ECUAȚII CU MODULE

Rezolvați:

1) $|5-x|-|3-x|=x+6$

2) $|2-x|-|x+3|=3x+1$

3) $|5-x|-|x-4|+|x+1|=3x+2$

4) $|x-2|=4$

5) $2x^2-3|x|+1=0$

6) $|2+|x-3|+x|=7x-8$

7) $|4x-5+|2-3x||=4x+3$

8) $|x|+|x-1|+|x-2|=3$

9) $|x-1|+|x+3|+3x-8=0$

10) $x+1-|5x-2|+|3x-5|=9x-4$

11) $|1-x|-|x-4|=5$

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Rezolvați:

$$|5-x|-|3-x|=x+6$$

E1) $5-x=0 \Rightarrow x=5$

$3-x=0 \Rightarrow x=3$

E2)

x		3		5	
$5-x$	$+$	$+$	$+$	0	$-$
$3-x$	$+$	0	$-$	$-$	$-$

E3) Caz I: $x \in (-\infty; 3]$

$\Rightarrow (5-x)-(3-x)=x+6 \Rightarrow 5-x-3+x=x+6$

$\Rightarrow 2=x+6 \Rightarrow x=-4 \in (-\infty; 3] \Rightarrow S_I=\{-4\}$

Caz II: $x \in (3; 5]$

$\Rightarrow 5-x+(3-x)=x+6 \Rightarrow -2x+8=x+6$

$\Rightarrow -3x=-2 \Rightarrow x=\frac{2}{3} \notin (3; 5] \Rightarrow S_{II}=\emptyset$

Caz III: $x \in (5; \infty)$

$\Rightarrow -(5-x)+(3-x)=x+6 \Rightarrow -5+x+3-x=x+6$

$\Rightarrow -2=x+6 \Rightarrow x=-8 \notin (5; \infty) \Rightarrow S_{III}=\emptyset$

E4) $S_g = S_I \cup S_{II} \cup S_{III} = \{-4\}$

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Rezolvați:

$$|2-x|-|x+3|=3x+1$$

E_1) explicităm modulul

$2-x=0 \Rightarrow x=2$

$x+3=0 \Rightarrow x=-3$

x		-3		2	
$2-x$	$+\ +\ +\ +$	0	$-\ -\ -$		
$x+3$	$-\ -\ -\ -$	0	$+\ +\ +$		$+\ +\ +$

E_2) Cob I: $x \in (-\infty, -3]$

$\Rightarrow 2-x+x+3=3x+1 \Rightarrow 5=3x+1$

$\Rightarrow 3x=4 \Rightarrow x=\frac{4}{3} \notin (-\infty,-3]$

$\Rightarrow S_I=\emptyset$

Cob II: $x \in (-3, 2]$

$\Rightarrow (2-x)-(x+3)=3x+1 \Rightarrow -2x-1=3x+1$

$-5x=2 \Rightarrow x=-\frac{2}{5} \in (-3,2] \Rightarrow S_{II}=\{-\frac{2}{5}\}$

Cob III: $x \in (2, \infty)$

$\Rightarrow -(2-x)-(x+3)=3x+1$

$-2+x-x-3=3x+1 \Rightarrow -3x=6$

$x=-2 \notin (2,\infty) \Rightarrow S_{III}=\emptyset$

E_3) $S_f = S_I \cup S_{II} \cup S_{III} = \{-\frac{2}{5}\}$

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9.11. Rezolvați:

$$|5-x|-|x-4|+|x+1|=3x+2$$

E_1) $5-x=0 \Rightarrow x=5$

$x-4=0 \Rightarrow x=4$

$x+1=0 \Rightarrow x=-1$

E_2)

x		-1		4		5	
$5-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$x-4$	$-$	$-$	$-$	$-$	0	$+$	$+$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$

E_3] Cob I: $x \in (-\infty; -1] \Rightarrow (5-x)+(x-4)-(x+1) = 3x+2$

$5-x+x-4-x-1-3x-2=0 \Rightarrow -4x=2 \Rightarrow x=-\frac{1}{2} \notin (-\infty; -1]$

