PROBLEME NONSTANDARD DE ȘIRURI, PROGR. ARITMETICE ȘI GEOMETRICE

1) Dacă a_n p.a., să se arate

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{k-1}}+\sqrt{a_k}} = \frac{k-1}{\sqrt{a_1}+\sqrt{a_k}}$$

2) Dacă a_n p.a., să se arate

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_n a_{n+1}} = \frac{n}{a_1 a_{n+1}}$$

3) Fie $1; 4; 10; 19; 31; \dots$
Se cere al 50-lea termen și suma primilor 50 de termeni

Prof. Ovidiu Bădescu

Dacă a_n progr aritm, arătați că

$$S_n = \frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{k-1}}+\sqrt{a_k}} = \frac{k-1}{\sqrt{a_1}+\sqrt{a_k}}$$

$\cdot \dfrac{\sqrt{a_1}-\sqrt{a_2}}{\sqrt{a_1}-\sqrt{a_2}}$ $\cdot \dfrac{\sqrt{a_2}-\sqrt{a_3}}{\sqrt{a_2}-\sqrt{a_3}}$ $\cdot \dfrac{\sqrt{a_{k-1}}-\sqrt{a_k}}{\sqrt{a_{k-1}}-\sqrt{a_k}}$

E1) $S_k = \dfrac{1}{\sqrt{a_1}+\sqrt{a_2}} + \dfrac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dots + \dfrac{1}{\sqrt{a_{k-1}}+\sqrt{a_k}}$

E2) $S_k = \dfrac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \dfrac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + \dots + \dfrac{\sqrt{a_{k-1}}-\sqrt{a_k}}{a_{k-1}-a_k}$

E3) $S_k = \dfrac{\sqrt{a_1}-\sqrt{a_2}}{-r} + \dfrac{\sqrt{a_2}-\sqrt{a_3}}{-r} + \dots + \dfrac{\sqrt{a_{k-1}}-\sqrt{a_k}}{-r}$

E4) $S_k = \dfrac{(\sqrt{a_1}-\sqrt{a_2})+(\sqrt{a_2}-\sqrt{a_3})+\dots+(\sqrt{a_{k-1}}-\sqrt{a_k})}{-r}$

$\cdot \dfrac{\sqrt{a_1}+\sqrt{a_k}}{\sqrt{a_1}+\sqrt{a_k}}$

$$S_k = \frac{\sqrt{a_1}-\sqrt{a_k}}{-r} = \frac{a_1-a_k}{-r(\sqrt{a_1}+\sqrt{a_k})}$$

E5) $$S_k = \frac{a_1-(a_1+(k-1)r)}{-r(\sqrt{a_1}+\sqrt{a_k})} = \frac{-(k-1)r}{-r(\sqrt{a_1}+\sqrt{a_k})}$$

$$S_k = \frac{k-1}{\sqrt{a_1}+\sqrt{a_k}}$$

Prof. Ovidiu Bădescu

Dacă (a_n) progresie aritmetică de rație r, calculați:

$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_n a_{n+1}} = \frac{n}{a_1 a_{n+1}}$$

E1) $S_n = \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_n a_{n+1}}$

E2) $S_n = \frac{a_2 - a_1}{a_1 a_2} \cdot \frac{1}{r} + \frac{a_3 - a_2}{a_2 a_3} \cdot \frac{1}{r} + \dots + \frac{a_{n+1} - a_n}{a_n a_{n+1}} \cdot \frac{1}{r}$

E3) $S_n = \frac{1}{r}\left(\frac{a_2}{a_1 a_2} - \frac{a_1}{a_1 a_2} + \frac{a_3}{a_2 a_3} - \frac{a_2}{a_2 a_3} + \dots + \frac{a_{n+1}}{a_n a_{n+1}} - \frac{a_n}{a_n a_{n+1}}\right)$

E4) $S_n = \frac{1}{r}\left(\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \dots + \frac{1}{a_n} - \frac{1}{a_{n+1}}\right)$

E5) $S_n = \frac{1}{r}\left(\frac{1}{a_1} - \frac{1}{a_{n+1}}\right) = \frac{1}{r} \cdot \frac{a_{n+1} - a_1}{a_1 a_{n+1}} =$

$$= \frac{1}{r} \cdot \frac{a_1 + (n+1-1)r - a_1}{a_1 a_{n+1}} = \frac{nr}{r a_1 a_{n+1}}$$

$$S_n = \frac{n}{a_1 a_{n+1}}$$

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Fie $1, 4, 10, 19, 31, \ldots$

Care este al 50-lea termen și calculați suma primilor 50 de termeni

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E1) $a_1 = 1$, $a_2 = a_1 + 3$

$a_3 = a_2 + 6$

$a_4 = a_3 + 9$

$\ldots \quad \ldots \quad \ldots$

$a_{50} = a_{49} + 3 \cdot 49$

$a_{50} = a_1 + 3(1+2+3+\ldots+49)$

$a_{50} = 1 + 3 \cdot \frac{49 \cdot 50}{2} = 1 + 3 \cdot \frac{2450}{2}$

$= 1 + 3 \cdot 1225 = 3676$

E2) $S_{50} = a_1 + a_2 + \ldots + a_{50} = \sum_{k=1}^{50} a_k$

Dar, $a_k = a_1 + 3 \cdot (1+2+\ldots+(k-1)) = a_1 + 3 \cdot \frac{(k-1)k}{2}$

$S_{50} = \sum_{k=1}^{50} \left(a_1 + \frac{3}{2}(k-1)k\right) = \sum_{k=1}^{50} a_1 + \frac{3}{2} \sum_{k=1}^{50} (k^2 - k)$

$= 50 a_1 + \frac{3}{2}\left(\sum_{1}^{50} k^2 - \sum_{1}^{50} k\right) = 50 \cdot 1 + \frac{3}{2}\left(\frac{50 \cdot 51 \cdot 101}{6} - \frac{50 \cdot 51}{2}\right)$

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