ECUAȚII EXPONENȚIALE CE SE REZOLVĂ PRIN ÎMPĂRȚIRE

Rezolvați:

1) $3 \cdot 4^x - 6^x = 2 \cdot 9^x$

2) $3 \cdot 2^{2x} + 6^x - 2 \cdot 3^{2x} = 0$

3) $2 \cdot 25^x - 10^x - 10 \cdot 4^x = 0$

4) $25^x - 2 \cdot 15^x + 9^x = 0$

5) $4^x - 2 \cdot 6^x + 9^x = 0$

6) $6 \cdot 9^{\frac{1}{x}} - 13 \cdot 6^{\frac{1}{x}} + 6 \cdot 4^{\frac{1}{x}} = 0$

7) $4^x + 3^x \cdot 2^{x+1} = 3 \cdot 9^x$

8) $9^x - 3 \cdot 6^x + 2 \cdot 4^x = 0$

9) $25^x - 3 \cdot 10^x + 2 \cdot 4^x = 0$

10) $25^x - 6 \cdot 15^x + 8 \cdot 9^x = 0$

11) $3 \cdot 16^x + 2 \cdot 81^x = 5 \cdot 36^x$

12) $3 \cdot 16^x + 37 \cdot 36^x = 28 \cdot 9^{2x}$

Prof. Ovidiu Bădescu

Rezolvați:

$$3 \cdot 4^x - 6^x = 2 \cdot 9^x$$

E_1) ec.: $3 \cdot 2^{2x} - 2^x \cdot 3^x - 2 \cdot 3^{2x} = 0 \mid : 3^{2x}$

$$\Rightarrow \frac{3 \cdot 2^{2x}}{3^{2x}} - \frac{2^x \cdot 3^x}{3^{2x} \, 3^x} - \frac{2 \cdot 3^{2x}}{3^{2x}} = 0$$

$$\Rightarrow 3 \cdot \left(\frac{2}{3}\right)^{2x} - \left(\frac{2}{3}\right)^x - 2 = 0$$

E_2) not $t = \left(\frac{2}{3}\right)^x > 0$

$$3t^2 - t - 2 = 0 \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1+24}}{2 \cdot 3} \begin{cases} \frac{6}{6} = 1 \\ -\frac{4}{6} = -\frac{2}{3} \end{cases}$$

E_3) $t_1 = 1 \Rightarrow \left(\frac{2}{3}\right)^x = 1 \Rightarrow \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^0$

deoarece $f(x) = \left(\frac{2}{3}\right)^x$ e inj $\Rightarrow x = 0$

$t_2 = -\frac{2}{3} \Rightarrow \left(\frac{2}{3}\right)^x = -\frac{2}{3}$ fals pt că

$a > 0 \Rightarrow a^x > 0$

Prof. Ovidiu Bădescu

Rezolvați:

$$3\cdot 2^{2x}+6^{x}-2\cdot 3^{2x}=0$$

$E_1)\ 3\cdot 2^{2x}+6^{x}-2\cdot 3^{2x}=0 \mid :3^{2x}$

$$\Rightarrow 3\cdot\left(\frac{2}{3}\right)^{2x}+\left(\frac{6^{x}}{3^{x}}\right)-2=0$$

$$\Rightarrow 3\cdot\left(\frac{2}{3}\right)^{2x}+\left(\frac{2}{3}\right)^{x}-2=0$$

$E_2)\ t=\left(\frac{2}{3}\right)^{x}>0 \Rightarrow \left(\frac{2}{3}\right)^{2x}=t^2$

$E_3)\ 3t^2+t-2=0 \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{1+24}}{2\cdot 3}\Rightarrow$

$$t_{1,2}=\frac{-1\pm 5}{6}\begin{cases}\frac{4}{6}=\frac{2}{3}\\ -\frac{6}{6}=-1\end{cases}$$

$E_4)\ t_1=\frac{2}{3}\Rightarrow\left(\frac{2}{3}\right)^{x}=\frac{2}{3}$

$f(x)=\left(\frac{2}{3}\right)^{x}$ inj $\Rightarrow x=1$

$t_2=-1\Rightarrow\left(\frac{2}{3}\right)^{x}=-1$ fals $\Rightarrow x\in\emptyset$

$$S_f: x=1$$

Prof. Ovidiu Badescu

Rezolvați: $2 \cdot 25^x - 10^x - 10 \cdot 4^x = 0$

$E_1)$ $2 \cdot 25^x - 10^x - 10 \cdot 4^x = 0 \mid : 4^x$

$2 \cdot \left(\frac{25}{4}\right)^x - \left(\frac{10}{4}\right)^x - 10 = 0$

$2 \cdot \left(\frac{5}{2}\right)^{2x} - \left(\frac{5}{2}\right)^x - 10 = 0$

$E_2)$ not $\left(\frac{5}{2}\right)^x = t \Rightarrow 2t^2 - t - 10 = 0$

$t_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 2 \cdot (-10)}}{2 \cdot 2} = \frac{1 \pm 9}{4} \begin{cases} \frac{10}{4} = \frac{5}{2} \\ -\frac{8}{4} = -2 \end{cases}$

