CONDIȚIA CA O EXPRESIE DE GRADUL II SĂ PĂSTREZE SEMN CONSTANT

Găsiți $m \in \mathbb{R}$ dacă următoarele:

1) $(m-3)x^2 - mx - m < 0, \forall x \in \mathbb{R}$

2) $x^2 + mx + 4 \geq 0, \forall x \in \mathbb{R}$

3) $(m-1)x^2 + (m-3)x - m + 3 \leq 0, \forall x \in \mathbb{R}$

4) $(m+3)x^2 - (m+3)x + m - 4 > 0, \forall x \in \mathbb{R}$

5) $(m-2)x^2 + (m-1)x + m - 1 < 0, \forall x \in \mathbb{R}$

6) $(m+3)x^2 - (m-1)x + m - 1 \geq 0, \forall x \in \mathbb{R}$

7) $mx^2 - 3x + 2 \leq 0, \forall x \in \mathbb{R}$

8) $-x^2 + (3m-1)x + 4 < 0, \forall x \in \mathbb{R}$

9) $x^2 - (1-2m)x + m - 3 < 0, \forall x \in \mathbb{R}$

10) $(m+2)x^2 - (2m-1)x + 1 - 2m \geq 0, \forall x \in \mathbb{R}$

11) $mx^2 - 2mx + m - 1 \leq 0, \forall x \in \mathbb{R}$

12) $x^2 + 3x + m \geq 0, \forall x \in \mathbb{R}$

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Găsiți $m \in \mathbb{R}$ dacă

$(m-3)x^2 - mx - m < 0, \forall x \in \mathbb{R}$

$E_1)$ Cond: $\begin{cases} a < 0 \\ \Delta < 0 \end{cases}$

$E_2)$ $a < 0 \Rightarrow m-3 < 0 \Rightarrow m \in (-\infty; 3) = I_1$

$E_3)$ $\Delta < 0 \Rightarrow (-m)^2 + 4(m-3)m < 0 \Leftrightarrow$

$m^2 + 4m^2 - 12m < 0 \Leftrightarrow 5m^2 - 12m < 0$

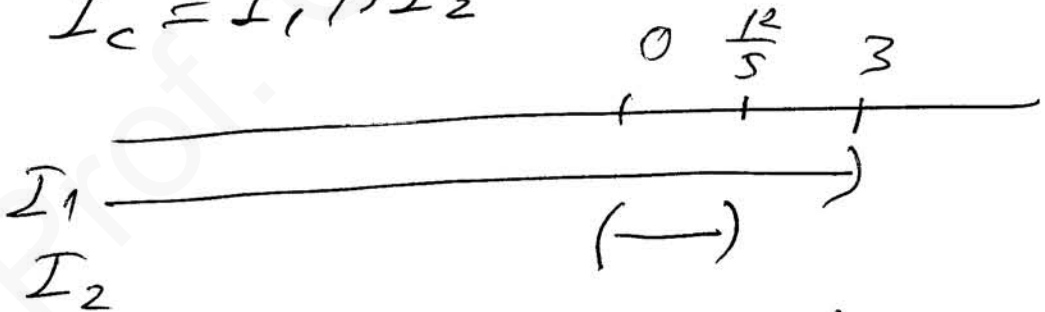
$5m^2 - 12m = 0 \Rightarrow m(5m-12) = 0 \Rightarrow \begin{cases} m_1 = 0 \\ m_2 = \frac{12}{5} \end{cases}$

$E_4)$

		0		$\frac{12}{5}$	
$5m^2-12m$	$+ \; +$	0	$- - -$	0	$+ \; +$

$5m^2 - 12m < 0 \Leftrightarrow m \in (0; \frac{12}{5}) = I_2$

$E_5)$ $I_c = I_1 \cap I_2$



$I_1 \cap I_2 = (0; \frac{12}{5})$

$\Rightarrow m \in (0; \frac{12}{5})$

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Găsiți $m \in \mathbb{R}, m \neq 1$ dacă

