c = I_1\cap I_2=(-\infty;5)\cap(4;\infty)=(4;5)$

$E_3)$ $\log_2(\log_{\frac{1}{4}}(5-x))<-4 \Leftrightarrow$

$\Leftrightarrow \log_2(\log_{\frac{1}{4}}(5-x)) < \log_2 2^{-4}$, $b=2>1$

$\Leftrightarrow \log_{\frac{1}{4}}(5-x)<2^{-4} \Leftrightarrow \log_{\frac{1}{4}}(5-x)<\frac{1}{16}$

$E_4)$ scriem $\frac{1}{16}$ ca $\log_{\frac{1}{4}} \Rightarrow \frac{1}{16}=\log_{\frac{1}{4}}\left(\frac{1}{4}\right)^{\frac{1}{16}}$

$E_5)$ ineq: $\log_{\frac{1}{4}}(5-x)<\log_{\frac{1}{4}}\left(\frac{1}{4}\right)^{\frac{1}{16}}$, $b\in(0;1)$

$\Rightarrow 5-x>\left(\frac{1}{4}\right)^{\frac{1}{16}} \Rightarrow x<5-\left(\frac{1}{4}\right)^{\frac{1}{16}} \Rightarrow x\in(-\infty;5-\left(\frac{1}{4}\right)^{\frac{1}{16}})=I_3$

$E_6)$ $S_f=I_3\cap I_c=(-\infty;5-\left(\frac{1}{4}\right)^{\frac{1}{16}})\cap(4;5)=(4;5-\left(\frac{1}{4}\right)^{\frac{1}{16}})$

Prof. Ovidiu Bătescu

Rezolvați: $\log_2 x + \log_4 x > 3$

E_1) Cond.: $x > 0 \Rightarrow x \in (0;\infty) = I_C$

E_2) Dar, $\log_4 x = \log_{2^2} x = \frac{1}{2}\log_2 x \Rightarrow$ inecuația

E_3) $\log_2 x + \frac{1}{2}\log_2 x > 3$

$\Rightarrow \log_2 x \left(1 + \frac{1}{2}\right) > 3 \Rightarrow (\log_2 x)\cdot\frac{3}{2} > 3$

$\Rightarrow \log_2 x > \frac{6}{3} \Rightarrow \log_2 x > 2$

E_4) La inecații este obligatorie să avem comparare de logaritmi în aceeași bază

$2 = \log_2 2^2 = \log_2 4$

E_5) inec.: $\left.\begin{array}{l}\log_2 x > \log_2 4 \\ \text{baza } b = 2 > 1\end{array}\right\} \Rightarrow x > 4 \Rightarrow x \in (4;\infty)$

E_6) $x \in (4;\infty) \cap I_C = (4;\infty)$

Prof. Ovidiu Bădescu

Rezolvați: $\log_{\frac{1}{3}}(x^2-4)>0$

E_1) orice inecuație cu logaritmi: Folosește:

$\log_b x < \log_b y$ și $b\in(0;1) \Leftrightarrow x>y$

$\log_b x < \log_b y$ și $b\in(1;\infty) \Leftrightarrow x<y$

E_2) La orice inecuație cu logaritmi trebuie puse condiții și găsit I_C (la numere nu apar condiții)

$\log_b A = N \Rightarrow$ condiții $\begin{cases} b>0 \\ b\neq 1 \\ A>0 \end{cases}$

E_3) $\log_{\frac{1}{3}}(x^2-4)>0$

cond.: $x^2-4>0$

$x^2-4=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2$

x		-2		2	
x^2-4	$+$	0	$-$	0	$+$

Pt ca $x^2-4>0 \Rightarrow \boxed{x\in(-\infty;-2)\cup(2;\infty)=I_C}$

E_4) se folosește proprietatea $\log_b 1 = 0$

$\Rightarrow$ inec: $\log_{\frac{1}{3}}(x^2-4) > \log_{\frac{1}{3}} 1$ și deoarece

$b=\frac{1}{3}\in(0;1) \Rightarrow x^2-4<1 \Rightarrow x^2-5<0$

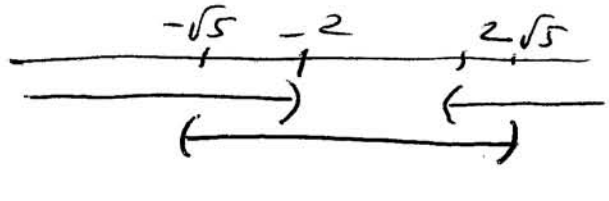
x		$-\sqrt{5}$		$\sqrt{5}$	
x^2-5	$+$	0	$-$	0	$+$

