CALCULUL ARIILOR ȘI AL VOLUMELOR PENTRU TRIUNGHI ȘI PARALELOGRAM

1) $a=4, b=6, c=10$. Se cere R

2) $\sin A=\frac{1}{2}$, $\sin B=1$, $BC=4$, se cere $\mathcal{A}_{\Delta ABC}$

3) $BC=8\sqrt{3}$, $m(\hat{A})=30^\circ$, $m(\hat{C})=60^\circ$ se cere $\mathcal{A}_{\Delta ABC}$

4) $ABCD$ paralelogram cu $AB=6$, $AD=8$, $m(A\hat{D}C)=105^\circ$ Se cere $\mathcal{A}_{ABCD}$

5) ΔABC dreptunghic cu ipotenuza a, se cere $a^2 \sin B \sin C = 2S$

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ΔABC, $a=4$, $b=6$, $c=10$. Se cere $r=?$

$E_1)$ $r=\frac{S}{p}$ unde S aria, p semiperimetrul

$E_2)$ $S=\sqrt{p(p-a)(p-b)(p-c)}$ unde $p=\frac{a+b+c}{2}$

$\Rightarrow p=\frac{4+6+10}{2}=10$

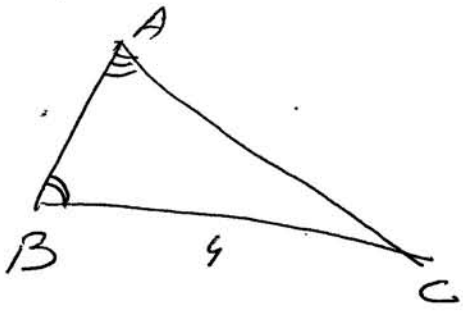
$S=\sqrt{10\cdot 6\cdot 4\cdot 0}=0$

$E_3)$ $r=\frac{S}{p}=\frac{0}{10}=0$

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ΔABC cu $\sin A = \frac{1}{2}$, $\sin B = 1$, $BC = 4$.

Se cere $A_{\Delta ABC}$



E_1) încercăm T. sinusurilor

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

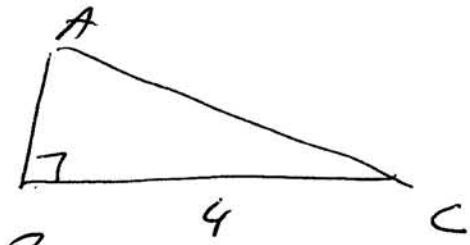
$$\Rightarrow \frac{4}{\frac{1}{2}} = \frac{b}{1} = \frac{c}{\sin C} = 2R$$

E_2) $\Rightarrow 4 \cdot 1 = b \cdot \frac{1}{2} \Rightarrow 4 = \frac{b}{2} \Rightarrow \boxed{b = 8}$

$\Rightarrow$ nu ajută

E_3) Dacă $\sin B = 1$ deducem că $B = 90^\circ$

$\Rightarrow \Delta ABC$ dreptunghic



E_4) $\sin A = \frac{1}{2} \Rightarrow \frac{BC}{AC} = \frac{1}{2}$

$\Rightarrow \frac{4}{AC} = \frac{1}{2} \Rightarrow \boxed{AC = 8}$

E_5) T. Pitagora $\Rightarrow AC^2 = AB^2 + BC^2$

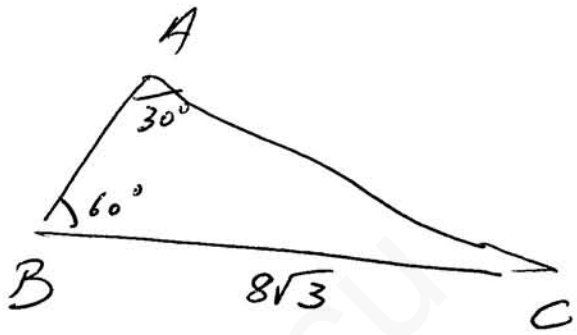
$\Rightarrow 64 = AB^2 + 16 \Rightarrow AB^2 = 48 \Rightarrow \boxed{AB = 4\sqrt{3}}$

E_6) $A_{\Delta ABC} = \frac{AB \cdot BC \sin B}{2} = \frac{4\sqrt{3} \cdot 4}{2} = 8\sqrt{3}$