$E_3)$ $t_1 = \frac{5}{2} \Rightarrow \left(\frac{5}{2}\right)^x = \frac{5}{2}$

$f(x) = \left(\frac{5}{2}\right)^x$ inj $\Rightarrow \boxed{x = 1}$

$t_2 = -2 \Rightarrow \left(\frac{5}{2}\right)^x = -2 \Rightarrow x \in \emptyset$

$S_f : x = 1$

Prof. Ovidiu Badescu

Rezolvați:

$$25^x - 2\cdot 15^x + 9^x = 0$$

$E_1)$ $25^x - 2\cdot 15^x + 9^x = 0 \;/:9^x$

$$\left(\frac{25}{9}\right)^x - 2\cdot\left(\frac{15}{9}\right)^x + 1 = 0$$

$$\left(\frac{5}{3}\right)^{2x} - 2\cdot\left(\frac{5}{3}\right)^x + 1 = 0$$

$E_2)$ not $t = \left(\frac{5}{3}\right)^x \Rightarrow t^2 - 2t + 1 = 0$

$$\Rightarrow t_{1,2} = \frac{2 \pm \sqrt{4-4}}{2\cdot 1} = \frac{2}{2} = 1$$

$E_3)$ $t_1 = 1 \Rightarrow \left(\frac{5}{3}\right)^x = 1 \Rightarrow \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^0$ $\Big\} \Rightarrow$

Cum $f(x) = \left(\frac{5}{3}\right)^x$ inj

$\Rightarrow x = 0$ soluție

Prof. Ovidiu Bădescu

Rezolvați:

$$25^x - 6 \cdot 15^x + 8 \cdot 9^x = 0$$

$E_1)\ 25^x - 6 \cdot 15^x + 8 \cdot 9^x = 0 \Leftrightarrow$

$\Leftrightarrow 5^{2x} - 6 \cdot 3^x \cdot 5^x + 8 \cdot 3^{2x} = 0 \mid :5^{2x}$

$E_2)\ \frac{5^{2x}}{5^{2x}} - \frac{6 \cdot 3^x \cdot 5^x}{5^x \cdot 5^x} + 8 \cdot \frac{3^{2x}}{5^{2x}} = 0$

$$1 - 6 \cdot \left(\frac{3}{5}\right)^x + 8 \cdot \left(\frac{3}{5}\right)^{2x} = 0$$

$E_3)$ not $\left(\frac{3}{5}\right)^x = t \Rightarrow 8t^2 - 6t + 1 = 0$

$$t_{1,2} = \frac{6 \pm \sqrt{36-32}}{2 \cdot 8} = \frac{6 \pm 2}{16} \begin{cases} \frac{8}{16} = \frac{1}{2} \\ \frac{4}{16} = \frac{1}{4} \end{cases}$$

$E_4)\ t_1 = \frac{1}{2} \Rightarrow \left(\frac{3}{5}\right)^x = \frac{1}{2} \mid \lg(\cdot\cdot)$

$\Rightarrow x \lg \frac{3}{5} = \lg \frac{1}{2} \Rightarrow \boxed{x = \frac{\lg \frac{1}{2}}{\lg \frac{3}{5}}}$

$t_2 = \frac{1}{4} \Rightarrow \left(\frac{3}{5}\right)^x = \frac{1}{4} \mid \lg(\cdot\cdot)$

$x \lg \frac{3}{5} = \lg \frac{1}{4} \Rightarrow \boxed{x = \frac{\lg \frac{1}{4}}{\lg \frac{3}{5}}}$

Prof. Ovidiu Badescu

Rezolvați: $3 \cdot 16^x + 2 \cdot 81^x = 5 \cdot 36^x$ 11

$E_1)$ $3 \cdot 4^{2x} + 2 \cdot 9^{2x} - 5 \cdot (4 \cdot 9)^x = 0 \,/ : 9^{2x}$

$E_2)$ $3 \cdot \left(\frac{4}{9}\right)^{2x} + 2 \cdot 1 - 5 \cdot \frac{4^x \cdot 9^x}{9^x \cdot 9^x} = 0$

$3 \cdot \left(\frac{4}{9}\right)^{2x} - 5 \cdot \left(\frac{4}{9}\right)^x + 2 = 0$

$E_3)$ not $\left(\frac{4}{9}\right)^x = t \Rightarrow 3t^2 - 5t + 2 = 0$

$t_{1,2} = \frac{5 \pm \sqrt{25 - 24}}{2 \cdot 3} = \frac{5 \pm 1}{6} \begin{matrix} 1 \\ \angle \\ \frac{4}{6} = \frac{2}{3} \end{matrix}$

$E_4)$ Caz I: $\left(\frac{4}{9}\right)^x = 1 \Rightarrow \left(\frac{4}{9}\right)^x = \left(\frac{4}{9}\right)^0$, deoarece

$f(x) = \left(\frac{4}{9}\right)^x$ inj $\Rightarrow x = 0$

Caz II: $\left(\frac{4}{9}\right)^x = \frac{2}{3} \Rightarrow \left(\frac{2}{3}\right)^{2x} = \frac{2}{3}$, deoarece

$f(x) = \left(\frac{2}{3}\right)^{2x}$ inj $\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$E_5)$ S_f: $x \in \{\frac{1}{2}; 0\}$

Prof. Viviu Bădescu