# IDENTITĂȚI TRIGONOMETRICE NONSTANDARD

Arătați că:

1) $\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}=\frac{2}{3}$

2) $2(\sin^6 x+\cos^6 x)-3(\sin^4 x+\cos^4 x)=-1$

3) $\sin 10^\circ \sin 50^\circ \sin 70^\circ \sin 80^\circ=\frac{\cos 10^\circ}{8}$

4) $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ=\frac{1}{16}$

5) dacă $\sin a+\sin b=1$, și $\cos a+\cos b=\frac{1}{2}$, să se calculeze $\cos(a-b)$

6) $\sin^2 1^\circ+\sin^2 2^\circ+\ldots+\sin^2 90^\circ=\frac{91}{2}$

7) $\cos 1^\circ+\cos 2^\circ+\ldots+\cos 179^\circ=0$

8) $\operatorname{tg} 1^\circ \operatorname{tg} 2^\circ \operatorname{tg} 3^\circ \ldots \operatorname{tg} 89^\circ=1$

9) $\operatorname{ctg} 41^\circ \operatorname{ctg} 42^\circ \ldots \operatorname{ctg} 49^\circ=1$

10) $\frac{\sin 22^\circ \cos 8^\circ+\cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ+\cos 157^\circ \sin 187^\circ}=1$

Prof. Ovidiu Bădescu

Arătați că:

$$\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}=\frac{2}{3}$$

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$E_1)$ $\sin^4 x+\cos^4 x=(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x \Rightarrow$

$$\boxed{\sin^4 x+\cos^4 x=1-2\sin^2 x\cos^2 x} \quad (1)$$

$E_2)$ $\sin^6 x+\cos^6 x=(\sin^2 x)^3+(\cos^2 x)^3=$

$=(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)=$

$=1\cdot(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)$ (2)

Pentru $(\sin^2 x)^3+(\cos^2 x)^3$ s-a folosit formula

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$E_3)$ Dar, $\sin^4 x+\cos^4 x=1-2\sin^2 x\cos^2 x$ /din (1)

și înlocuim în (2) $\Rightarrow$

$$\sin^6 x+\cos^6 x=1-2\sin^2 x\cos^2 x-\sin^2 x\cos^2 x$$

$$\Rightarrow \boxed{\sin^6 x+\cos^6 x=1-3\sin^2 x\cos^2 x}$$

$E_4)$ Înlocuim în relația din enunț $\Rightarrow$

$$\frac{\sin^4 x+\cos^4 x-1}{\sin^6 x+\cos^6 x-1}=\frac{1-2\sin^2 x\cos^2 x-1}{1-3\sin^2 x\cos^2 x-1}=\frac{2\sin^2 x\cos^2 x}{3\sin^2 x\cos^2 x}=\frac{2}{3}$$

Profesor Dănescu

Arătați că:

$$2(\sin^6 x+\cos^6 x)-3(\sin^4 x+\cos^4 x)=-1$$

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$E_1)\ \sin^6 x+\cos^6 x=(\sin^2 x)^3+(\cos^2 x)^3=$

$=(\sin^2 x+\cos^2 x)(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)=$

Știu folosit: $a^3+b^3=(a+b)(a^2-ab+b^2)$ (*)

$=1\cdot(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x)$

$E_2)\ \sin^4 x+\cos^4 x=(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$

$=1-2\sin^2 x\cos^2 x$

$E_3)\ \sin^6 x+\cos^6 x=1\cdot(1-2\sin^2 x\cos^2 x-\sin^2 x\cos^2 x)$

$\sin^6 x+\cos^6 x=1-3\sin^2 x\cos^2 x$

$E_4)\ \sin^4 x+\cos^4 x=1-2\sin^2 x\cos^2 x$ (din $E_2$)

$E_5)\ M_S=2(1-3\sin^2 x\cos^2 x)-3(1-2\sin^2 x\cos^2 x)$

$M_S=2-6\sin^2 x\cos^2 x-3+6\sin^2 x\cos^2 x$

$M_S=-1$

Prof Ovidiu Bagesu

Arătați că:

$$\sin 10^\circ \sin 50^\circ \sin 70^\circ \sin 80^\circ = \frac{\cos 10^\circ}{8}$$

