DEDUCEREA FORMULELOR TRIGONOMETRICE

Deduceți formulele care te

1) $tg(a+b) = \frac{tg a + tg b}{1 - tg a \, tg b}$

2) $tg(a-b) = \frac{tg a - tg b}{1 + tg a \, tg b}$

3) $ctg(a+b) = \frac{ctg a \, ctg b - 1}{ctg b + ctg a}$

4) $ctg(a-b) = \frac{ctg a \, ctg b + 1}{ctg b - ctg a}$

5) $\sin 2x = 2 \sin x \cos x$

6) $\cos 2x = \cos^2 x - \sin^2 x$

7) $\cos 2x = 2\cos^2 x - 1$

8) $\cos 2x = 1 - 2\sin^2 x$

9) $tg\, 2x = \frac{2 tg\, x}{1 - tg^2 x}$

10) $ctg\, 2x = \frac{ctg^2 x - 1}{2 ctg\, x}$

11) $\cos^2 x = \frac{1 + \cos 2x}{2}$

12) $\sin^2 x = \frac{1 - \cos 2x}{2}$

13) $\sin a = \frac{2 tg \frac{a}{2}}{1 + tg^2 \frac{a}{2}}$

14) $\cos a = \frac{1 - tg^2 \frac{a}{2}}{1 + tg^2 \frac{a}{2}}$

15) $tg\, a = \frac{2 tg \frac{a}{2}}{1 + tg^2 \frac{a}{2}}$

16) $\sin a \cos b = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$

17) $\cos a \sin b = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$

18) $\cos a \cos b = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$

19) $\sin a \sin b = -\frac{1}{2}\left(\cos(a+b) - \cos(a-b)\right)$

20) $tg\, a + tg\, b = \frac{\sin(a+b)}{\cos a \cos b}$

21) $tg\, a - tg\, b = \frac{\sin(a-b)}{\cos a \cos b}$

22) $ctg\, a + ctg\, b = \frac{\sin(a+b)}{\sin a \sin b}$

23) $ctg\, a - ctg\, b = \frac{\sin(a-b)}{\sin a \sin b}$

24) $\sin a + \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$

25) $\sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$

26) $\cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$

27) $\cos a - \cos b = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$

Prof. Ovidiu Badescu

Arătați că: $tg(a+b) = \frac{tg a + tg b}{1 - tg a \cdot tg b}$

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met I:

$E_1)\ tg(a+b) = \frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b}$ (1)

$E_2)\ M_d = \frac{tg a + tg b}{1 - tg a \cdot tg b} = \frac{\frac{\sin a}{\cos a} + \frac{\sin b}{\cos b}}{1 - \frac{\sin a}{\cos a} \cdot \frac{\sin b}{\cos b}} =$

$= \frac{\frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b}}{\frac{\cos a \cos b - \sin a \sin b}{\cos a \cos b}} = \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b}$ (2)

Din (1) și (2) $\Rightarrow$ egalitate

met II:

$E_1)\ tg(a+b) = \frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b}$

$E_2)\ tg x = \frac{\sin x}{\cos x} \Rightarrow \sin x = tg x \cdot \cos x$

$E_3)\ tg(a+b) = \frac{tg a \cdot \cos a \cdot \cos b + \cos a \cdot tg b \cdot \cos b}{\cos a \cos b - tg a \cdot \cos a \cdot tg b \cos b} =$

$= \frac{\cos a \cos b (tg a + tg b)}{\cos a \cos b (1 - tg a \cdot tg b)} = \frac{tg a + tg b}{1 - tg a \cdot tg b}$

met III:

$E_1)\ tg(a+b) = \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b} \quad | : \cos a \cos b$

$\frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b} \Big/ \frac{\cos a \cos b - \sin a \sin b}{\cos a \cos b}$

$= \frac{tg a + tg b}{1 - tg a \cdot tg b}$, s-a folosit $\frac{A}{B} = \frac{\frac{A}{C}}{\frac{B}{C}}$ și $\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

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Deduceți că:

$$\operatorname{tg}(a-b) = \frac{\operatorname{tg} a - \operatorname{tg} b}{1 + \operatorname{tg} a \cdot \operatorname{tg} b}$$

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Deducerea formulei

$\sin a + \sin b$ în funcție de $\frac{a+b}{2}$, $\frac{a-b}{2}$

$E_1)$ $a = \frac{a+b}{2} + \frac{a-b}{2}$ și $b = \frac{a+b}{2} - \frac{a-b}{2}$

$E_2)$ $\sin a + \sin b = \sin\left(\frac{a+b}{2} + \frac{a-b}{2}\right) + \sin\left(\frac{a+b}{2} - \frac{a-b}{2}\right)$

$E_3)$ $\sin\left(\frac{a+b}{2} + \frac{a-b}{2}\right) = \sin\frac{a+b}{2}\cos\frac{a-b}{2} + \cos\frac{a+b}{2}\sin\frac{a-b}{2}$

$\sin\left(\frac{a+b}{2} - \frac{a-b}{2}\right) = \sin\frac{a+b}{2}\cos\frac{a-b}{2} - \cos\frac{a+b}{2}\sin\frac{a-b}{2}$

$E_4)$ adunând $\Rightarrow \sin\left(\frac{a+b}{2} + \frac{a-b}{2}\right) + \sin\left(\frac{a+b}{2} - \frac{a-b}{2}\right) =$

$= 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$

$\Rightarrow \boxed{\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}}$

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