E_5) $x^2-5=0 \Rightarrow x=\pm\sqrt{5}$

Pt ca $x^2-5<0 \Rightarrow \boxed{x\in(-\sqrt{5};\sqrt{5})=I_1}$

E_6) $x\in I_1\cap I_C =$

$=(-\sqrt{5};-2)\cup(2;\sqrt{5})$



Prof. Ovidiu Badescu

Rezolvați: $\log_3 x + \log_9 x > 3$

E_1) Cond: $x > 0 \Rightarrow x \in (0;\infty) = I_C$

E_2) $\log_3 x + \log_{3^2} x > 3 \Leftrightarrow \log_3 x + \frac{1}{2}\log_3 x > 3$

$\Leftrightarrow \log_3 x \left(1 + \frac{1}{2}\right) > 3 \Leftrightarrow$

$\frac{3}{2} \cdot \log_3 x > 3 \Leftrightarrow \log_3 x > \frac{6}{3}$

$\Rightarrow \log_3 x > 2$

E_3) $2 = \log_3 3^2 = \log_3 9 \Rightarrow$ inecuația:

$$\left.\begin{array}{l} \log_3 x > \log_3 9 \\ b = 3 > 1 \end{array}\right\} \Rightarrow x > 9 \Rightarrow x \in (9;\infty) = I_1$$

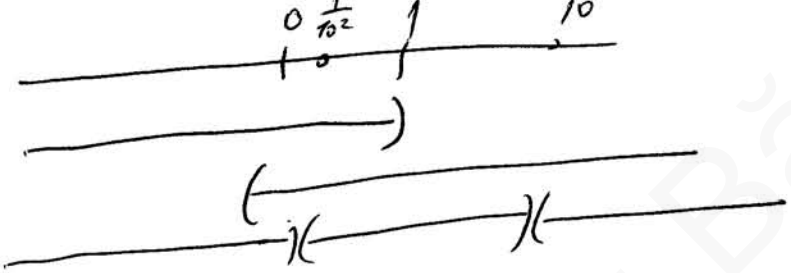
E_4) $x \in I_1 \cap I_C = (9;\infty)$

Prof. Ovidiu Bădescu

Rezolvați: $\frac{\lg(1-x)}{4-\lg^2 x} \geq 0$

E1) cond: $\begin{cases} 1-x>0 \Rightarrow x<1 \Rightarrow x\in(-\infty;1) \\ 4-\lg^2 x \neq 0 \Leftrightarrow \lg^2 x \neq 4 \Rightarrow \lg x \neq \pm 2 \\ x>0 \Rightarrow x\in(0;\infty) \end{cases}$ $\begin{matrix} x\neq 10^2 \\ x\neq 10^{-2} \end{matrix}$

$I_c = (-\infty;1)\cap(0;\infty)\cap\left(\left(-\infty;\frac{1}{10^2}\right)\cup\left(\frac{1}{10^2};10^2\right)\cup(10^2;\infty)\right)$



$I_c = \left(0;\frac{1}{10^2}\right)\cup\left(\frac{1}{10^2};1\right)$

E2) $\lg(1-x)=0 \Leftrightarrow 1-x=10^0 \Leftrightarrow 1-x=1 \Rightarrow x=0$

$4-\lg^2 x=0 \Leftrightarrow \lg^2 x=4 \Leftrightarrow \lg x=\pm 2 \Rightarrow x\in\left\{\frac{1}{100};100\right\}$

E3)

x	$(0$		$\frac{1}{100}$		$1)$		100	
$\lg(1-x)$		$-$		$-$		$-$		$-$
$4-\lg^2 x$		$-$	0	$+$		$+$		$+$
$F(x)$		$+$	$\nexists$	$-$		$+$		$-$

semnul pt $\lg(1-x)$ pe $(0;1)\setminus\left\{\frac{1}{100}\right\}$, $x=\frac{1}{2} \Rightarrow \lg\frac{1}{2}\neq 0$

semnul pt $4-\lg^2 x$: $x=\frac{1}{1000} \Rightarrow 4-\lg^2 10^{-3} = 4-(\lg 10^{-3})^2 < 0$

