# NUMERE COMPLEXE SUB FORMĂ TRIGONOMETRICĂ

Scrieți trigonometric pe
a) $z_1$ b) $z_2$ c) $\frac{z_1}{z_2}$ d) $z_1 \cdot z_2$
e) $z_1^3$ și rezolvați ecuația
$z^4 = z_2$ în fiecare din situațiile

1) $z_1 = -\sqrt{3} + i$ ; $z_2 = -5i$

2) $z_1 = 1 + \sqrt{3}i$ ; $z_2 = 1 - i$

3) $z_1 = -1 + i$ ; $z_2 = \sqrt{8} + \sqrt{8}i$

4) $z_1 = 3$ ; $z_2 = -1 - i$

5) $z_1 = -5i$ ; $z_2 = 3i$

6) $z_1 = \sqrt{3} - i$ ; $z_2 = -4$

7) $z_1 = \sqrt{2} - \sqrt{2}i$ ; $z_2 = 1 + i$

8) $z_1 = \sqrt{3} + i$ ; $z_2 = -\sqrt{2} + \sqrt{2}i$

9) $z_1 = 2i$ ; $z_2 = 1 - i$

10) $z_1 = -4$ ; $z_2 = -\sqrt{3} + i$

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Scrieți trigonometrică $\frac{z_1}{z_2}$, $z_1 \cdot z_2$, $z_1^3$ și rezolv $z^4 = z_2$

pentru $z_1 = -\sqrt{3} + i$, $z_2 = -5i$

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$E_1)$ $z_1 = -\sqrt{3} + i$, $r = \sqrt{a^2+b^2} = \sqrt{(-\sqrt{3})^2+1^2} = \sqrt{4} = 2$

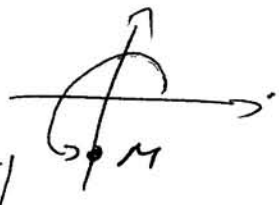
$M(-\sqrt{3}, 1) \in C_d II \Rightarrow \varphi = \operatorname{arctg}\frac{b}{a} + 2k\pi$, $k = 1$

$\Rightarrow \varphi = \operatorname{arctg}\frac{1}{-\sqrt{3}} + \pi = -\operatorname{arctg}\frac{1}{\sqrt{3}} + \pi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$

și $z = r(\cos\varphi + i\sin\varphi) \Rightarrow \boxed{z_1 = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)}$

$E_2)$ $z_2 = -5i$, $r = \sqrt{0^2+(-5)^2} = \sqrt{25} = 5$

$M(0, -5) \in Oy_-$ $\Rightarrow \varphi = \frac{3\pi}{2}$




$\Rightarrow z_2 = r(\cos\varphi + i\sin\varphi) \Rightarrow \boxed{z_2 = 5\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)}$

$E_3)$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos(\varphi_1-\varphi_2) + i\sin(\varphi_1-\varphi_2)\right) = \frac{2}{5}\left(\cos\left(\frac{5\pi}{6} - \frac{3\pi}{2}\right) + i\sin\left(\frac{5\pi}{6} - \frac{3\pi}{2}\right)\right)$

$\Rightarrow \frac{z_1}{z_2} = \frac{2}{5}\left(\cos\left(-\frac{4\pi}{6}\right) + i\sin\left(-\frac{4\pi}{6}\right)\right) = \frac{2}{5}\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$

Dar, $-\frac{2\pi}{3} \notin [0; 2\pi)$ $\Rightarrow$ îl aducem la cercul trigonometric

$\Rightarrow \frac{z_1}{z_2} = \frac{2}{5}\left(\cos\left(2\pi - \frac{2\pi}{3}\right) + i\sin\left(2\pi - \frac{2\pi}{3}\right)\right) \Rightarrow \boxed{\frac{z_1}{z_2} = \frac{2}{5}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)}$

$E_4)$ $z_1 \cdot z_2 = r_1 r_2\left(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)\right) = 10\left(\cos\left(\frac{5\pi}{6} + \frac{3\pi}{2}\right) + i\sin\left(\frac{5\pi}{6} + \frac{3\pi}{2}\right)\right)$

$\Rightarrow z_1 \cdot z_2 = 10\left(\cos\frac{14\pi}{6} + i\sin\frac{14\pi}{6}\right) = 10\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right)$, dar $\frac{7\pi}{3} \notin [0; 2\pi)$

$\Rightarrow z_1 \cdot z_2 = 10\left(\cos\left(2\pi + \frac{\pi}{3}\right) + i\sin\left(2\pi + \frac{\pi}{3}\right)\right) \Rightarrow \boxed{z_1 \cdot z_2 = 10\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

$E_5)$ $z_1^3 = r^3(\cos 3\varphi + i\sin 3\varphi) = 8\left(\cos\frac{15\pi}{6} + i\sin\frac{15\pi}{6}\right) = 8\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right)$