$\Rightarrow S_I = \emptyset$

Cob II: $x \in (-1; 4] \Rightarrow (5-x)+(x-4)+(x+1)=3x+2$

$5-x+x-4+x+1-3x-2=0 \Rightarrow -2x=0 \Rightarrow x=0 \in (-1; 4]$

$\Rightarrow S_{II} = \{0\}$

Cob III: $x \in (4; 5] \Rightarrow (5-x)-(x-4)+(x+1)=3x+2$

$\Rightarrow 5-x-x+4+x+1-3x-2=0 \Rightarrow -4x=-8$

$\Rightarrow x=2 \notin (4; 5] \Rightarrow S_{III} = \emptyset$

Cob IV: $x \in (5; \infty) \Rightarrow -(5-x)-(x-4)+(x+1)=3x+2$

$\Rightarrow -5+x-x+4+x+1-3x-2=0 \Rightarrow -2x=2 \Rightarrow$

$\Rightarrow x=-1 \notin (5; \infty) \Rightarrow S_{IV} = \emptyset$

$\Rightarrow S = S_I \cup S_{II} \cup S_{III} \cup S_{IV} = \{0\}$

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Rezolvați: $|x-2|=4$

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met I (nu e generală)

E_1) folosim: $|x|=\alpha \Rightarrow x=\pm\alpha$

E_2) $|x-2|=4 \Rightarrow x-2=\pm 4$

E_3) $x-2=4 \Rightarrow x=6$

$x-2=-4 \Rightarrow x=-2$

$S_f: x\in\{-2;6\}$

met II (generală)

E_1) $x-2=0 \Rightarrow x=2$

E_2)

x		2	
$x-2$	$-\ -\ -\ -\ -$	0	$+\ +\ +$

E_3) Caz I: $x\in(-\infty;2] \Rightarrow -(x-2)=4$

$\Rightarrow -x+2=4 \Rightarrow -x=2 \mid\cdot(-1) \Rightarrow x=-2\in(-\infty;2]$

$\Rightarrow S_I=\{-2\}$

Caz II: $x\in(2;\infty) \Rightarrow x-2=4 \Rightarrow x=6\in(2;\infty)$

$\Rightarrow S_{II}=\{6\}$

E_4) $S_f=S_I\cup S_{II}=\{-2;6\}$

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Rezolvați:

$$2x^2 - 3|x| + 1 = 0$$

E_1) Caz I: $x \leq 0$

x		0	
x	$---$	0	$++$

$\Rightarrow |x| = -x$

$\Rightarrow 2x^2 + 3x + 1 = 0 \Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{4} = \frac{-3 \pm 1}{4}$

$\Rightarrow \begin{cases} x_1 = -\frac{2}{4} = -\frac{1}{2} < 0 \Rightarrow x_1 = -\frac{1}{2} \text{ sol.} \\ x_2 = -\frac{4}{4} = -1 < 0 \Rightarrow x_2 = -1 \text{ sol.} \end{cases}$

E_2) Caz II: $x > 0 \Rightarrow |x| = x$

$\Rightarrow 2x^2 - 3x + 1 = 0 \Rightarrow x_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \begin{cases} 1 > 0 \\ \frac{1}{2} > 0 \end{cases}$

E_3) $S_f : x \in \{-1; -\frac{1}{2}; \frac{1}{2}; 1\}$

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Rezolvați: $|2+|x-3|+x|=7x-8$