$x=\frac{1}{10} \Rightarrow 4-\lg^2\frac{1}{10} = 4-(\lg 10^{-1})^2 = 4-1=3>0$

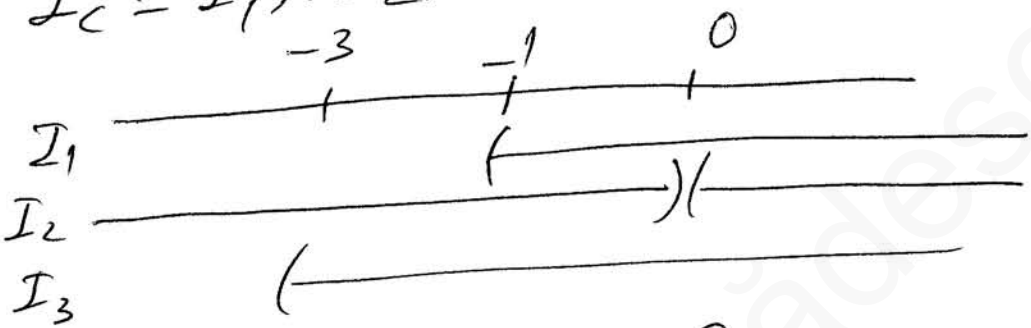
E4) $F(x)\geq 0 \Rightarrow x\in\left(0;\frac{1}{100}\right)$

Proiectul Badescu

Rezolvați: $\log_{x+1}(x+3) \geq 2$

E1) cond.: $\begin{cases} x+1>0 \Rightarrow x\in(-1;\infty)=I_1 \\ x+1\neq 1 \Rightarrow x\in(-\infty;0)\cup(0;\infty)=I_2 \\ x+3>0 \Rightarrow x\in(-3;\infty)=I_3 \end{cases}$

$I_C = I_1 \cap I_2 \cap I_3 = (-1;0)\cup(0;\infty)$



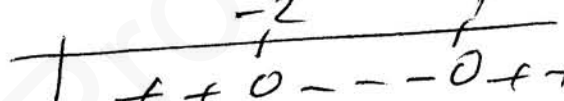
E2) folosim $a = \log_b b^a \Rightarrow$

$\text{dar: } \log_{x+1}(x+3) \geq 2 \Leftrightarrow \log_{x+1}(x+3) \geq \log_{x+1}(x+1)^2$

E3) Cob I: $x+1\in(0;1) \Leftrightarrow x\in(-1;0)=I_4$

$\Rightarrow x+3 \leq (x+1)^2 \Rightarrow x+3 \leq x^2+2x+1 \Leftrightarrow$

$\Leftrightarrow x^2+x-2 \geq 0 \Rightarrow x_{1,2} = \frac{-1\pm\sqrt{1+8}}{2} \begin{matrix} 1 \\ -2 \end{matrix}$



$\Rightarrow$ ok $x^2+x-2\geq 0$

$\Leftrightarrow x\in(-\infty;-2]\cup[1;\infty)=I_5$

$S_I = I_5 \cap I_4 = \emptyset$

Cob II: $x+1\in(1;\infty) \Leftrightarrow x\in(0;\infty)=I_6$

$\Rightarrow x+3 \geq (x+1)^2 \Leftrightarrow x^2+x-2 \leq 0 \Leftrightarrow x\in[-2;1]=I_7$

$S_{II} = I_7 \cap I_6 = (0;1]$

E4) $S_f = S_I \cup S_{II} = \emptyset \cup [0;1] = (0;1]$

Prof. Ovidiu Bădescu

Rezolvați: $x^{\log_4 x+1} > 256x$

E_1) Cond.: $x>0 \Rightarrow x\in(0;\infty)$

E_2) $x^{\log_4 x+1} > 256x \mid \log_4(\dots)$

E_3) $\log_4 x^{\log_4 x+1} > \log_4(256x)$

$\Leftrightarrow (\log_4 x+1)\log_4 x > \log_4 256 + \log_4 x$

E_4) not $\log_4 x = t \Rightarrow (t+1)t > \log_4 4^4 + t$

$\Leftrightarrow t^2+t > 4+t \Rightarrow t^2>4 \Leftrightarrow t^2-4>0$

$t^2-4=0 \Rightarrow t=\pm 2$

t^2-4	$+$	0	$-$	$-$	$-$	0	$+$	$+$
		-2				2		

$t^2-4>0 \Leftrightarrow t\in(-\infty;-2)\cup(2;\infty)$

E_5) $t<-2 \Leftrightarrow \log_4 x < -2 \Leftrightarrow \log_4 x < \log_4 4^{-2}$