$\Rightarrow z_1^3 = 8\left(\cos\left(2\pi + \frac{\pi}{2}\right) + i\sin\left(2\pi + \frac{\pi}{2}\right)\right) \Rightarrow \boxed{z_1^3 = 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}$

$E_6)$ $z_k = \sqrt[4]{r_2}\left(\cos\frac{\varphi + 2k\pi}{4} + i\sin\frac{\varphi + 2k\pi}{4}\right) = \sqrt[4]{5}\left(\cos\frac{\frac{3\pi}{2} + 2k\pi}{4} + i\sin\frac{\frac{3\pi}{2} + 2k\pi}{4}\right)$,

... valori lui $k$ de la 0 la 3 și scriem $z_0 = \dots$, $z_1 = \dots$, $z_2 = \dots$, $z_3 = \dots$

Scrieți trigonometric pe a) $z_1$, b) $z_2$ c) $\frac{z_1}{z_2}$, d) $z_1 \cdot z_2$ e) $z_1^3$ și rezolvați ecuația $z^4 = z_2$ pentru

$z_1 = 1+\sqrt{3}i$ și $z_2 = 1-i$

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$E_1)$ $z_1 = 1+\sqrt{3}i$, $r = \sqrt{a^2+b^2} = \sqrt{1+3} = 2$

$M(1;\sqrt{3}) \in Cd\ I \Rightarrow \varphi = \operatorname{arctg}\frac{b}{a} + k\pi$, $k=0$

$\Rightarrow \varphi = \operatorname{arctg}\sqrt{3} + 0 = \frac{\pi}{3} \Rightarrow z_1 = r(\cos\varphi + i\sin\varphi) \Rightarrow \boxed{z_1 = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})}$

$E_2)$ $z_2 = 1-i$, $r = \sqrt{a^2+b^2} = \sqrt{2}$; $M(1,-1) \in Cd\ IV \Rightarrow \varphi = \operatorname{arctg}\frac{b}{a} + k2\pi$, $k=2$

$\Rightarrow \varphi_{z_2} = \operatorname{arctg}\frac{-1}{1} + 2\pi = -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4} \Rightarrow \boxed{z_2 = \sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})}$

$E_3)$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\varphi_1-\varphi_2) + i\sin(\varphi_1-\varphi_2)) = \frac{2}{\sqrt{2}}(\cos(\frac{\pi}{3} - \frac{7\pi}{4}) + i\sin(\frac{\pi}{3} - \frac{7\pi}{4})) =$

$\Rightarrow \frac{z_1}{z_2} = \sqrt{2}(\cos(-\frac{17\pi}{12}) + i\sin(-\frac{17\pi}{12}))$. Deoarece $-\frac{17\pi}{12} \notin [0;2\pi)$ și

$-\frac{17\pi}{12} + 2\pi = \frac{7\pi}{12} \in [0;2\pi) \Rightarrow$

$\Rightarrow \boxed{\frac{z_1}{z_2} = \sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}),\ \varphi = \frac{7\pi}{4} \in [0;2\pi)}$

$E_4)$ $z_1 \cdot z_2 = r_1 r_2(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)) = 2\sqrt{2}(\cos(\frac{\pi}{3} + \frac{7\pi}{4}) + i\sin(\frac{\pi}{3} + \frac{7\pi}{4}))$

$z_1 z_2 = 2\sqrt{2}(\cos\frac{25\pi}{12} + i\sin\frac{25\pi}{12})$, $\varphi = \frac{25\pi}{12} \notin [0;2\pi) \Rightarrow \frac{25\pi}{12} - 2\pi = \frac{\pi}{12} \in [0;2\pi)$

$\Rightarrow \boxed{z_1 z_2 = 2\sqrt{2}(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}),\ \varphi = \frac{\pi}{12} \in [0;2\pi)}$

$E_5)$ $z_1^3 = r_1^3(\cos(3\varphi_1) + i\sin(3\varphi_1)) = 8(\cos\pi + i\sin\pi) \Rightarrow \pi \in [0;2\pi)$

$\Rightarrow \boxed{z_1^3 = 8(\cos\pi + i\sin\pi),\ \pi \in [0;2\pi)}$

$E_6)$ $z^4 = \sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}) \Rightarrow z_k = \sqrt[n]{r}(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n})_{k=\overline{0,n-1}}$

$\Rightarrow z_k = \sqrt[4]{\sqrt{2}}(\cos\frac{\frac{7\pi}{4}+2k\pi}{4} + i\sin\frac{\frac{7\pi}{4}+2k\pi}{4})$, $k=\overline{0,3}$

$\Rightarrow z_k = \sqrt[8]{2}(\cos\frac{7\pi+8k\pi}{16} + i\sin\frac{7\pi+8k\pi}{16})$, $k=\overline{0,3}$