INECUAȚII REZOLVATE CU TABEL DE SEMNE (FRACȚII ȘI PRODUSE)

1) $\frac{(x^2-3x+2)(-x+4)}{2-x} \leq 0$

2) $\frac{16x-8}{x^2+3x+2} \geq 0$

3) $\frac{(5-x)(x+2)}{(x^2-3x+2)(x^2+4x+4)} \geq 0$

4) $\frac{(3-x)(x^2-7x+6)}{(x^2-4x+4)(-x^2+x-1)} \leq 0$

5) $\frac{3+x}{(1-x)(x^2-1)(x^2-5x+6)} > 0$

6) $\frac{16x-8}{x^2+3x+2} \geq 2$

7) $1 \leq \frac{x}{x-3}$

8) $-1 < \frac{x+1}{x^2-4x+3}$

9) $\frac{x+1}{x^2-4x+3} > 2$

10) $\frac{(x+1)(x^2+2x+2)}{x^3-2x^2-5x+6} \leq 0$

11) $\frac{x^3-9x^2+23x-15}{(x+1)(x^2-4x+4)} < 0$

12) $\frac{x^3-2x^2-5x+6}{x^4-16x^3+86x^2-176x+105} > 0$

13) $(-2+x)(x-5)(x^2-7x+6) \geq 0$

14) $(x^2-x+1)(-x^2+2x-1) \leq 0$

15) $(x-3)(4-x)(x^2-3x+2) \leq 0$

16) $\frac{x^2-3x+2}{4-2x} \leq 2$

17) $\frac{6x-1}{1-3x} \geq \frac{9x+2}{1-6x}$

Prof. Ovidiu Bădescu

Rezolvați: $\frac{16x-8}{x^2+3x+2} \geq 0$

$E_1)$ $16x-8=0 \Rightarrow 16x=8 \Rightarrow x=\frac{8}{16}=\frac{1}{2}$

$x^2+3x+2=0 \Rightarrow x_{1,2}=\frac{-3\pm\sqrt{9-8}}{2\cdot 1}=\frac{-3\pm 1}{2}\begin{matrix}-1\\-2\end{matrix}$

$E_2)$

		-2		-1		$1/2$		
$16x-8$	$---$		$---$		$---$	0	$+++$	
x^2+3x+2	$+$	0	$--$	0	$+$	$+$	$+$	$+$
$F(x)$	$--$	$\|$	$+$	$\|$	$--$	0	$+++$	

$E_3)$ $F(x)\geq 0 \Leftrightarrow x\in(-2,-1]\cup[1/2,\infty)$

Prof. Ovidiu Băgescu

Rezolvați: $\frac{(5-x)(x+2)}{(x^2-3x+2)(x^2+4x+4)} \geq 0$

E_1) $5-x=0 \Rightarrow x=5$

$x+2=0 \Rightarrow x=-2$

$x^2-3x+2=0 \Rightarrow \begin{cases} x_1=1 \\ x_2=2 \end{cases}$ mL

$x^2+4x+4=0 \Rightarrow x_{1,2}=\frac{-4\pm\sqrt{16-16}}{2\cdot 1}=-2$

E_2)

x		-2		1		2		5	
$5-x$	+	+	+	+	+	+	0	−	−
$x+2$	−	−	0	+	+	+	+	+	+
(x^2-3x+2)	+	+	+	0	−	−	0	+	+
x^2+4x+4	+	+	0	+	+	+	+	+	+
$F(x)$	−	−	|	+	|	−	|	+	0	−	−

E_3) $F(x) \geq 0 \Rightarrow x \in (-2;1) \cup (2;5]$

Obs.!

Dacă se cere $F(x)>0 \Rightarrow x \in (-2;1) \cup (2;5)$

Dacă se cere $F(x) \leq 0 \Rightarrow x \in (-\infty;-2) \cup \{1;2\} \cup [5;\infty)$

Dacă se cere $F(x)<0 \Rightarrow x \in (-\infty;-2) \cup (1;2) \cup (5;\infty)$

Prof. Ovidiu Bagescu

Rezolvați: $\frac{(3-x)(x^2-7x+6)}{(x^2-4x+4)(-x^2+x-1)} \leq 0$ 4

E1) $3-x=0 \Rightarrow x=3$

$x^2-7x+6=0 \Rightarrow x_{1,2}=\frac{7\pm\sqrt{49-24}}{2\cdot 1}=\frac{7\pm 5}{2}\begin{matrix}6\\1\end{matrix}$

