ECUAȚII LOGARITMICE CE SE REZOLVĂ PRIN NOTAȚII

Rezolvați:

1) $\log_2^2 x - 3\log_2 x + 2 = 0$

2) $\log_2^2 x - \log_2 x - 2 = 0$

3) $\log_3^2 x - \log_3 x - 6 = 0$

4) $\log_2^2 x - \log_2(4x) + 2 = 0$

5) $\lg^2 x - \lg x = 2$

6) $1 + 6\lg^2 x = 5\lg x$

7) $\ln^2 x - \ln x - 2 = 0$

8) $2\lg^2(x^3) - 3\lg x - 1 = 0$

9) $\log_x 2 + \log_2 x = \frac{10}{3}$

10) $4\log_2^2 x - 5\log_2 x^2 + 4 = 0$

11) $13\log_x 3 = \log_3 x + \log_x 27$

12) $\log_5^2 x + \log_{5x} \frac{5}{x} = 1$

13) $\sqrt{\log_x \sqrt{3x} \cdot \log_3 x} = -1$

Prof. Ovidiu Bădescu

Rezolvați: $\log_2^2 x - \log_2 x - 2 = 0$

E_1) cond: $x > 0 \Rightarrow x \in (0, \infty) = I_C$

E_2) $\log_2 x = t \Rightarrow \log_2^2 x = (\log_2 x)^2 = t^2$

E_3) $t^2 - t - 2 = 0 \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1+8}}{2 \cdot 1} = \frac{1 \pm 3}{2}$

$\begin{cases} t_1 = 2 \\ t_2 = -1 \end{cases}$

E_4) Cob I: $t_1 = 2 \Rightarrow \log_2 x = 2 \Rightarrow x = 2^2 = 4 \in I_C$

Cob II: $t_2 = -1 \Rightarrow \log_2 x = -1 \Rightarrow x = 2^{-1} = \frac{1}{2} \in I_C$

$$S_g = \left\{ 4; \frac{1}{2} \right\}$$

Prof. Ovidiu Bădescu

Rezolvați: $2\lg^2(x^3) - 3\lg x - 1 = 0$

E_1) cond.: $\begin{cases} x^3 > 0 \Rightarrow x > 0 \Rightarrow x \in (0;\infty) = I_1 \\ x > 0 \Rightarrow x \in (0;\infty) = I_2 \end{cases}$

$I_c = I_1 \cap I_2 = (0;\infty)$

E_2) $\lg^2(x^3) = (\lg x^3)^2 = (3\lg x)^2 = 9(\lg x)^2$

E_3) ec. devine: $2 \cdot 9(\lg x)^2 - 3\lg x - 1 = 0$

not $\lg x = t \Rightarrow 18t^2 - 3t - 1 = 0$

$t_{1,2} = \dfrac{3 \pm \sqrt{9+72}}{2 \cdot 18} = \dfrac{3 \pm 9}{36} \begin{cases} \frac{12}{36} = \frac{1}{3} \\ -\frac{6}{36} = -\frac{1}{6} \end{cases}$

E_4) $t_1 = \frac{1}{3} \Rightarrow \lg x = \frac{1}{3} \Rightarrow x = 10^{\frac{1}{3}} \Rightarrow x = \sqrt[3]{10} \in I_c$

$t_2 = -\frac{1}{6} \Rightarrow \lg x = -\frac{1}{6} \Rightarrow x = 10^{-\frac{1}{6}} \Rightarrow x = \frac{1}{10^{\frac{1}{6}}}$

$\Rightarrow x = \dfrac{1}{\sqrt[6]{10}} \in I_c$

$S_f: x \in \left\{ \sqrt[3]{10}; \dfrac{1}{\sqrt[6]{10}} \right\}$

Prof. Ovidiu Băcescu

Rezolvați: $13\log_x 3 = \log_3 x + \log_x 27$

$E_1)$ cond: $\begin{cases} x>0 \\ x\neq 1 \end{cases} \Rightarrow x\in(0;1)\cup(1;\infty)$

$E_2)$ folosim $\log_a b = \frac{1}{\log_b a}$

$\Rightarrow 13\log_x 3 = \frac{1}{\log_x 3} + \log_x 3^3$

$13\log_x 3 = \frac{1}{\log_x 3} + 3\log_x 3$

$E_3)$ not $t = \log_x 3 \Rightarrow 13t = \frac{1}{t} + 3t \mid \cdot t$

$\Rightarrow 13t^2 = 1 + 3t^2 \Rightarrow 10t^2 = 1 \Rightarrow t^2 = \frac{1}{10}$

$t = \pm\frac{1}{\sqrt{10}}$

$E_4)$ $t_1 = \frac{1}{\sqrt{10}} \Rightarrow \log_x 3 = \frac{1}{\sqrt{10}} \Rightarrow 3 = x^{\frac{1}{\sqrt{10}}} \mid ()^{\sqrt{10}}$

