ECUAȚII DE GRAD SUPERIOR CU FACTOR SAU SCHEMA LUI HORNER

1) $x^3-6x^2+11x-6=0$ $\{1;2;3\}$

2) $x^3-7x+6=0$ $\{1;2;-3\}$

3) $x^3-9x^2+23x-15=0$ $\{1;3;5\}$

4) $x^3-12x^2+44x-48=0$ $\{2;4;6\}$

5) $x^4-10x^3+35x^2-50x+24=0$
$\{1;2;3;4\}$

6) $x^4-16x^3+86x^2-176x+105=0$
$\{1;3;5;7\}$

7) $x^4-20x^3+140x^2-400x+384=0$
$\{2;4;6;8\}$

8) $8x^3-36x^2+46x-15=0$ dacă are
rād $\in \mathbb{Q}\setminus\mathbb{Z}$ $\{\frac{1}{2};\frac{3}{2};\frac{5}{2}\}$

9) $24x^3+10x^2-13x-6=0$ dacă
are rād $\in \mathbb{Q}\setminus\mathbb{Z}$ $\{-\frac{1}{2};-\frac{2}{3};\frac{3}{4}\}$

10) $36x^3-39x^2+x+6=0$ dacă
are rād $\in \mathbb{Q}\setminus\mathbb{Z}$ $\{-\frac{1}{3};\frac{2}{3};\frac{3}{4}\}$

11) $x^4-2x^3-13x^2+38x-24=0$
$\{1;2;3;-4\}$

12) $72x^4-42x^3-36x^2+13x+6=0$
$\{-\frac{1}{3};\frac{2}{3};\frac{3}{4};-\frac{1}{2}\}$

13) $16x^4+16x^3-32x^2-100x+75=0$

14) $x^3-4\sqrt{3}x^2+3x+18\sqrt{3}=0$
dacă are și rădăcini $\in \mathbb{R}\setminus\mathbb{Q}$

Prof. Ovidiu Bădescu

Rezolvați: $x^3-6x^2+11x-6=0$

E_1) $D_{-6}=\{\pm1;\pm2;\pm3;\pm6\}$

E_2)

	x^3	x^2	x	x^0
	1	-6	11	-6
1	1	$1\cdot1+(-6)=-5$	$1\cdot(-5)+11=6$	$16+(-6)=0$

E_3) ec. devine: $(x-1)(1\cdot x^2-5x+6)=0$

$$\Rightarrow \begin{cases} x-1=0 \Rightarrow x_1=1 \\ \text{sau} \\ x^2-5x+6=0 \Rightarrow x_{2,3}=\dfrac{5\pm\sqrt{25-24}}{2\cdot1} \begin{matrix} 3 \\ 2 \end{matrix} \end{cases}$$

E_4) $x\in\{1;2;3\}$

Prof. Ovidiu Băcsu

Rezolvați: $x^3 - 7x + 6 = 0$

$E_1)$ $D_6 = \{\pm 1; \pm 2; \pm 3; \pm 6\}$

$E_2)$

	x^3	x^2	x	x^0
	1	0	-7	6
1	1	$1 \cdot 1 + 0 = 1$	$1 \cdot 1 - 7 = -6$	$1 \cdot (-6) + 6 = \boxed{0}$

$E_3)$ se descrie:

$$(x-1)(1 \cdot x^2 + 1 \cdot x - 6) = 0$$

$$\Rightarrow \begin{cases} x - 1 = 0 \Rightarrow x = 1 \\ \text{sau} \\ x^2 + x - 6 = 0 \Rightarrow x_{1,2} = \dfrac{-1 \pm \sqrt{1+24}}{2 \cdot 1} \end{cases}$$

$$= \frac{-1 \pm 5}{2} \begin{matrix} \nearrow 2 \\ \searrow -3 \end{matrix}$$

$E_4)$ $x \in \{-3; 1; 2\}$

Prof. Ovidiu Bădescu

Rezolvați: $x^3 - 12x^2 + 44x - 48 = 0$

E1) $D_{-48} = \{\pm 1, \pm 2, \pm 3, \pm 4, \dots, \pm 48\}$

E2)

	x^3	x^2	x	x^0
	1	-12	44	-48
~~1~~	~~1~~	~~$1\cdot 1+(-12)$ $=-11$~~	~~$1\cdot(-11)+44$ $=33$~~	~~$1\cdot 33+(-48)$ $=-15\neq 0$~~
2	1	$2\cdot 1+(-12)=$ $=-10$	$2\cdot(-10)+44$ $=24$	$2\cdot 24+(-48)$ $=0$

