MODULUL ȘI CONJUGATUL UNUI NUMĂR COMPLEX

Calculați $|z|$ dacă

1) $z_1 = 3 - \sqrt{5}i$; $z_2 = 4 - i$

2) $z_1 = 2i$; $z_2 = -1 + i$

3) $z_1 = \frac{1}{2} - i\frac{\sqrt{3}}{2}$; $z_2 = \frac{1-i}{4i}$

4) $z_1 = (1-i)^{200}$; $z_2 = \frac{4-3i}{3+4i}$

5) $z = \left(\frac{\sqrt{2}+i\sqrt{3}}{\sqrt{3}-i\sqrt{2}}\right)^{2012}$

6) $z = \left(\sqrt{3+\sqrt{5}} + i\sqrt{3-\sqrt{5}}\right)^{2012}$

7) $z = \frac{(1+i)^2(1-i)^3}{(1-i\sqrt{3})^4(1-i\sqrt{3})}$

8) Studiați dacă $\exists z \in \mathbb{C}$ cu $|z-i| = |z+2i| = |iz-1|$

Calculați $\bar{z}$ pentru

1) $z_1 = 3 - \sqrt{5}i$; $z_2 = 4 + i$

2) $z_1 = 2i$; $z_2 = -3$

3) $z_1 = \frac{1}{4+3i}$; $z_2 = \frac{1-i}{1+i}$

4) arăt. că $z \in \mathbb{R}$ pentru $z = (-1+2i)^n + (-1-2i)^n$

5) arăt că $z \in \mathbb{R}^*$ i (pur imaginar) dacă $z = (5-4i)^n - (5+4i)^n$

6) arăt. că $z \in \mathbb{R}$ pentru $z = (4-i)^n + (4+i)^n$

7) arăt. că $z \in \mathbb{R}^*$ i dacă $z = (5-7i)^n - (5+7i)^n$

8) dacă $z = \frac{a+2i}{2+ai}$, se cere $a = ?$, $a \in \mathbb{R}$ dacă $z \in \mathbb{R}$

Prof. Ovidiu Bădescu

Calculați:

a) $|(1-i)^{200}|$

b) $\left|\frac{4-3i}{3+4i}\right|$

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a) Folosim $|z^n| = |z|^n \Rightarrow$

$\Rightarrow |(1-i)^{200}| = |1-i|^{200} =$

$= \left(\sqrt{1^2+(-1)^2}\right)^{200} = (\sqrt{2})^{200} = 2^{100}$

b) Folosim $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$\Rightarrow \left|\frac{4-3i}{3+4i}\right| = \frac{|4-3i|}{|3+4i|} = \frac{\sqrt{4^2+(-3)^2}}{\sqrt{3^2+4^2}} =$

$= \frac{\sqrt{25}}{\sqrt{25}} = \frac{5}{5} = 1$

Prof. Ovidiu Badescu

Se cere $|z|=?$, $z=\left(\frac{\sqrt{2}+i\sqrt{3}}{\sqrt{3}-i\sqrt{2}}\right)^{2012}$

$E_1)\ |z|=\left|\left(\frac{\sqrt{2}+i\sqrt{3}}{\sqrt{3}-i\sqrt{2}}\right)^{2012}\right|=\left|\frac{\sqrt{2}+i\sqrt{3}}{\sqrt{3}-i\sqrt{2}}\right|^{2012}$

$E_2)\ |z|=\left(\frac{|\sqrt{2}+i\sqrt{3}|}{|\sqrt{3}-i\sqrt{2}|}\right)^{2012}=\left(\frac{\sqrt{2+3}}{\sqrt{3+2}}\right)^{2012}$

$=1^{2012}=1$

s-a folosit: $|z^n|=|z|^n$

$$\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}$$

$$|z|=\sqrt{a^2+b^2}$$

Prof. Ovidiu Bădescu

Fie $z=\left(\sqrt{3+\sqrt{5}}+i\sqrt{3-\sqrt{5}}\right)^{2012}$

Se cere $|z|=?$

$E_1)$ folosim $|z^n|=|z|^n$

$E_2)\ |z|=\left|\left(\sqrt{3+\sqrt{5}}+i\sqrt{3-\sqrt{5}}\right)^{2012}\right|=$