$\Rightarrow 3^{\sqrt{10}} = x \Rightarrow \boxed{x = 3^{\sqrt{10}}}$

$t_2 = -\frac{1}{\sqrt{10}} \Rightarrow \log_x 3 = -\frac{1}{\sqrt{10}} \Rightarrow 3 = x^{-\frac{1}{\sqrt{10}}} \mid ()^{-\sqrt{10}}$

$\Rightarrow 3^{-\sqrt{10}} = x \Rightarrow \boxed{x = 3^{-\sqrt{10}}}$

Prof. Ovidiu Bădescu

Rezolvați:

$$\log_5^2 x + \log_{5x} \frac{5}{x} = 1$$

E_1) cond.: $\begin{cases} x>0 \Rightarrow x\in(0;\infty) \\ 5x>0 \Rightarrow x\in(0;\infty) \\ 5x\neq 1 \Rightarrow x\neq \frac{1}{5} \\ \frac{5}{x}>0 \Rightarrow x\in(0;\infty) \end{cases}$ $\quad I_c=(0;\infty)\setminus\{\frac{1}{5}\}$

E_2) $\log_5^2 x = (\log_5 x)^2$ și aducem toți logaritmii în aceeași bază folosind $\log_b a = \frac{\log_c a}{\log_c b}$

$$\log_{5x}\frac{5}{x} = \frac{\log_5 \frac{5}{x}}{\log_5(5x)} = \frac{\log_5 5 - \log_5 x}{\log_5 5 + \log_5 x}$$

$$\Rightarrow \log_{5x}\frac{5}{x} = \frac{1-\log_5 x}{1+\log_5 x}$$

E_3) ec. devine: $(\log_5 x)^2 + \frac{1-\log_5 x}{1+\log_5 x} = 1$, not $\log_5 x = t$

$$\Rightarrow t^2 + \frac{1-t}{1+t} = 1 \;\Big|\cdot(1+t) \Rightarrow t^2(1+t)+1-t = 1+t$$

$$\Rightarrow t^2+t^3-2t=0 \Rightarrow t(t^2+t-2)=0 \Rightarrow \begin{cases} t_1=0 \\ \text{sau} \\ t^2+t-2=0 \end{cases}$$

$$\Rightarrow \begin{cases} t_1=0 \\ t_{2,3}=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm 3}{2} \end{cases} \Rightarrow \begin{matrix} -2 \\ +1 \end{matrix}$$

E_4) $t_1=0 \Rightarrow \log_5 x=0 \Rightarrow x=5^0 \Rightarrow x=1\in I_c$

$t_2=-2 \Rightarrow \log_5 x=-2 \Rightarrow x=5^{-2} \Rightarrow x=\frac{1}{25}\in I_c$

$t_3=1 \Rightarrow \log_5 x=1 \Rightarrow x=5 \Rightarrow x=5\in I_c$

$S_f=\{1;\frac{1}{25};5\}$

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Rezolvați:

$$\sqrt{\log_x \sqrt{3x} \cdot \log_3 x} = -1$$

$E_1)$ cond: $\begin{cases} x > 0 \\ x \neq 1 \\ \sqrt{3x} > 0 \\ x > 0 \end{cases} \Rightarrow x \in (0;1) \cup (1;\infty)$

$E_2)$ $\sqrt{\log_x \sqrt{3x} \cdot \log_3 x} = -1 \mid (\,)^2$

$$\log_x \sqrt{3x} \cdot (\log_3 x)^2 = 1$$

$E_3)$ $$\frac{\log_3 \sqrt{3x}}{\log_3 x} \cdot (\log_3 x)^2 = 1$$

$E_4)$ $\frac{1}{2} \log_3 (3x) \cdot \log_3 x = 1 \mid \cdot 2$

$\Leftrightarrow (\log_3 3 + \log_3 x) \log_3 x = 2$, not $\log_3 x = t$

$\Leftrightarrow (1+t)t = 2 \Rightarrow t^2 + t - 2 = 0$

$\Rightarrow t_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2 \cdot 1} \begin{cases} \frac{-1+3}{2} = 1 \\ \frac{-1-3}{2} = -2 \end{cases}$

$E_5)$ $t_1 = 1 \Rightarrow \log_3 x = 1 \Rightarrow x = 3$ sol posibilă

$t_2 = -2 \Rightarrow \log_3 x = -2 \Rightarrow x = 3^{-2} = \frac{1}{9}$ sol posibilă

$E_6)$ le verificăm, $x_1 = 3 \Rightarrow \sqrt{\log_3 \sqrt{9} \cdot \log_3 3} = -1 \Leftrightarrow 1 = -1$ fals

$x_2 = \frac{1}{9} \Rightarrow \sqrt{\log_{\frac{1}{9}} \sqrt{\frac{1}{3}} \cdot \log_3 \frac{1}{9}} = -1 \Leftrightarrow \sqrt{\frac{1}{2} \cdot \frac{1}{2} \cdot (-2)} = -1$ Ok.

$S_f: \boxed{x = \frac{1}{9}}$

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