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$E_1$) din $\sin 2x = 2\sin x \cos x \Rightarrow \sin x \cos x = \frac{\sin 2x}{2}$

$E_2$) din $\sin x = \cos(90^\circ - x) \Rightarrow \sin 80^\circ = \cos(90^\circ - 80^\circ) = \cos 10^\circ$

$E_3$) $M_S = \sin 10^\circ \sin 50^\circ \sin 70^\circ \sin 80^\circ =$

$$= \sin 10^\circ \sin 50^\circ \sin 70^\circ \cos 10^\circ =$$

$$= \frac{\sin(2 \cdot 10^\circ)}{2} \sin 50^\circ \sin 70^\circ = \frac{\sin 20^\circ \sin 50^\circ \sin 70^\circ}{2}$$

$E_4$) dar, $\sin 70^\circ = \cos(90^\circ - 70^\circ) = \cos 20^\circ$

$$\Rightarrow M_S = \frac{\sin 20^\circ \sin 50^\circ \cos 20^\circ}{2} = \frac{\frac{\sin(2 \cdot 20^\circ)}{2} \sin 50^\circ}{2} =$$

$$= \frac{\sin 40^\circ \sin 50^\circ}{4}$$

$E_5$) Dar, $\sin 50^\circ = \cos(90^\circ - 50^\circ) = \cos 40^\circ \Rightarrow$

$$M_S = \frac{\sin 40^\circ \cos 40^\circ}{4} = \frac{\frac{\sin(2 \cdot 40^\circ)}{2}}{4} = \frac{\sin 80^\circ}{8}$$

$E_6$) Cum $\sin 80^\circ = \cos(90^\circ - 80^\circ) = \cos 10^\circ$

$$\Rightarrow M_S = \frac{\cos 10^\circ}{8}$$

Prof. Ovidiu Bădescu

Arătați că:

$$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$$

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E1) $N = \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ \mid \cdot 2\cos 10^\circ$

$\Rightarrow 2N\cos 10^\circ = 2\sin 10^\circ \cos 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

E2) Dar, $2\sin x \cos x = \sin 2x$

$\Rightarrow 2\sin 10^\circ \cos 10^\circ = \sin(2\cdot 10^\circ) = \sin 20^\circ$

E3) $2N\cos 10^\circ = \sin 20^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

E4) $\sin 20^\circ = \sin(90^\circ - 70^\circ) = \cos 70^\circ$

$\Rightarrow 2N\cos 10^\circ = \cos 70^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

E5) Dar, $\sin x \cos x = \frac{\sin 2x}{2} \Rightarrow$

$\Rightarrow \sin 70^\circ \cos 70^\circ = \frac{\sin(2\cdot 70^\circ)}{2} = \frac{\sin 140^\circ}{2}$

E6) $2N\cos 10^\circ = \frac{\sin 140^\circ}{2} \cdot \sin 30^\circ \sin 50^\circ$

E7) $\sin 140^\circ = \sin(90^\circ + 50^\circ) = \sin 90^\circ \cos 50^\circ + \cos 90^\circ \sin 50^\circ$

$= 1\cdot \cos 50^\circ + 0 \cdot \sin 50^\circ = \cos 50^\circ$

E8) $2N\cos 10^\circ = \frac{\cos 50^\circ}{2} \cdot \sin 30^\circ \sin 50^\circ =$

$= \frac{1}{2} \cdot \frac{\sin(2\cdot 50^\circ)}{2} \cdot \frac{1}{2} = \frac{1}{8}\sin 100^\circ = \frac{1}{8}\sin(90^\circ + 10^\circ)$

$= \frac{1}{8}\cos 10^\circ \Rightarrow N = \frac{1}{16}$

Prof. Ovidiu Badescu

Dacă $\sin a + \sin b = 1$ și $\cos a + \cos b = \frac{1}{2}$, se cere $\cos(a-b)$

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$E_1)$ $\cos(a-b) = \cos a \cos b + \sin a \sin b$

