EXPRIMAREA UNOR VECTORI ÎN FUNCȚIE DE DOI VECTORI DAȚI

Fie $ABCDEF$ un hexagon regulat și punctele $M, N, P, Q, R, S, T, U, V, G, H, I, J, K, L$ astfel încât $\overrightarrow{AB}=\frac{2}{3}\overrightarrow{BM}$; $\overrightarrow{BC}=-\frac{1}{5}\overrightarrow{CN}$; $\overrightarrow{CP}=-\frac{3}{2}\overrightarrow{PD}$; $\overrightarrow{DQ}=-\frac{1}{4}\overrightarrow{QE}$; $\overrightarrow{EF}=\frac{1}{4}\overrightarrow{FR}$; $\overrightarrow{SF}=-\frac{2}{3}\overrightarrow{FA}$; $\overrightarrow{TA}=\frac{1}{2}\overrightarrow{TB}$; $\overrightarrow{UC}=-\frac{2}{3}\overrightarrow{UD}$; $\overrightarrow{VE}=\frac{1}{4}\overrightarrow{ED}$; $\overrightarrow{EG}=\frac{5}{3}\overrightarrow{GF}$; $\overrightarrow{HF}=\frac{1}{3}\overrightarrow{HA}$; $\overrightarrow{IB}=\frac{2}{3}\overrightarrow{AI}$; $\overrightarrow{BJ}=\frac{2}{3}\overrightarrow{JC}$; $\overrightarrow{DK}=-\frac{3}{2}\overrightarrow{DE}$ și $\overrightarrow{EL}=\frac{4}{3}\overrightarrow{DL}$. Situând în fiecare caz doar punctele care ne interesează, exprimați pe:

1) $\overrightarrow{MN}=?\overrightarrow{AF}+?\overrightarrow{AE}$
2) $\overrightarrow{PQ}=?\overrightarrow{AD}+?\overrightarrow{AB}$
3) $\overrightarrow{RS}=?\overrightarrow{BC}+?\overrightarrow{BD}$
4) $\overrightarrow{UT}=?\overrightarrow{AB}+?\overrightarrow{CD}$
5) $\overrightarrow{GV}=?\overrightarrow{CD}+?\overrightarrow{CE}$
6) $\overrightarrow{HI}=?\overrightarrow{DE}+?\overrightarrow{BC}$
7) $\overrightarrow{JK}=?\overrightarrow{EB}+?\overrightarrow{AE}$
8) $\overrightarrow{LA}=?\overrightarrow{ED}+?\overrightarrow{AC}$
9) $\overrightarrow{MS}=?\overrightarrow{BC}+?\overrightarrow{BD}$
10) $\overrightarrow{UH}=?\overrightarrow{DE}+?\overrightarrow{AC}$
11) $\overrightarrow{VR}=?\overrightarrow{AD}+?\overrightarrow{AF}$

1) $ABCDEF$ hexagon regulat cu $\overrightarrow{AM}=-\frac{3}{2}\overrightarrow{MB}$, $\overrightarrow{AN}=\frac{1}{4}\overrightarrow{AF}$, să se scrie $\overrightarrow{MN}=?\overrightarrow{BD}+?\overrightarrow{BE}$