E3) ec.: $(x-2)(1\cdot x^2 - 10x + 24) = 0$

$$\Rightarrow \begin{cases} x_1 = 2 \\ x^2 - 10x + 24 = 0 \Rightarrow x_{2,3} = \dfrac{10 \pm \sqrt{100-96}}{2\cdot 1} \end{cases}$$

$$\begin{cases} \frac{10+2}{2} = 6 \\ \frac{10-2}{2} = 4 \end{cases}$$

E4) $S_p: x \in \{2, 4, 6\}$

Prof. Ovidiu Bădescu

Rezolvați:

$$x^4 - 16x^3 + 86x^2 - 176x + 105 = 0$$

$E_1)$ $D_{105} = \{ \pm 1; \pm 3; \pm 5; \dots \}$

$E_2)$

	x^4	x^3	x^2	x	x^0
	1	−16	86	−176	105
1	1	−15	71	−105	0

$\Rightarrow (x-1)(1 \cdot x^3 - 15x^2 + 71x - 105) = 0$

$\Rightarrow$ $x - 1 = 0 \Rightarrow x = 1$ sau $x^3 - 15x^2 + 71x - 105 = 0$

$E_3)$ $x^3 - 15x^2 + 71x - 105 = 0$

	x^3	x^2	x	x^0
	1	−15	71	−105
~~1~~	~~1~~	~~−14~~	~~57~~	~~−48 ≠ 0~~
3	1	−12	71 − 36 = 35	0

$\Rightarrow (x-3)(x^2 - 12x + 35) = 0$

$\Rightarrow$ $x - 3 = 0 \Rightarrow x = 3$ sau $x^2 - 12x + 35 = 0$

$E_4)$ $x^2 - 12x + 35 = 0$

$$x_{1,2} = \frac{12 \pm \sqrt{144 - 140}}{2 \cdot 1} = \frac{12 \pm 2}{2} \begin{matrix} 7 \\ 5 \end{matrix}$$

$E_5)$ $S_f : x \in \{1; 3; 5; 7\}$

Prof. Ovidiu Bădescu

Rezolvați: $x^4 - 20x^3 + 140x^2 - 400x + 384 = 0$

$E_1)$ $D_{384} = \{\pm 1; \pm 2; 3; \pm 4; \dots\}$

$E_2)$

	x^4	x^3	x^2	x	x^0
	1	−20	140	−400	384
1	1	−19	121	−279	105 ≠ 0
2	1	−18	−36+140 = 104	208−400 = −192	0

$\Rightarrow (x-2)(x^3 - 18x^2 + 104x - 192) = 0$

$\Rightarrow \begin{cases} x-2=0 \Rightarrow x=2 \\ \text{sau} \\ x^3 - 18x^2 + 104x - 192 = 0 \end{cases}$

$E_3)$

	x^3	x^2	x	x^0
	1	−18	104	−192
1	1	−17	87	−105 ≠ 0
2	1	−16	104−32 = 72	144−192 ≠ 0
3	1	−15	−45+104 = 59	177−192 ≠ 0
4	1	−14	−56+104 = 48	192−192 = 0 ⇒ ok.

$\Rightarrow (x-4)(1 \cdot x^2 - 14x + 48) = 0$

$\Rightarrow \begin{cases} x-4=0 \Rightarrow x=4 \\ \text{sau} \\ x^2 - 14x + 48 = 0 \end{cases}$

$E_4)$ $x^2 - 14x + 48 = 0 \Rightarrow x_{1,2} = \dfrac{14 \pm \sqrt{196-192}}{2 \cdot 1} \Rightarrow$

$\Rightarrow x_{1,2} = \dfrac{14 \pm 2}{2} \begin{matrix} \nearrow 8 \\ \searrow 6 \end{matrix}$

E_5 / S_f: $x \in \{2; 4; 6; 8\}$

Prof. Ovidiu Bădescu

Rezolvați în $\mathbb{Q}\setminus\mathbb{Z}$ ecuația

$$8x^3-36x^2+46x-15=0$$

$E_1)$ $\frac{D_{-15}}{D_8}=\frac{\{\pm1;\pm3;\pm5;\pm15\}}{\{\pm1;\pm2;\pm4;\pm8\}}=$

$=\{\pm1;\pm3;\pm5;\pm15;\pm\frac{1}{2};\pm\frac{3}{2};\pm\frac{5}{2};\pm\frac{15}{2};\ldots;\pm\frac{15}{8}\}$

$E_2)$ n-ar soluții din $\mathbb{Q}\setminus\mathbb{Z}$ deci nu încercăm divizorii întregi

	x^3	x^2	x	x^0
	8	-36	46	-15
$\frac{1}{2}$	8	$8\cdot\frac{1}{2}-36=$ $=4-36=-32$	$\frac{1}{2}\cdot(-32)+46$ $=-16+46=30$	$\frac{1}{2}\cdot30+(-15)$ $=15-15=$ **0**

