PROBLEME FINALE DE TRIGONOMETRIE

Arătați că în $\triangle ABC$ avem:

1) $\cos A+\cos B+\cos C = 1+\frac{r}{R}$

2) $\sin A+\sin B+\sin C = \frac{p}{R}$

3) $\operatorname{tg} A+\operatorname{tg} B+\operatorname{tg} C = \operatorname{tg} A \operatorname{tg} B \operatorname{tg} C$

4) $r = 4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

5) $p = 4R \cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

6) $p-a = 4R \cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

7) $S = 2R^2 \sin A \sin B \sin C$

8) $r = \frac{r_a \cdot r_b \cdot r_c}{2}$

9) $r = \frac{r_a r_b r_c}{r_a r_b + r_b r_c + r_c r_a}$

10) $r_a r_b + r_b r_c + r_c r_a = p^2$

11) $\frac{a^2}{r_a}+\frac{b^2}{r_b}+\frac{c^2}{r_c} = 4(r+R)$

12) $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2} = 1-\frac{r}{2R}$

13) $r_a+r_b+r_c = r+4R$

Prof. Ovidiu Bădescu

Se cere:

$$\sin^4 x + \cos^4 x \geq \frac{1}{4}, \quad \forall x \in (0; \pi)$$

$E_1)$ $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x =$

$$= 1 - 2\sin^2 x \cos^2 x$$

$E_2)$ Dar, $\sin^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \cos^2 \alpha = 1 - \sin^2 \alpha$

$\Rightarrow \sin^4 x + \cos^4 x = 1 - 2\sin^2 \alpha (1 - \sin^2 \alpha)$

$E_3)$ Ar trebui arătat că

$1 - 2\sin^2 \alpha (1 - \sin^2 \alpha) \geq \frac{1}{4} \quad | \cdot 4$

$4 - 8\sin^2 \alpha (1 - \sin^2 \alpha) \geq 1$

$\Leftrightarrow 4 - 8\sin^2 \alpha + 8\sin^4 \alpha - 1 \geq 0$

$\Leftrightarrow 8\sin^4 \alpha - 8\sin^2 \alpha + 4 - 1 \geq 0$

$\Leftrightarrow 2(4\sin^4 \alpha - 4\sin^2 \alpha + 1) - 2 + 3 \geq 0$

$\Leftrightarrow 2(2\sin^2 \alpha - 1)^2 + 1 \geq 0$

adev at că $(2\sin^2 \alpha - 1)^2 \geq 0$ (pătrat perfect)

Prof. Ovidiu Radescu

Calculați valoarea lui $\cos\frac{\pi}{5}$ și a lui $\sin\frac{\pi}{5}$.

$E_1)\ \sin\frac{2\pi}{5}=2\sin\frac{\pi}{5}\cos\frac{\pi}{5}$

$E_2)\ \sin\frac{2\pi}{5}=\sin\left(\pi-\frac{2\pi}{5}\right)=\sin\frac{3\pi}{5}=3\sin\frac{\pi}{5}-4\sin^3\frac{\pi}{5}$

$E_3)$ Egalând $\Rightarrow 2\sin\frac{\pi}{5}\cos\frac{\pi}{5}=3\sin\frac{\pi}{5}-4\sin^3\frac{\pi}{5}\ /:\sin\frac{\pi}{5}\neq 0$

$\Rightarrow 2\cos\frac{\pi}{5}=3-4\sin^2\frac{\pi}{5}\Rightarrow 2\cos\frac{\pi}{5}=3-4\left(1-\cos^2\frac{\pi}{5}\right)$

$E_4)$ not $\cos\frac{\pi}{5}=t\Rightarrow 2t=3-4+4t^2\Rightarrow 4t^2-2t-1=0$

$$t_{1,2}=\frac{2\pm\sqrt{4+16}}{8}=\frac{2\pm 2\sqrt{5}}{8}=\frac{1\pm\sqrt{5}}{4}$$

$E_5)\ \frac{\pi}{5}\in C_dI\Rightarrow\cos\frac{\pi}{5}>0\Rightarrow\cos\frac{\pi}{5}=\frac{1+\sqrt{5}}{4}$

Obs.: s-a folosit $\sin 3x=3\sin x-4\sin^3 x$

not π:

$E_1)\ \cos\frac{2\pi}{5}=2\cos^2\frac{\pi}{5}-1$

$E_2)\ \cos\frac{2\pi}{5}=-\cos\left(\pi-\frac{2\pi}{5}\right)=-\cos\frac{3\pi}{5}=-\left(4\cos^3\frac{\pi}{5}-3\cos\frac{\pi}{5}\right)$

$E_3)$ Egalând și notând $\cos\frac{\pi}{5}=t\Rightarrow 2t^2-1=-4t^3+3t$

$\Rightarrow 4t^3+2t^2-3t-1=0$,

	t^3	t^2	t	t^0
	4	2	−3	−1
−1	4	−2	−1	0

$\Rightarrow (t+1)(4t^2-2t-1)=0$

$E_4)$ $t=-1\Rightarrow\cos\frac{\pi}{5}=-1$ fals

sau

$4t^2-2t-1=0\Rightarrow t_{1,2}=\frac{2\pm 2\sqrt{5}}{8}=\frac{1\pm\sqrt{5}}{4}\xrightarrow{C_dI}\frac{1+\sqrt{5}}{4}$

Rf $\sin\frac{\pi}{5}$, $\sin^2\frac{\pi}{5}+\cos^2\frac{\pi}{5}=1\Rightarrow\sin^2\frac{\pi}{5}=1-\frac{1+2\sqrt{5}+5}{16}=\frac{10-\sqrt{5}}{16}$

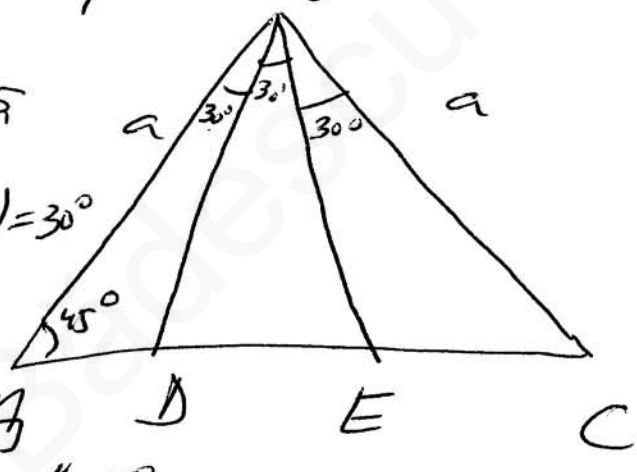
$\sin\frac{\pi}{5}=\sqrt{\frac{5-\sqrt{5}}{8}}$ C_dI

ΔABC dreptunghic isoscel, $AB = AC = a$.

$m(B\hat{A}\Delta) = m(\Delta\hat{A}E) = m(E\hat{A}C)$

Se cere $\Delta E = ?$

(Se știe $\sin 75^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$)



$E_1)$ $m(\hat{A}) = 90^\circ \Rightarrow$ ipoteză

$m(B\hat{A}\Delta) = m(\Delta\hat{A}E) = m(E\hat{A}C) = 30^\circ$

$E_2)$ ΔABC dreptunghic isoscel $\Rightarrow m(A\hat{B}C) = m(A\hat{C}B) = 45^\circ$

$E_3)$ în $\Delta AB\Delta$ $\xrightarrow{T\ sinus}$ $\frac{B\Delta}{\sin 30^\circ} = \frac{a}{\sin 105^\circ}$

Dar, $\sin 105^\circ = \sin(180^\circ - 105^\circ) = \sin 75^\circ$

$\Rightarrow \frac{B\Delta}{\frac{1}{2}} = \frac{a}{\frac{\sqrt{6}+\sqrt{2}}{4}} \Rightarrow B\Delta = \frac{a \cdot \frac{1}{2}}{\frac{\sqrt{6}+\sqrt{2}}{4}} \Rightarrow$

$\Rightarrow B\Delta = \frac{2a}{\sqrt{6}+\sqrt{2}} = \frac{2a(\sqrt{6}-\sqrt{2})}{4} = \frac{a(\sqrt{6}-\sqrt{2})}{2}$

$E_4)$ ΔABC dreptunghic isoscel $\Rightarrow BC = a\sqrt{2}$

$\Rightarrow \Delta E = BC - 2B\Delta = a\sqrt{2} - a(\sqrt{6}-\sqrt{2})$

$= a\sqrt{2} - a\sqrt{6} + a\sqrt{2} = 2a\sqrt{2} - a\sqrt{6}$

Prof. Ovidiu Bădescu