SCRIEREA TRIGONOMETRICĂ A NUMERELOR COMPLEXE CU ARGUMENT VARIABIL

Scrieți sub formă trigon.:

1) $z = -\cos a + i \sin a$, $a \in (0; \frac{\pi}{2})$

2) $z = \sin a + i \cos a$, $a \in (\frac{\pi}{2}; \pi)$

3) $z = -\cos a + i \sin a$, $a \in (\frac{3\pi}{2}; 2\pi)$

4) $z = \sin a - i \cos a$, $a \in (0; \frac{\pi}{2})$

5) $z = \sin a - i \cos a$, $a \in (\pi; \frac{3\pi}{2})$

6) $z = 1 - \cos a + i \sin a$, $a \in (0; \frac{\pi}{2})$

~~7)~~ $z = 1 - \cos a + i \sin a$, $a \in (\frac{\pi}{2}; \pi)$

8) $z = 1 + \cos a + i \sin a$, $a \in (\frac{3\pi}{2}; 2\pi)$

9) $z = 1 - \cos a - i \sin a$, $a \in (\pi; \frac{3\pi}{2})$

10) $z = 1 + \cos a - i \sin a$, $a \in (\frac{3\pi}{2}; 2\pi)$

11) $z = 1 - i \operatorname{tg} a$, $a \in (\frac{\pi}{2}; \pi)$

12) $z = 1 + i \operatorname{tg} a$, $a \in (\pi; \frac{3\pi}{2})$

~~13)~~ $z = \sin x + (1 + \cos x) i$, $x \in (\pi; \frac{3\pi}{2})$

~~14)~~ $z = -1 + \cos x + i \sin x$, $x \in (\frac{3\pi}{2}; 2\pi)$

~~15)~~

21) Rezolvați: $(z+a)^n - (z-a)^n = 0$, $a \in \mathbb{R}^*$, $n \in \mathbb{N}$, $n \geq 2$

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Scrieți trigonometric $z = 1 - \cos a + i \sin a$, $a \in \left(\frac{\pi}{2}; \pi\right)$

$E_1)$ $r = \sqrt{(1-\cos a)^2 + \sin^2 a} = \sqrt{1 - 2\cos a + \cos^2 a + \sin^2 a} =$

$= \sqrt{2 - 2\cos a} = \sqrt{2(1-\cos a)}$

Dar, $\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow 1 - \cos a = 2\sin^2 \frac{a}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2} \Rightarrow 1 + \cos a = 2\cos^2 \frac{a}{2}$

$r = \sqrt{2 \cdot 2\sin^2 \frac{a}{2}} = 2\left|\sin \frac{a}{2}\right|$

$E_2)$ $a \in \left(\frac{\pi}{2}; \pi\right) \Rightarrow \frac{a}{2} \in \left(\frac{\pi}{4}; \frac{\pi}{2}\right) \Rightarrow \frac{a}{2} \in C_1 I \Rightarrow$

$\Rightarrow \sin \frac{a}{2} > 0 \Rightarrow \left|\sin \frac{a}{2}\right| = \sin \frac{a}{2} \Rightarrow \boxed{r = 2\sin \frac{a}{2}}$

$E_3)$ $M(1 - \cos a, \sin a)$, $a \in \left(\frac{\pi}{2}; \pi\right)$

$\Rightarrow 1 - \cos a > 0$, $\sin a > 0 \Rightarrow M \in C_1 I \Rightarrow k = 0$

$\Rightarrow \varphi = \operatorname{arctg}\left(\frac{\sin a}{1 - \cos a}\right) + 0 \cdot \pi$

$\varphi = \operatorname{arctg}\left(\frac{2\sin \frac{a}{2} \cos \frac{a}{2}}{2\sin^2 \frac{a}{2}}\right) = \operatorname{arctg}\left(\operatorname{ctg} \frac{a}{2}\right) =$

$= \operatorname{arctg}\left(\operatorname{tg}\left(\frac{\pi}{2} - \frac{a}{2}\right)\right) = \frac{\pi}{2} - \frac{a}{2} = \frac{\pi - a}{2}$

$E_4)$ $\boxed{z = 2\sin \frac{a}{2}\left(\cos \frac{\pi - a}{2} + i \sin \frac{\pi - a}{2}\right)}$

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Fie $z = \sin x + (1+\cos x)i$, $x \in (\pi, \frac{3\pi}{2})$

Scrie z trigonometric

$E_1) r = \sqrt{a^2+b^2} = \sqrt{\sin^2 x + (1+\cos x)^2} =$

$= \sqrt{\sin^2 x + 1 + 2\cos x + \cos^2 x} = \sqrt{2(1+\cos x)} \Rightarrow$

Dar, $1+\cos x = 2\cos^2 \frac{x}{2}$

$\Rightarrow r = \sqrt{4\cos^2 \frac{x}{2}} = 2|\cos \frac{x}{2}|$

$E_2) x \in (\pi, \frac{3\pi}{2}) \Rightarrow \frac{x}{2} \in (\frac{\pi}{2}, \frac{3\pi}{4}) \subset Cd\,II \Rightarrow$

$\Rightarrow \cos \frac{x}{2} < 0 \Rightarrow r = -2\cos \frac{x}{2}$

$E_3) \varphi_1 = \operatorname{arctg} \frac{b}{a} = \operatorname{arctg} \frac{1+\cos x}{\sin x} = \operatorname{arctg} \frac{2\cos^2 \frac{x}{2}}{2\sin \frac{x}{2}\cos \frac{x}{2}}$