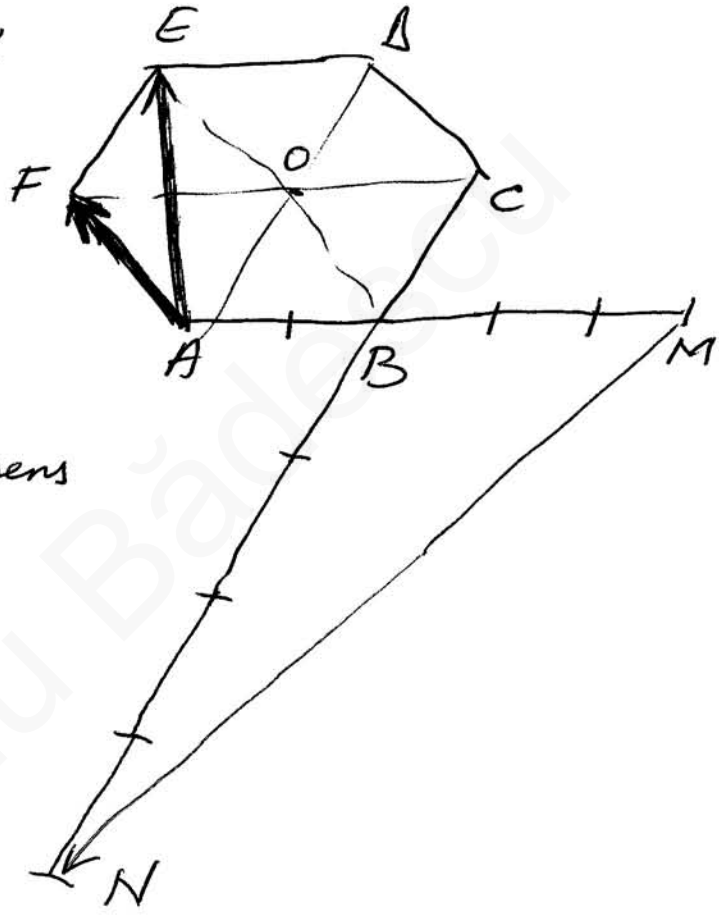
2) $ABCDEF$ hexagon regulat cu $\overrightarrow{CP}=\frac{2}{5}\overrightarrow{CD}$ și $\overrightarrow{DE}=\frac{5}{2}\overrightarrow{EQ}$, să se scrie $\overrightarrow{QP}=?\overrightarrow{CE}+?\overrightarrow{CF}$

Prof. Ovidiu Bădescu

Fie $ABCDEF$ hexagon regulat și M, N cu

$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{BM}$ și $\overrightarrow{BC} = -\frac{1}{5}\overrightarrow{CN}$.

Se cere $\overrightarrow{MN} = ?\,\overrightarrow{AF} + ?\,\overrightarrow{AE}$



$E_1)$ $\overrightarrow{AB} = \frac{2}{3}\overrightarrow{BM} \Rightarrow M \in AB$ și $\overrightarrow{AB}, \overrightarrow{BM}$ același sens și ordinea $A-B-M$

$E_2)$ $\overrightarrow{BC} = -\frac{1}{5}\overrightarrow{CN} \Rightarrow \overrightarrow{BC} = \frac{1}{5}\overrightarrow{NC}$

$\Rightarrow N \in BC$ și $\overrightarrow{BC}, \overrightarrow{NC}$ au ac sens și ordinea $N-B-C$

$E_3)$ $\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN} =$

$= \frac{3}{2}\overrightarrow{BA} + \frac{4}{1}\overrightarrow{CB} =$

$= \frac{3}{2}(\overrightarrow{BE} + \overrightarrow{EA}) + 4\overrightarrow{EF} =$

$= \frac{3}{2}(2\overrightarrow{BO} - \overrightarrow{AE}) + 4(-\overrightarrow{AE} + \overrightarrow{AF}) =$

$= 3\overrightarrow{BO} - \frac{3}{2}\overrightarrow{AE} - 4\overrightarrow{AE} + 4\overrightarrow{AF} =$

$= 3\overrightarrow{AF} - (\frac{3}{2} + 4)\overrightarrow{AE} + 4\overrightarrow{AF} = 7\overrightarrow{AF} - \frac{11}{2}\overrightarrow{AE}$

