DIVIZIBILITATE DEMONSTRATĂ PRIN INDUCȚIE MATEMATICĂ

Arătați că:

1) $(3^{2n} \cdot 7^n - 2^{3n} \cdot 5^n) \vdots 23, n \geq 1$

2) $(7 \cdot 25^n + 2 \cdot 6^{n+1}) \vdots 19, n \geq 0$

3) $(3 \cdot 5^{2n+1} + 2^{3n+1}) \vdots 17, n \geq 0$

4) $(6^{2n-1} + 1) \vdots 7, \forall n \geq 1$

5) $(7^n - 1) \vdots 6, \forall n \geq 1$

6) $(9^{n+1} - 8n - 9) \vdots 16, \forall n \geq 0$

7) $(3^{2n+2} - 2^{n+1}) \vdots 7, \forall n \geq 1$

8) $(10^n + 18n - 28) \vdots 27, \forall n \geq 1$

9) $(5^{n+1} - 1) \vdots 4, \forall n \geq 1$

10) $(2^{4n+1} - 2) \vdots 30, \forall n \geq 0$

11) $(4^n + 15n - 1) \vdots 9, \forall n \geq 1$

12) $(2^{n+1} \cdot 5^n + 7) \vdots 9, \forall n \geq 0$

13) $(3^{2n+1} + 2^{n+2}) \vdots 7, \forall n \geq 0$

14) $(9^n - 1) \vdots 8, \forall n \geq 1$

15) $(3^{2n+2} + 2^{2n+4}) \vdots 5, \forall n \geq 0$

16) $(3^n - 1)(10 \cdot 25^n + 9 \cdot 6^n) \vdots 38, n \geq 0$

Prof. Ovidiu Bădescu

Arătați că:

$$\left(3^{2n}\cdot 7^{n}-2^{3n}\cdot 5^{n}\right)\vdots 23,\ \forall n\geq 1$$

E1) I $P(1)$: $\left(3^{2}\cdot 7-2^{3}\cdot 5\right)\vdots 23 \Leftrightarrow$

$(63-40)\vdots 23$ adev.

E2) II Pp. $P(k)$ adev. $\Rightarrow$ $P(k+1)$ adev.

$P(k)$: $\left(3^{2k}\cdot 7^{k}-2^{3k}\cdot 5^{k}\right)\vdots 23$

$P(k+1)$: $\left(3^{2k+2}\cdot 7^{k+1}-2^{3k+3}\cdot 5^{k+1}\right)\vdots 23$

E3) Știm că $P(k)$ adev. $\Rightarrow \left(3^{2k}\cdot 7^{k}-2^{3k}\cdot 5^{k}\right)\vdots 23$

$\Rightarrow \exists c\in\mathbb{Z}$ a.î. $3^{2k}\cdot 7^{k}-2^{3k}\cdot 5^{k}=23c$

$$\Rightarrow 3^{2k}=\frac{23c+2^{3k}\cdot 5^{k}}{7^{k}}$$

E4) Înlocuim în $P(k+1)\Rightarrow$

$$3^{2k+2}\cdot 7^{k+1}-2^{3k+3}\cdot 5^{k+1}=3^{2}\cdot\frac{23c+2^{3k}\cdot 5^{k}}{7^{k}}\cdot 7^{k+1}-$$

$$-2^{3k+3}\cdot 5^{k+1}=$$

$$=63\left(23c+2^{3k}\cdot 5^{k}\right)-2^{3k+3}\cdot 5^{k+1}=$$

$$=63\cdot 23c+63\cdot 2^{3k}\cdot 5^{k}-2^{3k+3}\cdot 5^{k+1}$$

$$=63\cdot 23c+2^{3k}\cdot 5^{k}(63-40)$$

$$=\frac{63\cdot 23}{\vdots 23}c+2^{3k}\cdot 5^{k}\cdot 23\Rightarrow P(k+1)\text{ adev.}$$

$\Rightarrow P(n)$ adev., $\forall n\in\mathbb{N}^{*}$

Prof. Ovidiu Băcescu

Arătați că: $(7 \cdot 25^n + 2 \cdot 6^{n+1}) \vdots 19, \forall n \in \mathbb{N}$

E_1) notăm $P(n): (7 \cdot 25^n + 2 \cdot 6^{n+1}) \vdots 19$

E_2) $P(0): (7 \cdot 25^0 + 2 \cdot 6^1) \vdots 19 \Leftrightarrow 19 \vdots 19$ adev.