$E_2)$ Știm că: $\sin a + \sin b = 1 \mid ()^2$

$\sin^2 a + 2 \sin a \sin b + \sin^2 b = 1$

$E_3)$ Știm că: $\cos a + \cos b = \frac{1}{2} \mid ()^2$

$\cos^2 a + 2 \cos a \cos b + \cos^2 b = \frac{1}{4}$

$E_4)$ sistem $\begin{cases} \sin^2 a + 2 \sin a \sin b + \sin^2 b = 1 \\ \cos^2 a + 2 \cos a \cos b + \cos^2 b = \frac{1}{4} \end{cases}$

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$\underbrace{(\sin^2 a + \cos^2 a)}_{1} + 2(\sin a \sin b + \cos a \cos b) + \underbrace{(\sin^2 b + \cos^2 b)}_{1} = \frac{5}{4}$

$E_5)$ $2 + 2 \cdot \cos(a-b) = \frac{5}{4}$

$2\cos(a-b) = \frac{5}{4} - 2$

$2\cos(a-b) = -\frac{3}{4} \mid :2$

$\cos(a-b) = -\frac{3}{8}$

Prof. Ovidiu Badescu

Arătați că:

$$\sin^2 1^\circ + \sin^2 2^\circ + \ldots + \sin^2 90^\circ = \frac{91}{2}$$

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$E_1)$ $M_S = (\sin^2 1^\circ + \sin^2 89^\circ) + (\sin^2 2^\circ + \sin^2 88^\circ) + \ldots$

$$+ (\sin^2 44^\circ + \sin^2 46^\circ) + \sin^2 45^\circ + \sin^2 90^\circ$$

$E_2)$ dar, observăm că $1^\circ + 89^\circ = 90^\circ$, $2^\circ + 88^\circ = 90^\circ$ etc

și calculăm $\sin^2 1^\circ + \sin^2 89^\circ = \sin^2 1^\circ + \sin^2(90^\circ - 1^\circ) =$

$= \sin^2 1^\circ + \cos^2 1^\circ = 1$, deci în cas general

$$\sin^2 x + \sin^2(90^\circ - x) = \sin^2 x + \cos^2 x = 1$$

$E_3)$ $M_S = \underbrace{(1 + 1 + \ldots + 1}_{44 \text{ ori}}) + \left(\frac{\sqrt{2}}{2}\right)^2 + 1^2 =$

$$= 44 + \frac{2}{4} + 1 = 45 + \frac{1}{2} = \frac{91}{2}$$

Prof. Ovidiu Bădescu

Arătați că:

$$\cos 1^\circ + \cos 2^\circ + \dots + \cos 179^\circ = 0$$

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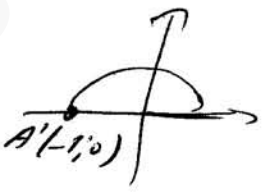
$E_1)$ $M_1 = (\cos 1^\circ + \cos 179^\circ) + (\cos 2^\circ + \cos 178^\circ) + \dots$

$+ (\cos 89^\circ + \cos 91^\circ) + \cos 90^\circ$

$E_2)$ dar, $\cos 1^\circ + \cos 179^\circ = \cos 1^\circ + \cos(180^\circ - 1^\circ) =$

$= \cos 1^\circ + \underbrace{\cos 180^\circ}_{-1} \cos 1^\circ + \underbrace{\operatorname{sen} 180^\circ}_{0} \operatorname{sen} 1^\circ =$

$= \cos 1^\circ + (-1)\cos 1^\circ + 0 = \cos 1^\circ - \cos 1^\circ = 0$




$E_3)$ analog, $\cos x + \cos(180^\circ - x) =$

$= \cos x + (\cos 180^\circ \cos x + \operatorname{sen} 180^\circ \operatorname{sen} x) =$