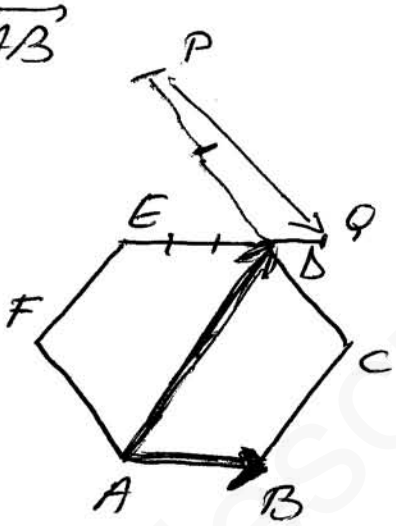
$E_4)$ $\overrightarrow{MN} = 7\overrightarrow{AF} - \frac{11}{2}\overrightarrow{AE}$

Prof. Ovidiu Bădescu

Fie $ABCDEF$ hexagon regulat și P, Q cu

$\overrightarrow{CP} = -\frac{3}{2}\overrightarrow{PD}$ și $\overrightarrow{DQ} = -\frac{1}{4}\overrightarrow{QE}$

Se cere $\overrightarrow{PQ} = ?\overrightarrow{AD} + ?\overrightarrow{AB}$



$E_1)\ \overrightarrow{CP} = -\frac{3}{2}\overrightarrow{PD} \Rightarrow \overrightarrow{CP} = \frac{3}{2}\overrightarrow{DP}$

$\Rightarrow P \in CD$ în ordine $C-D-P$

$E_2)\ \overrightarrow{DQ} = -\frac{1}{4}\overrightarrow{QE} \Rightarrow \overrightarrow{DQ} = \frac{1}{4}\overrightarrow{EQ}$

$\Rightarrow Q \in DE$ în ordine $E-D-Q$

$E_3)\ \overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DQ} = 2\overrightarrow{DC} + \frac{1}{3}\overrightarrow{ED} =$

$= 2(\overrightarrow{DA} + \overrightarrow{AB} + \overrightarrow{BC}) + \frac{1}{3}\overrightarrow{AB} =$

$= -2\overrightarrow{AD} + 2\overrightarrow{AB} + \overrightarrow{AD} + \frac{1}{3}\overrightarrow{AB}$

$= -\overrightarrow{AD} + \frac{7}{3}\overrightarrow{AB}$

Prof. Ovidiu Badescu

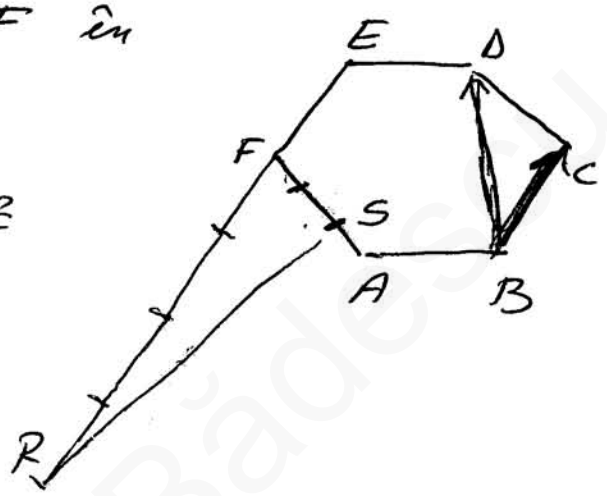
Fie $ABCDEF$ hexagon regulat și R, S cu $\overrightarrow{EF} = \frac{1}{4}\overrightarrow{FR}$ și $\overrightarrow{SF} = -\frac{2}{3}\overrightarrow{FA}$.