E_3) Pp. $P(k)$ adev. $\xrightarrow{(?)} P(k+1)$ adev.

$P(k): (7 \cdot 25^k + 2 \cdot 6^{k+1}) \vdots 19$

$P(k+1): (7 \cdot 25^{k+1} + 2 \cdot 6^{k+2}) \vdots 19$

E_4) Știm că $P(k)$ adev. $\Rightarrow$

$(7 \cdot 25^k + 2 \cdot 6^{k+1}) \vdots 19 \Rightarrow \exists c \in \mathbb{Z}$ a.î

$7 \cdot 25^k + 2 \cdot 6^{k+1} = 19c \Rightarrow$

$25^k = \frac{19c - 2 \cdot 6^{k+1}}{7}$

E_5) înlocuim în $P(k+1)$:

$7 \cdot 25^{k+1} + 2 \cdot 6^{k+2} = 7 \cdot 25 \cdot 25^k + 2 \cdot 6^{k+2} =$

$= 7 \cdot 25 \cdot \frac{19c - 2 \cdot 6^{k+1}}{7} + 2 \cdot 6^{k+2} =$

$= 25 \cdot 19c - 25 \cdot 2 \cdot 6^{k+1} + 2 \cdot 6^{k+2} =$

$= 25 \cdot 19c - 2 \cdot 6^{k+1}(25 - 6) =$

$= 25 \cdot 19c - 2 \cdot 6^{k+1} \cdot 19 = 19(25c - 2 \cdot 6^{k+1}) \vdots 19$

$E_1 \wedge E_2 \wedge E_3 \Rightarrow P(n)$ adev., $\forall n \in \mathbb{N}$

Prof. Ovidiu Badescu

Se cere: $(3 \cdot 5^{2n+1} + 2^{3n+1}) \vdots 17, \forall n \in \mathbb{N}$

I $P(0): (3 \cdot 5 + 2) \vdots 17 \Rightarrow 17 \vdots 17$ adev.

II Dp. $P(k)$ adev. $\overset{?}{\Rightarrow} P(k+1)$ adev.

$P(k): (3 \cdot 5^{2k+1} + 2^{3k+1}) \vdots 17$

$P(k+1): (3 \cdot 5^{2k+3} + 2^{3k+4}) \vdots 17$

Stim ca $P(k)$ adev. $\Rightarrow$

$(3 \cdot 5^{2k+1} + 2^{3k+1}) \vdots 17 \Rightarrow \exists c \in \mathbb{Z}$

a.i. $3 \cdot 5^{2k+1} + 2^{3k+1} = 17c$

$\Rightarrow 3 \cdot 5^{2k+1} = 17c - 2^{3k+1}$

$5^{2k+1} = \frac{17c - 2^{3k+1}}{3}$

Folosim in $P(k+1) \Rightarrow$

$3 \cdot 5^{2k+3} + 2^{3k+4} = 3 \cdot 5^2 \cdot 5^{2k+1} + 2^{3k+4} =$

$= \cancel{3} \cdot 5^2 \cdot \frac{17c - 2^{3k+1}}{\cancel{3}} + 2^{3k+4} =$

$= 5^2 \cdot 17c - 5^2 \cdot 2^{3k+1} + 2^{3k+4} =$

$= 25 \cdot 17c - 2^{3k+1}(\underbrace{25 - 2^3}_{17}) =$

$= 17(25c - 2^{3k+1}) \vdots 17 \Rightarrow P(k+1)$ adev. $\Rightarrow$

$\Rightarrow P(n)$ adev., $\forall n \geq 0$

Prof. Ovidiu Badescu

Se cere: $(6^{2n-1}+1) \vdots 7,\ \forall n \in \mathbb{N}^*$

E_1) I $P(1)$: $(6^1+1) \vdots 7 \Rightarrow 7 \vdots 7$ adev.