$= \cos x + (-1 \cdot \cos x + 0 \cdot \operatorname{sen} x) =$

$= \cos x - \cos x = 0$

$E_4)$ $M_1 = \underbrace{0 + 0 + \dots + 0}_{\text{89 ori}} + \cos 90^\circ$

Deoarece $\cos 90^\circ = 0 \Rightarrow M_1 = 0$

Prof. Ovidiu Badescu

Arătați că:

$$ctg\,41^\circ \cdot ctg\,42^\circ \cdot \ldots \cdot ctg\,49^\circ = 1$$

E1) $\boxed{ctg\,\alpha = \frac{1}{tg\,\alpha}} \Rightarrow$

$$\Rightarrow ctg\,41^\circ = \frac{1}{tg\,41^\circ} = \frac{1}{tg(90^\circ - 49^\circ)}$$

E2) folosim $\boxed{tg(90^\circ - \alpha) = ctg\,\alpha}$

$$\Rightarrow ctg\,41^\circ = \frac{1}{ctg\,49^\circ}$$

E3) analog, $ctg\,42^\circ = \frac{1}{ctg\,48^\circ}$, $ctg\,43^\circ = \frac{1}{ctg\,47^\circ}$

etc $\Rightarrow$

E4) $N = ctg\,41^\circ \cdot ctg\,42^\circ \cdots ctg\,49^\circ =$

$= (ctg\,41^\circ \cdot ctg\,49^\circ)(ctg\,42^\circ \cdot ctg\,48^\circ)(ctg\,43^\circ ctg\,47^\circ)$

$(ctg\,44^\circ \cdot ctg\,46^\circ) \cdot ctg\,45^\circ$

$N = \left(\frac{1}{ctg\,49^\circ} \cdot ctg\,49^\circ\right)\left(\frac{1}{ctg\,48^\circ} ctg\,48^\circ\right)\left(\frac{1}{ctg\,47^\circ} ctg\,47^\circ\right) \cdot$

$\cdot \left(\frac{1}{ctg\,46^\circ} ctg\,46^\circ\right) \cdot ctg\,45^\circ$

$N = 1 \cdot ctg\,45^\circ = 1 \cdot 1 = 1$

Prof. Ovidiu Bădescu

Arătați că:

$$\frac{\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ + \cos 157^\circ \sin 187^\circ} = 1$$

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$E_1)$ $\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ = N_1$

$\cos 158^\circ = \cos(180^\circ - 22^\circ) = \cos 180^\circ \cos 22^\circ + \sin 180^\circ \sin 22^\circ =$

$= -1 \cdot \cos 22^\circ + 0 \cdot \sin 22^\circ = -\cos 22^\circ$

$\cos 98^\circ = \cos(90^\circ + 8^\circ) = \cos 90^\circ \cos 8^\circ - \sin 90^\circ \sin 8^\circ =$

$= 0 \cdot \cos 8^\circ - 1 \cdot \sin 8^\circ = -\sin 8^\circ$

$E_2)$ $N_1 = \sin 22^\circ \cos 8^\circ + \cos 22^\circ \sin 8^\circ = \sin(22^\circ + 8^\circ) =$

$= \sin 30^\circ = \frac{1}{2}$

$E_3)$ $N_2 = \sin 23^\circ \cos 7^\circ + \cos 157^\circ \sin 187^\circ$

$\cos 157^\circ = \cos(180^\circ - 23^\circ) = \cos 180^\circ \cos 23^\circ + \sin 180^\circ \sin 23^\circ =$

$= -1 \cdot \cos 23^\circ + 0 \cdot \sin 23^\circ = -\cos 23^\circ$

$\sin 187^\circ = \sin(180^\circ + 7^\circ) = \sin 180^\circ \cos 7^\circ + \cos 180^\circ \sin 7^\circ =$

$= 1 \cdot \cos 7^\circ + (-1) \sin 7^\circ = -\sin 7^\circ$

$E_4)$ $N_2 = \sin 23^\circ \cos 7^\circ + (-\cos 23^\circ)(-\sin 7^\circ) =$

$= \sin 23^\circ \cos 7^\circ + \cos 23^\circ \sin 7^\circ = \sin(23^\circ + 7^\circ) =$

$= \sin 30^\circ = \frac{1}{2}$

$E_5)$ $N = \frac{N_1}{N_2} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

Prof. Ovidiu Bădescu