CALCULAREA EXPRESIILOR SIMETRICE REFERITOARE LA RĂDĂCINILE EC. DE GR II

Dacă x_1, x_2 sunt rădăcinile ecuației $2x^2 - x - 2 = 0$, fără a rezolva ecuația, calculați:

1) $E = \frac{x_1+2}{3x_1-1} + \frac{x_2+2}{3x_2-1}$

2) $E = \frac{1}{x_1+1} + \frac{1}{x_2+1}$

3) $E = \frac{1}{x_1^2+3x_1+2} + \frac{1}{x_2^2+3x_2+2}$

4) $E = \frac{x_1^2-5x_1+6}{x_1^2+4x_1+7} + \frac{x_2^2-5x_2+6}{x_2^2+4x_2+7}$

5) $E = \frac{x_1^3+x_1-1}{x_1^3+x_1+1} + \frac{x_2^3+x_2-1}{x_2^3+x_2+1}$

6) $E = \frac{x_1^4+x_1^2+2}{x_1^4-x_1^2+1} + \frac{x_2^4+x_2^2+2}{x_2^4-x_2^2+1}$

7) $E = \frac{x_1+3}{3x_1^2+x_1-1} + \frac{x_2+3}{3x_2^2+x_2-1}$

8) $E = \frac{2x_1^2-x_1+2}{x_2^2+3} + \frac{2x_2^2-x_2+2}{x_2^2+3}$

9) $E = \frac{3x_1^2+4x_1-1}{2x_1^2-x_1+2} + \frac{3x_2^2+4x_2-1}{2x_2^2-x_2+2}$

10) $E = \frac{2x_1+4}{x_1^2+x_1} + \frac{2x_2+4}{x_2^2+x_2}$

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Dacă x_1, x_2 sunt rădăcinile ecuației $2x^2-x-2=0$, fără a rezolva ecuația, calculați

$$E=\frac{x_1^4+x_1^2+2}{x_1^4-x_1^2-1}+\frac{x_2^4+x_2^2+2}{x_2^4-x_2^2+1}$$

$E_1)$ x_1 rad $\Rightarrow 2x_1^2-x_1-2=0\Rightarrow 2x_1^2=x_1+2\Rightarrow$

$$\Rightarrow \boxed{x_1^2=\frac{x_1}{2}+1} \quad (1)$$

$E_2)$ din $x_1^2=\frac{x_1}{2}+1 \mid \cdot x_1 \Rightarrow x_1^3=\frac{x_1^2}{2}+x_1 \overset{(1)}{\Rightarrow}$

$$x_1^3=\frac{\frac{x_1}{2}+1}{2}+x_1\Rightarrow x_1^3=\frac{x_1+2}{4}+x_1\Rightarrow \boxed{x_1^3=\frac{5x_1+2}{4}}$$

$E_3)$ din $x_1^3=\frac{5x_1+2}{4} \mid \cdot x_1 \Rightarrow x_1^4=\frac{5x_1^2+2x_1}{4} \overset{(1)}{\Rightarrow}$

$$x_1^4=\frac{5\left(\frac{x_1}{2}+1\right)+2x_1}{4}=\frac{\frac{5x_1+10}{2}+2x_1}{4}\Rightarrow \boxed{x_1^4=\frac{9x_1+10}{8}}$$

$E_4)$ analog, $x_2^2=\frac{x_2}{2}+1$, $x_2^3=\frac{5x_2+2}{2}$ și $x_2^4=\frac{9x_2+10}{8}\Rightarrow$

$E_5)$
$$E=\frac{\frac{9x_1+10}{8}+\frac{x_1}{2}+1}{\frac{9x_1+10}{8}-\frac{x_1}{2}+1}+\frac{\frac{9x_2+10}{8}+\frac{x_2}{2}+1}{\frac{9x_2+10}{8}-\frac{x_2}{2}+1}=\frac{9x_1+10+4x_1+8}{9x_1+10-4x_1+8}+\frac{9x_2+10+4x_2+8}{9x_2+10+4x_2+8}$$

$$E=\frac{13x_1+18}{5x_1+18}+\frac{13x_2+18}{5x_2+18}=\frac{(13x_1+18)(5x_2+18)+(13x_2+18)(5x_1+18)}{(5x_1+18)(5x_2+18)}$$

$$E=\frac{65x_1x_2+234x_1+90x_2+324+65x_2x_1+234x_2+90x_1+324}{25x_1x_2+90x_1+90x_2+324}$$

$$E=\frac{130x_1x_2+324(x_1+x_2)+648}{25x_1x_2+90(x_1+x_2)+324}\Rightarrow \boxed{E=\frac{130P+324S+648}{25P+90S+324}}$$

$E_6)$ Din VIETE, $S=\frac{1}{2}$, $P=-1\Rightarrow E=\frac{-130+162+648}{-25+45+324}=\frac{680^{(4}}{344}$

$$\Rightarrow E=\frac{170^{(2}}{86}=\frac{85}{43}\Rightarrow \boxed{E=\frac{85}{43}}$$