Se cere $\overrightarrow{RS} = ?\overrightarrow{BC} + ?\overrightarrow{BD}$

$E_1)$ $\overrightarrow{EF} = \frac{1}{4}\overrightarrow{FR} \Rightarrow R \in EF$ în ordinea $E-F-R$

$E_2)$ $\overrightarrow{SF} = -\frac{2}{3}\overrightarrow{FA} \Rightarrow \overrightarrow{SF} = \frac{2}{3}\overrightarrow{AF}$

$\Rightarrow S \in AF$ în ordinea $A-S-F$



$E_3)$ $\overrightarrow{RS} = \overrightarrow{RF} + \overrightarrow{FS} =$

$= 4\overrightarrow{FE} + \frac{2}{3}\overrightarrow{FA} = 4\overrightarrow{BC} + \frac{2}{3}\overrightarrow{DC} = 4\overrightarrow{BC} + \frac{2}{3}(-\overrightarrow{BD} + \overrightarrow{BC})$

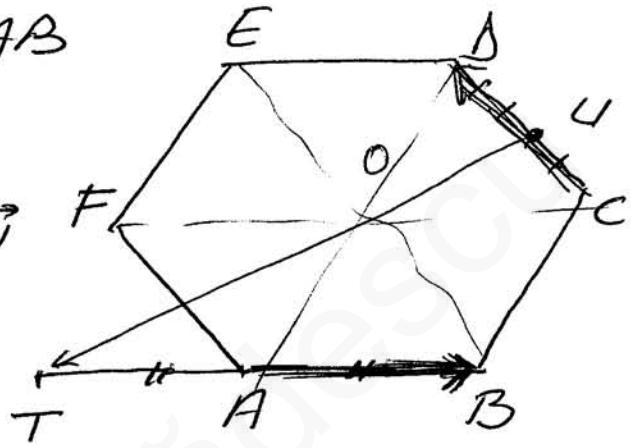
$= \frac{3}{4}\overrightarrow{BC} - \frac{2}{3}\overrightarrow{BD} + \frac{2}{3}\overrightarrow{BC} = \frac{14}{3}\overrightarrow{BC} - \frac{2}{3}\overrightarrow{BD}$

$$\overrightarrow{RS} = \frac{14}{3}\overrightarrow{BC} - \frac{2}{3}\overrightarrow{BD}$$

Prof. Ovidiu Bădescu

Fie $ABCDEF$ hexagon regulat și punctele U și T cu $\overrightarrow{TA}=\frac{1}{2}\overrightarrow{TB}$ și $\overrightarrow{UC}=-\frac{2}{3}\overrightarrow{UD}$.

Se cere $\overrightarrow{UT}=?\overrightarrow{AB}+?\overrightarrow{CD}$



$E_1)$ $\overrightarrow{TA}=\frac{1}{2}\overrightarrow{TB}\Rightarrow T\in AB$

în ordinea $T-A-B$

$E_2)$ $\overrightarrow{UC}=-\frac{2}{3}\overrightarrow{UB}\Rightarrow\overrightarrow{UC}=\frac{2}{3}\overrightarrow{DU}$

$\Rightarrow U\in CD$ în ordinea

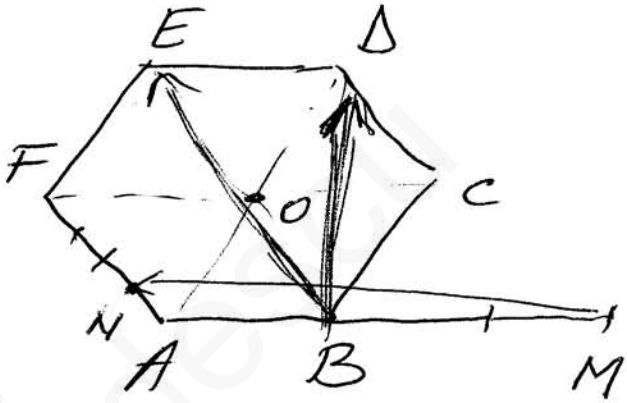
$E_3)$ $\overrightarrow{UT}=\overrightarrow{UC}+\overrightarrow{CB}+\overrightarrow{BT}=$

$=\frac{2}{5}\overrightarrow{DC}+(\overrightarrow{CO}+\overrightarrow{OB})+2\overrightarrow{BA}=-\frac{2}{5}\overrightarrow{CD}-\overrightarrow{AB}-\overrightarrow{CD}-2\overrightarrow{AB}=$ 5/