$E_3)$ ec este: $(x-\frac{1}{2})(8x^2-32x+30)=0$

$\Rightarrow$ $x-\frac{1}{2}=0\Rightarrow x=\frac{1}{2}$ sau $8x^2-32x+30=0\,/:2\Rightarrow 4x^2-16x+15=0$

$x_{1,2}=\frac{16\pm\sqrt{256-240}}{2\cdot4}=\frac{16\pm4}{8}$ $\begin{cases}\frac{20}{8}=\frac{5}{2}\\ \frac{12}{8}=\frac{3}{2}\end{cases}$

$E_4)$ $S_f: x\in\{\frac{1}{2};\frac{5}{2};\frac{3}{2}\}$

Prof. Ovidiu Badescu

Știind că ecuația are rădăcini raționale neîntregi, rezolvați:

$$24x^3+10x^2-13x-6=0$$

$E_1)$ $D_{-6}=\{\pm1,\pm2,\pm3,\pm6\}$

$E_2)$ dacă nu ne-ar fi spus că are răd. $\in\mathbb{Q}\setminus\mathbb{Z}$, atunci

	x^3	x^2	x	x^0
	24	10	−13	−6
1	24	34	21	15 ≠0

se încearcă toți divizorii lui −6 și nu iese rest 0 la niciunul, însă aici puteam direct E_3)

$E_3)$ $\dfrac{D_{-6}}{D_{24}}=\dfrac{\{\pm1,\pm2,\pm3,\pm6\}}{\{\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24\}}$

$=\{\pm1,\pm2,\pm3,\pm6,\pm\frac{1}{2},\pm\frac{2}{2},\pm\frac{3}{2},\pm\frac{6}{2},\pm\frac{1}{3}\}$

$E_4)$

	x^3	x^2	x	x^0
	24	10	−13	−6
$\frac{1}{2}$	24	22	−2	−7 ≠0
$-\frac{1}{2}$	24	−2	−12	0

$x+\frac{1}{2}=0$ sau $24x^2-2x-12=0\,|:2$

$\Rightarrow\left(x+\frac{1}{2}\right)(24x^2-2x-12)=0$

$\Rightarrow 12x^2-x-6=0\Rightarrow x_{2,3}=\dfrac{1\pm\sqrt{1+288}}{2\cdot12}=\dfrac{1\pm17}{24}$ $\begin{matrix}\frac{18}{24}=\frac{3}{4}\\ -\frac{16}{24}=-\frac{2}{3}\end{matrix}$ $\Rightarrow\{-\frac{1}{2},\frac{3}{4},-\frac{2}{3}\}$

Prof. Ciulu Badescu

Rezolvați:

$$x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$$

$E_1)$ $D_{-24} = \{\pm 1; \pm 2; \pm 3; \pm 6; \pm 8; \pm 12; \pm 24\}$

$E_2)$

	x^4	x^3	x^2	x	x^0
	1	-2	-13	38	-24
1	1	$1\cdot 1 - 2 = -1$	$1\cdot(-1) - 13 = -14$	$1\cdot(-14) + 38 = 24$	$24 - 24 = \boxed{0}$

$$\Rightarrow (x-1)(1\cdot x^3 - 1\cdot x^2 - 14x + 24) = 0$$

$$\Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x_1 = 1} \\ \text{sau} \\ x^3 - x^2 - 14x + 24 = 0 \end{cases}$$

$E_3)$

	x^3	x^2	x	x^0
	1	-1	-14	24
~~1~~	~~1~~	0	~~-14~~	~~$40 \neq 0$~~
2	1	1	-12	$\boxed{0}$

$$\Rightarrow (x-2)(1\cdot x^2 + 1\cdot x - 12) = 0$$

$$\Rightarrow \begin{cases} x - 2 = 0 \Rightarrow \boxed{x_2 = 2} \\ \text{sau} \\ x^2 + x - 12 = 0 \end{cases}$$

$E_4)$ $x^2 + x - 12 = 0 \rightarrow x_{1,2} = \dfrac{-1 \pm \sqrt{1 + 48}}{2 \cdot 1} = \dfrac{-1 \pm 7}{2}$

$$\Rightarrow \begin{cases} x_3 = \dfrac{6}{2} = 3 \\ x_4 = -\dfrac{8}{2} = -4 \end{cases} \Rightarrow S_p: x \in \{1; 2; 3; -4\}$$

Prof. Ovidiu Badescu