E_2) II Pp. $P(k)$ adev. $\overset{(?)}{\Rightarrow} P(k+1)$ adev.

$P(k)$: $(6^{2k-1}+1) \vdots 7$

$P(k+1)$: $(6^{2k+1}+1) \vdots 7$

E_3) Știm $P(k)$ adev. $\Rightarrow (6^{2k-1}+1) \vdots 7 \Rightarrow$

$\Rightarrow \exists c \in \mathbb{Z}$ a.î. $6^{2k-1}+1 = 7c \Rightarrow$

$\Rightarrow 6^{2k-1} = 7c-1$

E_4) înloc în $P(k+1) \Rightarrow 6^{2k+1}+1 = 6^{2k-1} \cdot 6^2 + 1 =$

$= (7c-1) \cdot 36 + 1 = 7c \cdot 36 - 36 + 1 =$

$= 7c \cdot 36 - 35 = 7(36c-5) \vdots 7$

$\Rightarrow P(k+1)$ adev $\Rightarrow P(n)$ adev., $\forall n \in \mathbb{N}^*$

Prof. Ovidiu Băgescu

Arătați că:

$$(9^{n+1} - 8n - 9) \vdots 16, \forall n \geq 0$$

E.I) I $P(0)$: $(9 - 0 - 9) : 16 \Leftrightarrow 0 : 16$ adev.

II Fie $P(k)$ adev $\xrightarrow{(?)}$ $P(k+1)$ adev unde

$P(k)$: $(9^{k+1} - 8k - 9) : 16$

$P(k+1)$: $(9^{k+2} - 8(k+1) - 9) : 16$

$P(k+1)$: $(9^{k+2} - 8k - 17) : 16$

E.II) Știm că $P(k)$ adev $\Rightarrow (9^{k+1} - 8k - 9) : 16$

$\Rightarrow \exists c \in \mathbb{Z}$ cu $9^{k+1} - 8k - 9 = 16c$

$\Rightarrow 9^{k+1} = 16c + 8k + 9$

Înlocuiesc în $P(k+1) \Rightarrow$

$$9^{k+2} - 8k - 17 = 9(16c + 8k + 9) - 8k - 17 =$$

$$= 9 \cdot 16c + 72k + 81 - 8k - 17 =$$

$$= 9 \cdot 16c + 64k + 64$$

$$= 16(9c + 4k + 4) : 16 \Rightarrow P(k+1) \text{ adev.}$$

$\Rightarrow P(n)$ adev., $\forall n \geq 0$

Prof. Ovidiu Bădescu

Rezolvați: $(3^{2n+2} + 2^{2n+4}) \vdots 5$, $\forall n \geq 0$

I $P(0)$: $(3^2 + 2^4) \vdots 5 \Leftrightarrow (9+16) \vdots 5$ adev.

II Pp. $P(k)$ adev. $\xrightarrow{(?)}$ $P(k+1)$ adev.

$P(k)$: $(3^{2k+2} + 2^{2k+4}) \vdots 5$

$P(k+1)$: $(3^{2k+4} + 2^{2k+6}) \vdots 5$

Știm că $P(k)$ adev. $\Rightarrow (3^{2k+2} + 2^{2k+4}) \vdots 5$

$\Rightarrow \exists c \in \mathbb{Z}$ a.î. $3^{2k+2} + 2^{2k+4} = 5c$

$\Rightarrow 2^{2k+4} = 5c - 3^{2k+2}$

Înlocuim în $P(k+1) \Rightarrow$

$3^{2k+4} + 2^{2k+6} = 3^{2k+4} + 4 \cdot (5c - 3^{2k+2}) =$

$= 3^{2k+4} + 4 \cdot 5c - 4 \cdot 3^{2k+2} =$

$= 3^{2k+2}(3^2 - 4) + 2 \cdot 5c =$

$= 3^{k+2} \cdot 5 + 2 \cdot 5c = 5(3^{k+2} + 2c) \vdots 5$

$\Rightarrow P(k+1)$ adev. $\Rightarrow P(n)$ adev., $\forall n \geq 0$

Prof. Ovidiu Badescu