$(m-1)x^2 + (m-3)x - m + 3 \leq 0, \forall x \in \mathbb{R}$

E_1) folosim $ax^2 + bx + c \leq 0, \forall x \in \mathbb{R}$

$\Rightarrow \begin{cases} a < 0 \\ \Delta \leq 0 \end{cases}$

E_2) $a < 0 \Rightarrow m - 1 < 0 \Rightarrow m < 1 \Rightarrow m \in (-\infty; 1) = I_1$

E_3) $\Delta \leq 0 \Rightarrow (m-3)^2 - 4(m-1)(-m+3) \leq 0$

$\Rightarrow m^2 - 6m + 9 - 4(-m^2 + 3m + m - 3) \leq 0$

$m^2 - 6m + 9 + 4m^2 - 16m + 12 \leq 0$

$5m^2 - 22m + 21 \leq 0$

E_4) $m_{1,2} = \frac{22 \pm \sqrt{484 - 420}}{2 \cdot 5} = \frac{22 \pm 8}{10} \begin{matrix} 3 \\ \swarrow \\ \frac{14}{10} = \frac{7}{5} \end{matrix}$

m		$\frac{7}{5}$		3	
$5m^2 - 22m + 21$	$+ + 0$	$- - - -$	0	$+ + +$	

Pt ca $5m^2 - 22m + 21 \leq 0 \Rightarrow m \in \left[\frac{7}{5}; 3\right] = I_2$

E_5) $m \in I_1 \cap I_2 = (-\infty; 1) \cap \left[\frac{7}{5}; 3\right] = \emptyset$

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Găsiți $m \in \mathbb{R}$ dacă

$$(m+3)x^2 - (m+3)x + m - 4 > 0, \forall x \in \mathbb{R}$$

E_1) $ax^2 + bx + c > 0, \forall x \in \mathbb{R}$

$$\Leftrightarrow \begin{cases} a > 0 \\ \Delta < 0 \end{cases}$$

E_2) $a > 0 \Rightarrow m + 3 > 0 \Rightarrow m > -3 \Rightarrow m \in (-3; \infty)$

E_3) $\Delta < 0 \Rightarrow [-(m+3)]^2 - 4(m+3)(m-4) < 0 \Leftrightarrow$

$\Leftrightarrow m^2 + 6m + 9 - 4(m^2 - 4m + 3m - 12) < 0$

$\Leftrightarrow m^2 + 6m + 9 - 4m^2 + 4m + 48 < 0$

$-3m^2 + 10m + 57 < 0 \mid \cdot(-1)$

$3m^2 - 10m - 57 > 0$

E_4) $m_{1,2} = \dfrac{10 \pm \sqrt{100 + 12 \cdot 57}}{2 \cdot 3} = \dfrac{10 \pm \sqrt{784}}{6} = \dfrac{10 \pm 28}{6}$

$\Rightarrow \begin{cases} m_1 = \frac{38}{6} = \frac{19}{3} \\ m_2 = -\frac{18}{6} = -3 \end{cases}$

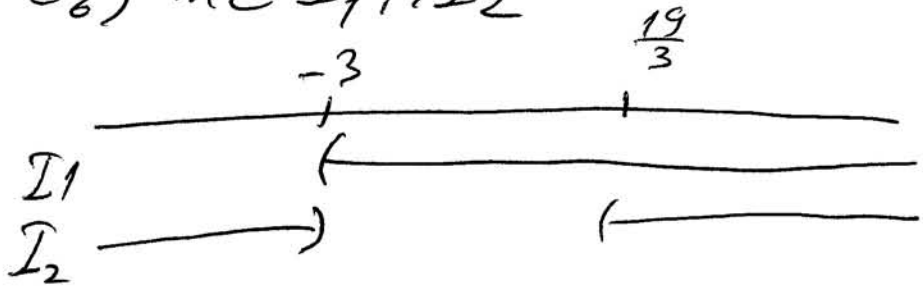
E_5)

m		-3		$\frac{19}{3}$	
$3m^2 - 10m - 57$	$+ +$	0	$- - -$	0	$+ +$

Deci $3m^2 - 10m - 57 > 0 \Rightarrow$

$$m \in (-\infty; -3) \cup \left(\frac{19}{3}; \infty\right) = I_2$$

E_6) $m \in I_1 \cap I_2$



$\Rightarrow m \in \left(\frac{19}{3}; \infty\right)$

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Găsiți $m=?$ dacă:

$(m+3)x^2-(m-1)x+m-1\geq 0,\ \forall x\in\mathbb{R}$

$E_1)$ cond: $\begin{cases} a>0 \\ \Delta\leq 0 \end{cases}$

$E_2)\ a>0 \Rightarrow m+3>0 \Rightarrow m\in(-3;\infty)=I_1$

$E_3)\ \Delta\leq 0 \Rightarrow (-(m-1))^2-4(m+3)(m-1)\leq 0$

$\Rightarrow m^2-2m+1-4(m^2-m+3m-3)\leq 0$

$m^2-2m+1-4m^2-8m+12\leq 0$

$-3m^2-10m+13\leq 0 \mid \cdot(-1)$

$3m^2+10m-13\geq 0$

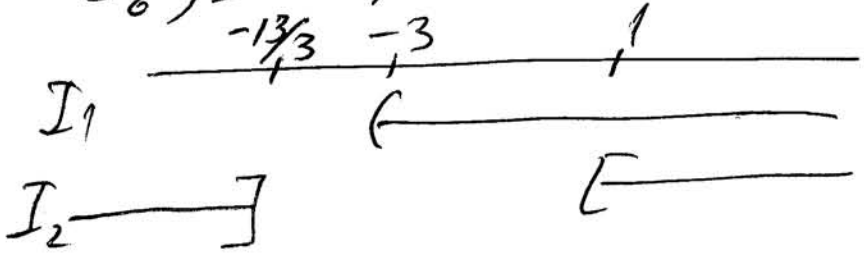
$E_4)\ m_{1,2}=\dfrac{-10\pm\sqrt{100+156}}{2\cdot 3}=\dfrac{-10\pm 16}{6}$ $\begin{matrix} 1 \\ -\frac{26}{6}=-\frac{13}{3} \end{matrix}$

$E_5)$

m		$-\frac{13}{3}$		1	
$3m^2+10m-13$	$+\ +$	0	$-\ -\ -$	0	$+\ +\ +$

$3m^2+10m-13\geq 0 \Rightarrow m\in(-\infty;-\frac{13}{3}]\cup[1;\infty)=I_2$

$E_6)\ I=I_1\cap I_2$



$I=I_1\cap I_2$

$I=[1;\infty)$

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Găsiți $m \in \mathbb{R}$ dacă:

$(m+2)x^2-(2m-1)x+1-2m \geq 0, \forall x \in \mathbb{R}$

$E_1)$ $ax^2+bx+c \geq 0, \forall x \in \mathbb{R}$

$\Leftrightarrow \begin{cases} a>0 \\ \Delta \leq 0 \end{cases}$

$E_2)$ În cazul nostru, $\begin{cases} m+2>0 \\ (-(2m-1))^2-4(m+2)(1-2m) \leq 0 \end{cases}$

$E_3)$ $m+2>0 \Rightarrow m \in (-2; \infty) = I_1$

$E_4)$ $4m^2-4m+1-4(m-2m^2+2-4m) \leq 0$

$\Leftrightarrow 4m^2-4m+1+12m+8m^2-8 \leq 0$

$\Leftrightarrow 12m^2+8m-7 \leq 0$

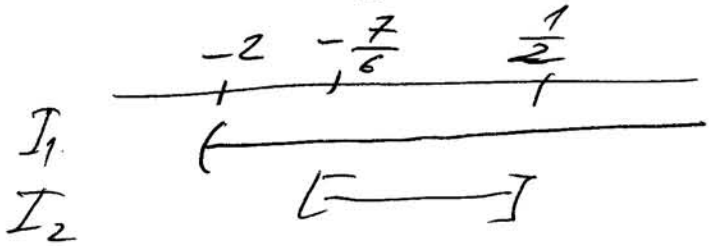
$E_5)$ $m_{1,2} = \dfrac{-8 \pm \sqrt{64+336}}{2 \cdot 12} = \dfrac{-8 \pm 20}{24}$ $\begin{matrix} \frac{12}{24}=\frac{1}{2} \\ -\frac{28}{24}=-\frac{7}{6} \end{matrix}$