$=\left|\sqrt{3+\sqrt{5}}+i\sqrt{3-\sqrt{5}}\right|^{2012}$

$E_3)\ |z|=\left(\sqrt{\left(\sqrt{3+\sqrt{5}}\right)^2+\left(\sqrt{3-\sqrt{5}}\right)^2}\right)^{2012}$

$=\left(\sqrt{3+\sqrt{5}+3-\sqrt{5}}\right)^{2012}=$

$=\left(\sqrt{6}\right)^{2012}=6^{\frac{2012}{2}}=6^{1006}$

Prof. Ovidiu Badescu

Studiați dacă există $z \in \mathbb{C}$ pentru care:

$$|z-i| = |z+2i| = |iz-1|$$

E_1) not. $z = a+bi$, $a, b \in \mathbb{R}$

$\Rightarrow |a+bi-i| = |a+bi+2i| = |i(a+bi)+1|$

$\Rightarrow |a+(b-1)i| = |a+(b+2)i| = |ai-b-1|$

$\Rightarrow |a+(b-1)i| = |a+(b+2)i| = |(-b-1)+ai|$

E_2) $\sqrt{a^2+(b-1)^2} = \sqrt{a^2+(b+2)^2} = \sqrt{(-b-1)^2+a^2}$ /()^2

$a^2+(b-1)^2 = a^2+(b+2)^2 = (-b-1)^2+a^2$

E_3) $\begin{cases} a^2+(b-1)^2 = a^2+(b+2)^2 \\ a^2+(b-1)^2 = (-b-1)^2+a^2 \end{cases} \Leftrightarrow \begin{cases} (b-1)^2 = (b+2)^2 \\ (b-1)^2 = (-b-1)^2 \end{cases}$

$\Rightarrow \begin{cases} b^2-2b+1 = b^2+4b+4 \Rightarrow -6b = 3 \Rightarrow b = -\frac{1}{2} \\ b^2-2b+1 = b^2+2b+1 \Rightarrow -4b = 0 \Rightarrow b = 0 \end{cases}$

E_4) Dar, b trebuie același $\Rightarrow \nexists b \Rightarrow \nexists z$

Prof. Ovidiu Radescu

Arătați că

$z=(5-4i)^n-(5+4i)^n$ este pur imaginar

$E_1)\ z\in\mathbb{R}^*i$ (pur imaginar) $\Leftrightarrow \overline{z}=-z$

$E_2)\ \overline{z}=\overline{(5-4i)^n-(5+4i)^n}=$

$=\overline{(5-4i)^n}-\overline{(5+4i)^n}=$

$=\left(\overline{5-4i}\right)^n-\left(\overline{5+4i}\right)^n=$

$=(5+4i)^n-(5-4i)^n=$

$=-z$

$E_3)\ \overline{z}=-z \Rightarrow z$ pur imaginar

Prof. Ovidiu Bădescu

Se cere conjugatul a) $\frac{1}{4+3i}$ b) $\frac{1-i}{1+i}$

$E_1)\ z = \frac{4-3i}{4+3i} = \frac{4-3i}{16+9} = \frac{4}{25} - \frac{3}{25}i$

$E_2)\ \bar{z} = \frac{4}{25} + \frac{3}{25}i$

$E_1)\ z = \frac{1-i}{1+i} = \frac{(1-i)^2}{(1+i)(1-i)} = \frac{1-2i+i^2}{1-i^2} = \frac{1-2i-1}{2} =$

$= -\frac{2i}{2} = -i$

$E_2)\ z = -i \Rightarrow \bar{z} = i$

Prof. Ovidiu Badescu

Fie $z = \frac{a+2i}{2+ai}$, $a \in \mathbb{R}$. Se cere $a = ?$ a.î.

$z \in \mathbb{R}$

Soluție:

$$z = \frac{a+2i}{2+ai} \overset{2-ai}{=} \frac{(a+2i)(2-ai)}{4+a^2} = \frac{2a-a^2i+4i+2a}{4+a^2}$$

$$z = \frac{4a+(4-a^2)i}{4+a^2} \Rightarrow z = \frac{4a}{4+a^2} + \frac{4-a^2}{4+a^2}\,i$$

Condiția ca $z \in \mathbb{R} \Leftrightarrow \frac{4-a^2}{4+a^2} = 0 \Rightarrow 4-a^2 = 0$

$\Rightarrow a^2 = 4 \Rightarrow a = \pm 2$

Prof. Ovidiu Bădescu