OPERAȚII CU RADICALI ȘI LOGARITMI

1) $100^{\lg 2} + \sqrt[3]{-27}$

2) $2^{2\log_2 5} + \sqrt{9}$

3) $25^{\log_5 3} + \sqrt[3]{-1}$

4) $49^{\frac{1}{2}\log_7 \frac{1}{4}} + \log_2 \frac{1}{8}$

5) $\lg\left(\lg \sqrt{\sqrt[5]{10}}\right)$

6) $81^{\frac{1}{4} - \frac{1}{2}\log_9 4} \cdot 49^{\log_7 2}$

7) $2\log_5 \sqrt[4]{5} + \frac{1}{2}\log_{\sqrt{5}} 25$

8) $\log_5^2 \sqrt{5} + \log_{\frac{1}{4}} 2$

9) $2\log_2 8 - \log_4 2$

10) $\log_{\sqrt{3}} 27 - \log_{\frac{1}{3}} 27 - \log_{\frac{\sqrt{3}}{2}}\left(\frac{64}{27}\right)$

11) Arătați că $a \in \mathbb{N}$ dacă
$a = \sqrt{\sqrt{3}-1} \cdot \sqrt[3]{\sqrt{6}+\sqrt{2}} \cdot \sqrt[6]{40\sqrt{3}+64}$

Prof. Ovidiu Bădescu

Calculați: $100^{\lg 2} + \sqrt[3]{-27}$

E1) $N = 100^{\lg 2} + \sqrt[3]{-27}$

$N = 2^{\lg 100} + \sqrt[3]{(-3)^3}$

$N = 2^{2} + (-3)$

$N = 4 - 3 \Rightarrow N = 1$

Prof. Ovidiu Bădescu

Calculati:

$$81^{\frac{1}{4}-\frac{1}{2}\log_9 4} \cdot 49^{\log_7 2}$$

$$E_1)\ N = \frac{81^{\frac{1}{4}}}{81^{\frac{1}{2}\log_9 4}} \cdot 2^{\log_7 49}$$

$$E_2)\ N = \frac{\sqrt[4]{3^4}}{\left(81^{\frac{1}{2}}\right)^{\log_9 4}} \cdot 2^{\log_7 7^2}$$

$$N = \frac{3}{\left(\sqrt{81}\right)^{\log_9 4}} \cdot 2^2 = \frac{3}{9^{\log_9 4}} \cdot 4 =$$

$$= \frac{3}{\log_9 9} \cdot 4 = \frac{3}{4} \cdot 4 = 3$$

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s-a folosit: $a^{b-c} = \frac{a^b}{a^c}$

$$a^{\log_b c} = c^{\log_b a}$$

Prof. Ovidiu Băbescu

Fie $a=\sqrt{\sqrt{3}-1}\cdot\sqrt[3]{\sqrt{6}+\sqrt{2}}\cdot\sqrt[6]{4\sqrt{3}+4}$

Arătați că $a\in\mathbb{N}$

E_1) Aducem radicalii la același ordin, ordinul 6

$E_2)\ \sqrt{\sqrt{3}-1}=\sqrt[6]{(\sqrt{3}-1)^3}=\sqrt[6]{(\sqrt{3})^3-3(\sqrt{3})^2+3\sqrt{3}-1^3}=$

$=\sqrt[6]{3\sqrt{3}-9+3\sqrt{3}-1}=\sqrt[6]{6\sqrt{3}-10}$

$E_3)\ \sqrt[3]{\sqrt{6}+\sqrt{2}}=\sqrt[6]{(\sqrt{6}+\sqrt{2})^2}=\sqrt[6]{6+2\sqrt{12}+2}=\sqrt[6]{8+4\sqrt{3}}$

$E_4)\ a=\sqrt[6]{(6\sqrt{3}-10)(8+4\sqrt{3})(4\sqrt{3}+4)}=\sqrt[6]{2(3\sqrt{3}-5)\cdot 4(2+\sqrt{3})\cdot 4(\sqrt{3}+1)}$

$a=\sqrt[6]{32(3\sqrt{3}-5)(2\sqrt{3}+2+3+\sqrt{3})}=\sqrt[6]{32(3\sqrt{3}-5)(3\sqrt{3}+5)}$

$a=\sqrt[6]{32\left((3\sqrt{3})^2-5^2\right)}=\sqrt[6]{32(27-25)}=\sqrt[6]{64}=\sqrt[6]{2^6}=2$

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mod II: grupăm

$a=\sqrt[6]{(\sqrt{3}-1)^3\left[\sqrt{2}(\sqrt{3}+1)\right]^2\cdot 4(\sqrt{3}+1)}=$

$=\sqrt[6]{(\sqrt{3}-1)^3\cdot 2(\sqrt{3}+1)^2\cdot 4(\sqrt{3}+1)}=\sqrt[6]{(\sqrt{3}-1)^3(\sqrt{3}+1)^3\cdot 8}$

$=\sqrt[6]{\left[(\sqrt{3}-1)(\sqrt{3}+1)\right]^3\cdot 2^3}=\sqrt[6]{\left[(\sqrt{3})^2-1\right]^3\cdot 2^3}=$

$=\sqrt[6]{2^3\cdot 2^3}=\sqrt[6]{2^6}=2\in\mathbb{N}$

Prof. Ovidiu Pădescu