$x^2-4x+4=0 \Rightarrow x_{1,2}=\frac{4\pm\sqrt{16-16}}{2\cdot 1}=\frac{4\pm 0}{2}=2$

$-x^2+x-1=0 \mid \cdot(-1) \Rightarrow x^2-x+1=0 \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4}}{2\cdot 1}$

$\Delta<0 \Rightarrow$ ec nu are rād reale

E2)

x		1		2		3				6		
$3-x$	+	+	+	+	+	0	−	−	−	−	−	−
x^2-7x+6	+	+	+	0	−	−	−	−	−	0	+	+
x^2-4x+4	+	+	+	0	+	+	+	+	+	+	+	+
$-x^2+x-1$	−	−	−	−	−	−	−	−	−	−	−	−
$F(x)$	−	−	−	0	+	|	+	0	−	−	−	0	+	+	+

E3) $F(x) \leq 0 \Leftrightarrow$

$x \in (-\infty; 1] \cup [3; 6]$

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Rezolvați:

$$\frac{3+x}{(1-x)(x^2-1)(x^2-5x+6)} > 0$$

E_1) $3+x=0 \Rightarrow x=-3$

$1-x=0 \Rightarrow x=1$

$x^2-1=0 \Rightarrow x=\pm 1$

$x^2-5x+6=0 \Rightarrow x_{1,2}=\frac{5\pm\sqrt{25-24}}{2\cdot 1}$ $\begin{matrix}3\\2\end{matrix}$

E_2)

x	-3	-1	1	2	3
$3+x$	$-$ 0	$+$	$+$	$+$	$+$
$1-x$	$+$	$+$	$+$	0 $-$	$-$
x^2-1	$+$	0 $-$	0	$+$	$+$
x^2-5x+6	$+$	$+$	0	$-$ 0	$+$
$F(x)$	$-$	0 $+$	$\nexists$	$-$ 0	$+$

$F(x)>0 \Leftrightarrow x\in(-3;-1)\cup(2;3)$

Prof. Ovidiu Badescu

Rezolvați: $\frac{16x-8}{x^2+3x+2} \geq 2$

$E_1)$ $\frac{16x-8}{x^2+3x+2} - 2 \geq 0 \Leftrightarrow \frac{16x-8-2x^2-6x-4}{x^2+3x+2} \geq 0$ $\quad / \cdot (x^2+3x+2)$

$\Leftrightarrow \frac{-2x^2+10x-12}{x^2+3x+2} \geq 0$

$E_2)$ $-2x^2+10x-12=0 \mid :(-2) \Rightarrow x^2-5x+6=0$

$x_{1,2} = \frac{5 \pm \sqrt{25-24}}{2 \cdot 1} = \frac{5 \pm 1}{2} \begin{cases} 3 \\ 2 \end{cases}$

$E_3)$ $x^2+3x+2=0 \Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2 \cdot 1} = \frac{-3 \pm 1}{2} \begin{cases} -1 \\ -2 \end{cases}$

$E_4)$

x		-2		-1		2		3	
$-2x^2+10x-12$	$---$	$\mid$	$---$	$\mid$	$---$	$\mid$	$+++$	$\mid$	$---$
x^2+3x+2	$+++$	$\mid$	$---$	$\mid$	$+++$	$\mid$	$+++$	$\mid$	$+++$
$F(x)$	$---$	$\mid$	$+++$	$\mid$	$---$	$\mid$	$+++$	$\mid$	$---$

$F(x) \geq 0 \Leftrightarrow x \in (-2, -1) \cup [2, 3]$

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Rezolvați: $\frac{x+1}{x^2-4x+3} > 2$

$E_1)$ $\frac{x+1}{x^2-4x+3} - 2 > 0 \Leftrightarrow \frac{x+1-2x^2+8x-6}{x^2-4x+3} > 0$ (x^2-4x+3)