$E_6)$

m		$-\frac{7}{6}$		$\frac{1}{2}$	
$12m^2+8m-7$	$+$	0	$-$	0	$+$

Așadar $12m^2+8m-7 \leq 0 \Leftrightarrow m \in \left[-\frac{7}{6}, \frac{1}{2}\right] = I_2$

$E_7)$ $I_f = I_1 \cap I_2$



$I_f = \left[-\frac{7}{6}, \frac{1}{2}\right]$

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Fie $f: \mathbb{R} \to \mathbb{R}$, $f(x) = mx^2 - 2mx + m - 1$, $m \in \mathbb{R}^*$. Se cere $m = ?$ a.î. care $f(x) \leq 0, \forall x \in \mathbb{R}$

$E_1)$ $mx^2 - 2mx + m - 1 \leq 0, \quad \forall x \in \mathbb{R}$

$\Rightarrow$ condiții: $\begin{cases} m < 0 \\ \Delta \leq 0 \end{cases}$

$E_2)$ $m < 0 \Rightarrow m \in (-\infty; 0) = I_1$

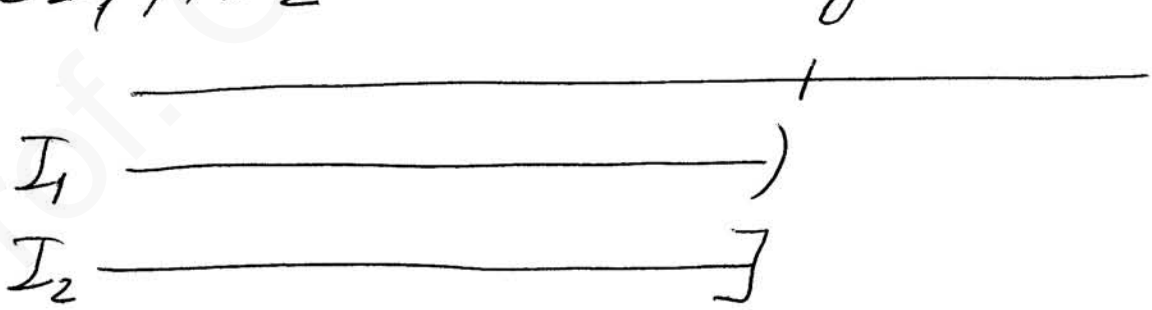
$E_3)$ $\Delta \leq 0 \Rightarrow b^2 - 4ac \leq 0 \Rightarrow$

$\Rightarrow (-2m)^2 - 4m(m-1) \leq 0$

$4m^2 - 4m^2 + 4m \leq 0$

$4m \leq 0 \mid :4 \Rightarrow m \leq 0 \Rightarrow m \in (-\infty; 0] = I_2$

$E_4)$ $m \in I_1 \cap I_2$



$I_1 \cap I_2 = (-\infty; 0)$

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$m = ?$ pt care $x^2 + 3x + m \geq 0, \forall x \in \mathbb{R}$

$E_1)$ $ax^2 + bx + c \geq 0, \forall x \in \mathbb{R} \Leftrightarrow$

$$\begin{cases} \Delta \leq 0 \\ a > 0 \end{cases}$$

$E_2)$ $x^2 + 3x + m \geq 0, \forall x \in \mathbb{R} \Leftrightarrow$

$$\begin{cases} \Delta \leq 0 \\ 1 > 0 \end{cases} \Leftrightarrow \begin{cases} 9 - 4m \leq 0 \Leftrightarrow 4m \geq 9 \\ \text{adev} \end{cases} \quad m \geq \frac{9}{4}$$

$E_3)$ $m \in \left[\frac{9}{4}; \infty\right)$

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