$=-3\overrightarrow{AB}-\frac{7}{5}\overrightarrow{CD}$

Prof. Ovidiu Bădescu

Fie $ABCDEF$ hexagon regulat, $\overrightarrow{AM} = -\frac{3}{2}\overrightarrow{MB}$, $\overrightarrow{AN} = \frac{1}{4}\overrightarrow{AF}$, să se scrie $\overrightarrow{MN} = ?\,\overrightarrow{BA} + ?\,\overrightarrow{BE}$



$E_1)$ $\overrightarrow{AM} = -\frac{3}{2}\overrightarrow{MB} \Rightarrow$

$\Rightarrow \overrightarrow{AM} = \frac{3}{2}\overrightarrow{BM} \Rightarrow M \in AB$ în exterior

$E_2)$ $\overrightarrow{AN} = \frac{1}{4}\overrightarrow{AF} \Rightarrow N \in AF$ în interior

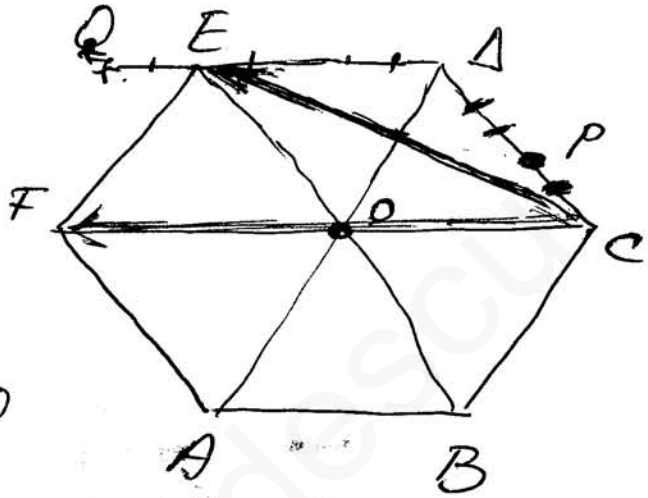
$E_3)$ $\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN} = 3\overrightarrow{BA} + \frac{1}{4}\overrightarrow{AF} =$

$= 3\overrightarrow{DE} + \frac{1}{4}\overrightarrow{CD} = 3(\overrightarrow{DB} + \overrightarrow{BE}) + \frac{1}{4}\cdot\frac{\overrightarrow{BE}}{2} =$

$= -3\overrightarrow{BD} + 3\overrightarrow{BE} + \frac{1}{8}\overrightarrow{BE} \overset{8/}{=} -3\overrightarrow{BD} + \frac{25}{8}\overrightarrow{BE}$

Prof. Ovidiu Bădescu

Fie $ABCDEF$ hexagon regulat cu $\overrightarrow{CP}=\frac{2}{5}\overrightarrow{CD}$ și $\overrightarrow{DE}=\frac{5}{2}\overrightarrow{EQ}$, exprimați $\overrightarrow{QP}$ în funcție de $\overrightarrow{CE}$ și $\overrightarrow{CF}$



$E_1)$ $\overrightarrow{CP}=\frac{2}{5}\overrightarrow{CD}\Rightarrow$

ordinea este $C-P-D$

$E_2)$ $\overrightarrow{DE}=\frac{5}{2}\overrightarrow{EQ}\Rightarrow$

ordinea este $D-E-Q$

$E_3)$ $\overrightarrow{QP}=\overrightarrow{QD}+\overrightarrow{DP}=\frac{7}{5}\overrightarrow{ED}+\frac{3}{5}\overrightarrow{DC}=$

$=\frac{7}{5}\cdot\left(-\frac{1}{2}\overrightarrow{CF}\right)+\frac{3}{5}\left(\overrightarrow{DE}+\overrightarrow{EC}\right)=$

$=-\frac{7}{10}\overrightarrow{CF}+\frac{3}{5}\left(\frac{1}{2}\overrightarrow{CF}-\overrightarrow{CE}\right)=$