$\Leftrightarrow \frac{-2x^2+9x-5}{x^2-4x+3} > 0$

$E_2)$ $-2x^2+9x-5=0 \mid \cdot(-1) \Rightarrow 2x^2-9x+5=0$

$\Rightarrow x_{1,2} = \frac{9 \pm \sqrt{81-40}}{2 \cdot 2} = \frac{9 \pm \sqrt{41}}{4} \begin{cases} \frac{9-\sqrt{41}}{4} \\ \frac{9+\sqrt{41}}{4} \end{cases}$

$E_3)$ $x^2-4x+3=0 \Rightarrow \begin{cases} x_1=1 \\ x_2=3 \end{cases}$

x		$\frac{9-\sqrt{41}}{4}$		1		3		$\frac{9+\sqrt{41}}{4}$	
$-2x^2+9x-5$	$-$	0	$+$	$+$	$+$	$+$	0	$-$	$-$
x^2-4x+3	$+$	$+$	$+$	0	$-$	$-$	0	$+$	$+$
$F(x)$	$-$	0	$+$	$\|$	$-$	$\|$	$+$	0	$-$

$E_4)$ $F(x) > 0 \Leftrightarrow x \in \left(\frac{9-\sqrt{41}}{4}, 1\right) \cup \left(3, \frac{9+\sqrt{41}}{4}\right)$

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Rezolvați: $\frac{(x+1)(x^2+2x+2)}{x^3-2x^2-5x+6} \leq 0$

$E_1)$ $x^3-2x^2-5x+6=0$

$D_6=\{\pm 1, \pm 2, \pm 3, \pm 6\}$

	x^3	x^2	x	x^0
	1	-2	-5	6
1	1	$1\cdot 1+(-2)=-1$	$1\cdot(-1)+(-5)=-6$	$1\cdot(-6)+6=0$

$\Rightarrow x^3-2x^2-5x+6=(x-1)(1\cdot x^2-x-6)$

$E_6)$ inec.: $\frac{(x+1)(x^2+2x+2)}{(x-1)(x^2-x-6)} \leq 0$

$E_7)$ $x+1=0 \Rightarrow x_1=-1$

$x^2+2x+2=0$, $x_{1,2}=\frac{-2\pm\sqrt{4-8}}{2\cdot 1} \notin \mathbb{R}$, $\Delta<0$

$x-1=0 \Rightarrow x=1$

$x^2-x-6=0 \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+24}}{2\cdot 1}=\frac{1\pm 5}{2} \begin{matrix} 3 \\ -2 \end{matrix}$

$E_8)$

x	-2	-1	1	3
$x+1$	$-$	0 $+$	$+$	$+$
x^2+2x+2	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	0 $+$	$+$
x^2-x-6	$+$	0 $-$	$-$	0 $+$
$F(x)$	$+$	$-0+$	$-$	$+$

$F(x) \leq 0 \Leftrightarrow \boxed{x \in (-2,-1] \cup (1,3)}$

Rezolvați: $\frac{x^3-9x^2+23x-15}{(x+1)(x^2-4x+4)} < 0$

$E_1)\ x^3-9x^2+23x-15=0$

$D_{-15}=\{\pm 1;\pm 3;\pm 5;\pm 15\}$

	x^3	x^2	x	x^0
	1	−9	23	−15
1	1	$1\cdot 1-9=-8$	$1\cdot(-8)+23=15$	$1\cdot 15+(-15)=0$ ⇒ OK

$\Rightarrow (x-1)(1\cdot x^2-8x+15)=0$

$E_2)\ \frac{x^3-9x^2+23x-15}{(x+1)(x^2-4x+4)} < 0 \Leftrightarrow$

$\Leftrightarrow \frac{(x-1)(x^2-8x+15)}{(x+1)(x^2-4x+4)} < 0$

$E_3)\ x-1=0 \Rightarrow x=1$

$x^2-8x+15=0 \Rightarrow x_{1,2}=\frac{8\pm\sqrt{64-4\cdot 15}}{2\cdot 1}=\frac{8\pm 2}{2}\begin{matrix}5\\3\end{matrix}$

$x+1=0 \Rightarrow x=-1$

$x^2-4x+4=0 \Rightarrow x_{1,2}=\frac{4\pm\sqrt{16-4\cdot 4}}{2\cdot 1}=\frac{4\pm 0}{2}=2$