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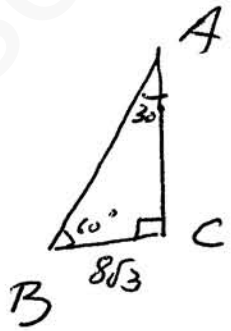
ΔABC cu $BC = 8\sqrt{3}$, $m(\hat{A}) = 30^\circ$, $m(\hat{C}) = 60^\circ$

Se cere $A_{\Delta ABC}$



E_1) $\hat{A} + \hat{B} + \hat{C} = 180^\circ \Rightarrow$

$m(\hat{C}) = 90^\circ \Rightarrow \Delta ABC$ dreptunghic în $C \Rightarrow$ refacem figura



E_2) T sinusurilor $\Rightarrow$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\Rightarrow \frac{8\sqrt{3}}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{1}$$

$$\Rightarrow 16\sqrt{3} = \frac{2b}{\sqrt{3}} = c \Rightarrow c = 16\sqrt{3}$$

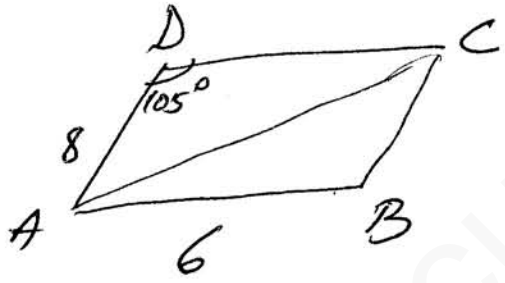
E_3) Din $\frac{2b}{\sqrt{3}} = c \Rightarrow b = \frac{c\sqrt{3}}{2} = \frac{16\sqrt{3} \cdot \sqrt{3}}{2} = 8 \cdot 3 = 24$

E_4) $A_{\Delta ABC} = \frac{BC \cdot AC \cdot \sin C}{2} = \frac{8\sqrt{3} \cdot 24 \cdot 1}{2} = 96\sqrt{3}$

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Se cere aria paralelogram $ABCD$ dacă

$AB=6,\ AD=8,\ m(\widehat{ADC})=105^\circ$



$E_1)$ $ABCD$ paralelogram

$AB=6 \Rightarrow DC=6$

$E_2)$ $\mathcal{A}_{ABCD}=2\mathcal{A}_{ADC}=2\cdot\dfrac{AD\cdot DC\cdot\sin D}{2}=$

$=8\cdot 6\cdot\sin 105^\circ$

$E_3)$ $\sin 105^\circ=\sin(60^\circ+45^\circ)=$

$=\sin 60^\circ\cos 45^\circ+\cos 60^\circ\sin 45^\circ=$

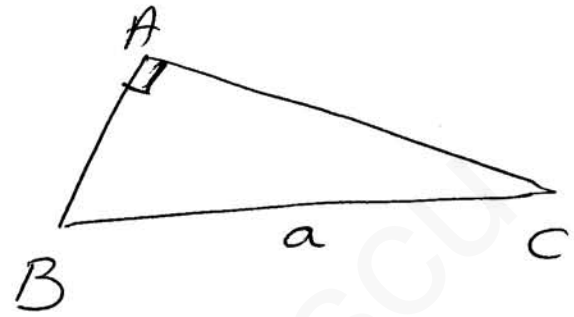
$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2}+\dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

$E_4)$ $\mathcal{A}_{ABCD}=48\cdot\dfrac{\sqrt{6}+\sqrt{2}}{4}=12(\sqrt{6}+\sqrt{2})$

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Arătați că dacă ΔABC are aria S, ipotenuza a avem:

$$a^2 \sin B \sin C = 2S$$



$E_1)$ $\sin B = \frac{AC}{a}$

$\sin C = \frac{AB}{a}$

$E_2)$ $a^2 \sin B \sin C = a^2 \cdot \frac{AC}{a} \cdot \frac{AB}{a} = AC \cdot AB \quad (1)$

$E_3)$ $S = \frac{AC \cdot AB \sin A}{2} = \frac{AC \cdot AB \sin 90^\circ}{2} = \frac{AC \cdot AB}{2}$

$2S = 2 \cdot \frac{AC \cdot AB}{2} = AC \cdot AB \quad (2)$

$E_4)$ Din $(1)+(2) \Rightarrow a^2 \sin B \sin C = 2S$

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