$\Leftrightarrow x < 4^{-2} \Leftrightarrow x < \frac{1}{16} \overset{cond}{\Rightarrow} x\in(0;\frac{1}{16}) = I_1$

$t>2 \Rightarrow \log_4 x > 2 \Rightarrow \log_4 x > \log_4 4^2$

$\Rightarrow x>16 \overset{cond}{\Rightarrow} x\in(16;\infty) = I_2$

E_6) $S_f = (0;\frac{1}{16})\cup(16;\infty)$

Prof. Ovidiu Badescu

Rezolvați: $\log_x 3 > \log_x 2$

E1) cond.: $\begin{cases} x > 0 \\ x \neq 1 \end{cases} \Rightarrow I_c = (0;1) \cup (1;\infty)$

E2) Cob I: $x \in (0;1)$ din $\log_x 3 > \log_x 2$

$\Rightarrow 3 < 2$ fals $\Rightarrow x \in \emptyset = S_I$

Cob II: $x \in (1;\infty)$, din $\log_x 3 > \log_x 2$

$\Rightarrow 3 > 2$ adev, $\forall x \in (1;\infty) = S_{II}$

E3) $S_f = S_I \cup S_{II} = (1;\infty)$

Prof. Ovidiu Băgescu

Rezolvați: $\log_{\frac{1}{2}}(3x^2-5x-3) < \log_{\frac{1}{2}}(4x-3)$

$E_1)$ Cond.: $\begin{cases} 3x^2-5x-3>0 \\ 4x-3>0 \end{cases}$

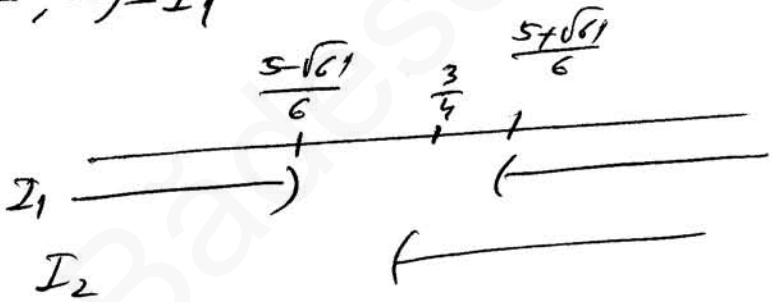
$3x^2-5x-3>0, \quad x_{1,2}=\frac{5\pm\sqrt{25+36}}{6}=\frac{5\pm\sqrt{61}}{6}$

$4x-3>0 \Rightarrow x>\frac{3}{4}$

	$\frac{5-\sqrt{61}}{6}$		$\frac{5+\sqrt{61}}{6}$	
	+	0	−	0 + +

$\Rightarrow x\in(-\infty;\frac{5-\sqrt{61}}{6})\cup(\frac{5+\sqrt{61}}{6};\infty)=I_1$

$x\in(\frac{3}{4};\infty)=I_2$



$I_C=I_1\cap I_2$

$\boxed{I_C=(\frac{5+\sqrt{61}}{6};\infty)}$

$E_2)$ $\log_{\frac{1}{2}}(3x^2-5x-3) < \log_{\frac{1}{2}}(4x-3) \Rightarrow$

$b=\frac{1}{2}\in(0;1)$

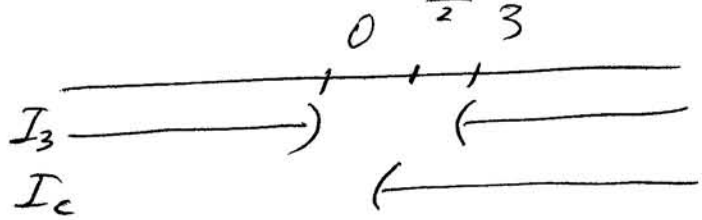
$\Rightarrow 3x^2-5x-3>4x-3$

$3x^2-9x>0 \;/:3 \Rightarrow x^2-3x>0$

$x_{1,2}=\frac{3\pm\sqrt{9-0}}{2\cdot1}=\frac{3\pm3}{2}\begin{matrix}3\\0\end{matrix}$

x		0		3	
x^2-3x	+	0	−	0	+

Rd ca $x^2-3x>0 \Rightarrow x\in(-\infty;0)\cup(3;\infty)=I_3$



$E_3)$ $S_f=I_3\cap I_C$

$\Rightarrow S_f=(3;\infty)$

Prof. Ovidiu Bănescu