$E_4)$

x	-1	1	2	3	5
$x-1$	− − − −	0	+	+	+ +
$x^2-8x+15$	+	+	+	0 − −	0 + +
$x+1$	− − −	0	+	+	+ +
x^2-4x+4	+	+	+	0	+ +
$F(x)$	+ + |	−	0 + |	+	0 − − 0 + +

$F(x) < 0 \Leftrightarrow$

$x \in (-1;1) \cup (1;3) \cup (3;5)$

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Rezolvați: $(-2+x)(x-5)(x^2-7x+6) \geq 0$ 13

$E_1)$ $-2+x=0 \Rightarrow x=2$

$x-5=0 \Rightarrow x=5$

$x^2-7x+6=0 \Rightarrow x_{1,2}=\frac{7\pm\sqrt{49-4\cdot1\cdot6}}{2\cdot1}=\frac{7\pm5}{2}\begin{cases}6\\1\end{cases}$

$E_2)$

x		1		2		5		6	
$-2+x$	$---$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$x-5$	$---$	$\|$	$---$	$---$	$---$	0	$+$	$+$	$+$
$(-2+x)(x-5)(x^2-7x+6)$	$+$	0	$---$	0	$+$	0	$---$	0	$++$
$P(x)$	$+$	0	$+$	0	$-$	0	$+$	0	$-0++$

$E_3)$ $P(x) \geq 0 \Leftrightarrow$

$\Leftrightarrow x \in (-\infty, 1] \cup [2, 5] \cup [6, \infty)$

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Rezolvați:

$$\frac{x^2-3x+2}{4-2x} \leq 2$$

$E_1)$ $\frac{x^2-3x+2}{4-2x} - 2 \leq 0$ $\quad / \cdot (4-2x)$

$E_2)$ $\frac{x^2-3x+2-2(4-2x)}{4-2x} \leq 0 \Leftrightarrow \frac{x^2-3x+2-8+4x}{4-2x} \leq 0$

$\Leftrightarrow \frac{x^2+x-6}{4-2x} \leq 0$

$E_3)$ $x^2+x-6=0 \Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2}$ $\begin{cases} \frac{-1+5}{2} = 2 \\ \frac{-1-5}{2} = -3 \end{cases}$

$E_4)$ $4-2x=0 \Rightarrow 2x=4 \Rightarrow x=2$

$E_5)$

x	-3		2	
x^2+x-6	$+ \; + \; 0$	$- \; - \; - \; - \; - \; 0$		$+ \; + \; +$
$4-2x$	$+ \; + \; +$	$+ \; + \; + \; + \; 0$		$- \; - \; -$
$F(x)$	$+ \; 0$	$- \; - \; -$	$\nexists$	$- \; - \; -$

$E_6)$ Rezultă $F(x) \leq 0 \Rightarrow x \in [-3, 2) \cup (2, \infty)$

Prof. Ovidiu Bădescu

Rezolvați:

$$\frac{6x-1}{1-3x} \geq \frac{9x+2}{1-6x}$$

$$E_1) \quad \frac{6x-1}{1-3x} - \frac{9x+2}{1-6x} \geq 0$$

$$E_2) \quad \frac{(1-6x)(6x-1) - (1-3x)(9x+2)}{(1-3x)(1-6x)} \geq 0$$

$$\frac{6x-1-36x^2+6x-(9x+2-27x^2-6x)}{(1-3x)(1-6x)} \geq 0$$

$$\frac{-36x^2+12x-1+27x^2-3x-2}{(1-3x)(1-6x)} \geq 0$$

$$\frac{-9x^2+9x-3}{(1-3x)(1-6x)} \geq 0 \Big| \cdot(-1) \Rightarrow \frac{9x^2-9x+3}{(1-3x)(1-6x)} \leq 0$$

$E_2)$

x		$\frac{1}{6}$		$\frac{1}{3}$	
$9x^2-9x+3$	$+$	$+$	$+$	$+$	$+$
$1-3x$	$+$	$+$	$+$	0	$-$
$1-6x$	$+$	0	$-$	$-$	$-$
$F(x)$	$+$	$\|\|$	$-$	$\|\|$	$+$

$\Delta = 81 - 4 \cdot 9 \cdot 3 = 81 - 108 < 0$

$Rd \quad F(x) \leq 0 \Leftrightarrow x \in \left(\frac{1}{6}; \frac{1}{3}\right)$

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