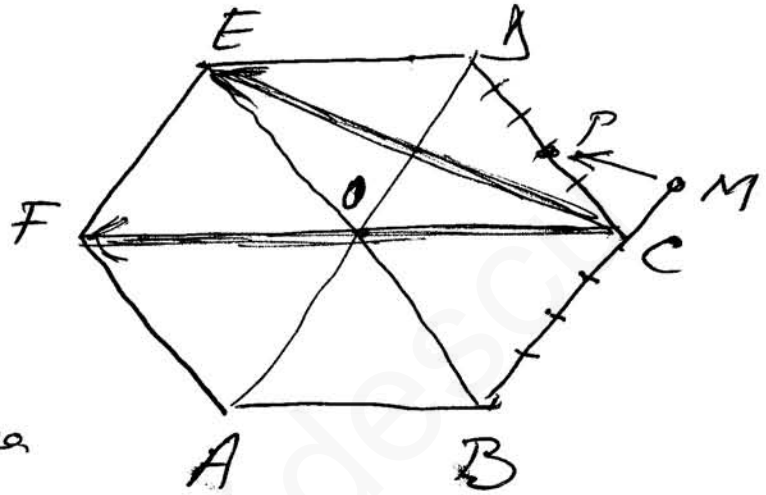
$=-\frac{7}{10}\overrightarrow{CF}+\frac{3}{10}\overrightarrow{CF}-\frac{3}{5}\overrightarrow{CE}=$

$=-\frac{4}{10}\overrightarrow{CF}-\frac{3}{5}\overrightarrow{CE}=-\frac{3}{5}\overrightarrow{CE}-\frac{2}{5}\overrightarrow{CF}$

$E_4)$ $\boxed{\overrightarrow{QP}=-\frac{3}{5}\overrightarrow{CE}-\frac{2}{5}\overrightarrow{CF}}$

Prof. Ovidiu Bădescu

Fie $ABCDEF$ hexagon regulat cu $\overrightarrow{BC} = -\frac{4}{5}\overrightarrow{MB}$ si $\overrightarrow{CP} = \frac{2}{5}\overrightarrow{CD}$. Exprimati $\overrightarrow{MP}$ in functie de $\overrightarrow{CE}$ si $\overrightarrow{CF}$



$E_1)$ $\overrightarrow{BC} = -\frac{4}{5}\overrightarrow{MB}$

$\Rightarrow \overrightarrow{BC} = \frac{4}{5}\overrightarrow{BM} \Rightarrow$ ordinea $B-C-M$

$\overrightarrow{CP} = \frac{2}{5}\overrightarrow{CD} \Rightarrow$ ordinea este $C-P-D$

$E_2)$ $\overrightarrow{MP} = \overrightarrow{MC} + \overrightarrow{CP} = \frac{1}{4}\overrightarrow{CB} + \frac{2}{5}\overrightarrow{CD} = \frac{1}{4}\overrightarrow{EF} + \frac{2}{5}(\overrightarrow{CE} + \overrightarrow{ED})$

$= \frac{1}{4}(-\overrightarrow{CE} + \overrightarrow{CF}) + \frac{2}{5}(\overrightarrow{CE} + (-\frac{1}{2}\overrightarrow{CF})) =$

$= -\frac{1}{4}\overrightarrow{CE} + \frac{1}{4}\overrightarrow{CF} + \frac{2}{5}\overrightarrow{CE} - \frac{1}{5}\overrightarrow{CF}$

$= (-\frac{5}{1}/\frac{1}{4} + \frac{4}{2}/\frac{2}{5})\overrightarrow{CE} + (\frac{1}{4} - \frac{1}{5})\overrightarrow{CF} \Rightarrow$

$$\boxed{\overrightarrow{MP} = \frac{3}{20}\overrightarrow{CE} + \frac{1}{20}\overrightarrow{CF}}$$

Prof. Dinu Radescu