$E_1)$ $x-3=0 \Rightarrow x=3$

	x		3	
$x-3$	$-\ -\ -\ -\ -$	0	$+\ +\ +$	

$E_2)$ Cazul I: $x\in(-\infty,3] \Rightarrow |2-(x-3)+x|=7x-8$

$|2-x+3+x|=7x-8 \Rightarrow 5=7x-8$

$\Rightarrow 7x=15 \Rightarrow x=\frac{15}{7}\in(-\infty;3]$

$\Rightarrow S_I=\{\frac{15}{7}\}$

Cazul II: $x\in(3;\infty) \Rightarrow |2+(x-3)+x|=7x-8$

$|2+2x-3|=7x-8 \Rightarrow |2x-1|=7x-8$

	x		$1/2$		3	
$2x-1$	$-\ -$	0	$+\ +$		$+\ +\ +$	

Dacă $x\in(3;\infty) \xrightarrow{\text{fals!}} 2x-1=7x-8$

$5x=7 \Rightarrow x=\frac{7}{5}\notin(3;\infty)$

$\Rightarrow S_{II}=\emptyset$

$S_f=S_I\cup S_{II}=\{\frac{15}{7}\}$

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Rezolvați: $|4x-5+|2-3x||=4x+3$ 7

$E_1)$ $2-3x=0 \Rightarrow 3x=2 \Rightarrow x=\frac{2}{3}$

$E_2)$

			$2/3$	
$2-3x$	$+ + + +$	0	$- - - -$	

$E_3)$ Caz I: $x \in (-\infty, \frac{2}{3}] \Rightarrow |4x-5+(2-3x)|=4x+3$

$|4x-5+2-3x|=4x+3 \Rightarrow |x-3|=4x+3$

			$2/3$		3	
x			$\|$			
$x-3$	$- - - -$		0		$+ + + +$	

Deci, $x \in (-\infty, \frac{2}{3}] \Rightarrow -(x-3)=4x+3$

$-x+4=4x+3 \Rightarrow 5x=1 \Rightarrow x=\frac{1}{5}$ N.

$x=\frac{1}{5} \in (-\infty, \frac{2}{3}] \Rightarrow S_I=\{\frac{1}{5}\}$

Caz II: $x \in (\frac{2}{3}, \infty) \Rightarrow |4x-5-(2-3x)|=4x+3$

$\Rightarrow |4x-5-2+3x|=4x+3 \Rightarrow |7x-7|=4x+3$

			$(2/3$		1	
$7x-7$	$- - - -$		0		$+ + + +$	

Caz II 1: $x \in (\frac{2}{3}; 1] \Rightarrow -7x+7=4x+3 \Rightarrow 11x=4$

$\Rightarrow x=\frac{4}{11}=0,36... \notin (\frac{2}{3}; 1] \Rightarrow S_{II,1}=\emptyset$

Caz II 2: $x \in (1; \infty) \Rightarrow 7x-7=4x+3 \Rightarrow 3x=10$

$\Rightarrow x=\frac{10}{3} \in (1; \infty) \Rightarrow S_{II\,2}=\{\frac{10}{3}\}$

$S_f = S_I \cup S_{II.1} \cup S_{II.2} = \{\frac{1}{5}; \frac{10}{3}\}$

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Rezolvați:

$$|1-x|-|x-4|=5$$

E1)

x		1		4	
$1-x$	$+\ +$	0	$-\ -$	$-\ -$	$-\ -$
$x-4$	$-\ -$	$-\ -$	$-\ -$	0	$+\ +$

E2) Cob I: $x \in (-\infty, 1] \Rightarrow$

$\Rightarrow (1-x)+(x-4)=5 \Rightarrow 1-x+x-4=5$

$-3=5$ fals $\Rightarrow S_I = \emptyset$

Cob II: $x \in (1, 4] \Rightarrow$

$-(1-x)+(x-4)=5 \Rightarrow -1+x+x-4=5$

$\Rightarrow 2x = 10 \Rightarrow x = 5 \notin (1, 4]$

$\Rightarrow S_{II} = \emptyset$

Cob III: $x \in (4, \infty)$

$\Rightarrow -(1-x)-(x-4)=5$

$-1+x-x+4=5$

$3=5$ fals $\Rightarrow S_{III} = \emptyset$

E3) $S_g = S_I \cup S_{II} \cup S_{III} = \emptyset$

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