$= \operatorname{arctg}(\operatorname{ctg} \frac{x}{2}) = \operatorname{arctg}(\operatorname{tg}(\frac{\pi}{2} - \frac{x}{2})) = \frac{\pi}{2} - \frac{x}{2}$

$E_4) M(\sin x, 1+\cos x)$, $x \in (\pi, \frac{3\pi}{2})$

$x_M = \sin x < 0$; $y_M = 1+\cos x > 0 \Rightarrow M \in Cd\,II$

$\Rightarrow \varphi = \pi + \varphi_1 = \pi + \frac{\pi}{2} - \frac{x}{2} = \frac{3\pi - x}{2}$

$E_5) z = -2\cos \frac{x}{2}\left(\cos \frac{3\pi - x}{2} + i \sin \frac{3\pi - x}{2}\right)$

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Scrieți trigonometric

$$z = -1 + \cos x + i \sin x, \quad x \in \left(\frac{3\pi}{2}; 2\pi\right)$$

E1) $M(-1+\cos x, \sin x) \in$ cadranul

E2) $r = \sqrt{(-1+\cos x)^2 + \sin^2 x} = \sqrt{1 - 2\cos x + \cos^2 x + \sin^2 x}$

$$= \sqrt{2 - 2\cos x} = \sqrt{2} \cdot \sqrt{1 - \cos x}$$

E3) Formulele:

$$\boxed{\begin{aligned} 1 - \cos x &= 2\sin^2 \frac{x}{2} \\ 1 + \cos x &= 2\cos^2 \frac{x}{2} \\ \sin x &= 2 \sin \frac{x}{2} \cos \frac{x}{2} \end{aligned}}$$

E4) $r = \sqrt{2} \cdot \sqrt{2\sin^2 \frac{x}{2}} = 2\left|\sin \frac{x}{2}\right|$

E5) $x \in \left(\frac{3\pi}{2}; 2\pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{3\pi}{4}; \pi\right) \Rightarrow \frac{x}{2} \in$ cadranul

$\Rightarrow \sin \frac{x}{2} > 0 \Rightarrow \boxed{r = 2\sin \frac{x}{2}}$

E6) $\varphi_1 = \operatorname{arctg} \frac{\sin x}{-1+\cos x} = \operatorname{arctg}\left(-\frac{\sin x}{1-\cos x}\right) =$

$$= -\operatorname{arctg} \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}} = -\operatorname{arctg}\left(\operatorname{ctg} \frac{x}{2}\right) =$$

$$= -\operatorname{arctg}\left(\operatorname{tg}\left(\frac{\pi}{2} - \frac{x}{2}\right)\right) = -\left(\frac{\pi}{2} - \frac{x}{2}\right) = \frac{x}{2} - \frac{\pi}{2}$$

E7) $A(-1+\cos x; \sin x) \Rightarrow M \in$ cadranul $\Rightarrow$

$\varphi = \pi + \varphi_1 = \frac{x}{2} + \frac{\pi}{2}$

$z = r(\cos(\frac{x}{2} + \frac{\pi}{2}) + i \sin(\frac{x}{2} + \frac{\pi}{2}))$

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Rezolvați: $(z+a)^n-(z-a)^n=0$,

$a\in\mathbb{R}^*$, $n\in\mathbb{N}$, $n\geq 2$

$E_1)\ (z+a)^n=(z-a)^n \mid :(z-a)^n$

$\Rightarrow \left(\frac{z+a}{z-a}\right)^n=1$

$E_2)$ Folosim $z^n=\alpha$, $\alpha=r(\cos\varphi+i\sin\varphi)$

$\Rightarrow z_k=\sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n}+i\sin\frac{\varphi+2k\pi}{n}\right), k=\overline{0,n-1}$

Dacă $z^n=1 \Rightarrow z_k=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$

$E_3)\ \left(\frac{z+a}{z-a}\right)^n=1 \Rightarrow \frac{z_k+a}{z_k-a}=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$

$E_4)$ not. $\frac{2k\pi}{n}=\alpha \Rightarrow z_k+a=(z_k-a)(\cos\alpha+i\sin\alpha)$

$z+a=z\cos\alpha+iz\sin\alpha-a\cos\alpha-ai\sin\alpha$

$z(1-\cos\alpha-i\sin\alpha)=-a\cos\alpha-a-ai\sin\alpha$

$E_5)\ z=\frac{-a(1+\cos\alpha+i\sin\alpha)}{1-\cos\alpha-i\sin\alpha}$

$$z=\frac{-a\left(2\cos^2\frac{\alpha}{2}+i\cdot 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)}{2\sin^2\frac{\alpha}{2}-i\cdot 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}=\frac{-2a\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}\right)}{2\sin\frac{\alpha}{2}\left(\sin\frac{\alpha}{2}-i\cos\frac{\alpha}{2}\right)}$$

$$=-a\cdot\operatorname{ctg}\frac{\alpha}{2}\cdot\frac{i\left(\sin\frac{\alpha}{2}-i\cos\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}-i\cos\frac{\alpha}{2}}=-ai\operatorname{ctg}\frac{\alpha}{2}$$

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