EGALITĂȚI DEMONSTRATE PRIN INDUCȚIE MATEMATICĂ

Arătați că:

1) $\frac{1}{5\cdot 8}+\frac{1}{8\cdot 5}+\ldots+\frac{1}{(3n+2)(3n+5)}=\frac{n}{5(3n+5)}, n\geq 1$

2) $1+5+5^2+\ldots+5^n=\frac{5^{n+1}-1}{4}, \forall n\geq 1$

3) $\frac{3}{4}\cdot\frac{8}{9}\cdot\ldots\cdot\frac{n(n+2)}{(n+1)^2}=\frac{n+2}{2n+2}, \forall n\geq 1$

4) $1+4+7+\ldots+(3n-2)=\frac{n(3n-1)}{2}$

5) $\frac{1}{3}+\frac{2}{3^2}+\ldots+\frac{n}{3^n}=\frac{3}{4}-\frac{2n+3}{4\cdot 3^n}, \forall n\geq 1$

6) $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{8}\right)\cdot\ldots\cdot\left(1+\frac{1}{n(n+2)}\right)=\frac{2n+2}{n+2}, \forall n\geq 1$

7) $\frac{1}{3}\cdot\frac{2}{5}\cdot\ldots\cdot\frac{n}{2n+1}=\frac{2^n\cdot(n!)^2}{(2n+1)!}, \forall n\geq 1$

8) $1^2+2^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}, \forall n\geq 1$

9) $8+88+\ldots+\underbrace{88\ldots 8}_{n \text{ ori}}=\frac{8}{81}\left(10^{n+1}-9n-10\right), n\geq 1$

10) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}+\frac{1}{n+1}=1, n\geq 1$

11) $1\cdot 2+2\cdot 3+\ldots+n(n+1)=\frac{n(n+1)(n+2)}{3}, n\geq 1$

12) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}, n\geq 1$

13) $\frac{1^2}{1\cdot 3}+\frac{2^2}{2\cdot 4}+\ldots+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}, n\geq 1$

14) $\frac{7}{1\cdot 8}+\frac{7}{8\cdot 15}+\ldots+\frac{7}{(7n-6)(7n+1)}+\frac{1}{7n+1}=1, n\geq 1$

15) $1\cdot 4+2\cdot 7+\ldots+n(3n+1)=n(n+1)^2, n\geq 1$

16) $\frac{2^3-1}{2^3+1}\cdot\frac{3^3-1}{3^3+1}\cdot\ldots\cdot\frac{n^3-1}{n^3+1}=\frac{2}{3}\cdot\frac{n^2+n+1}{n^2+n}, n\geq 2$

17) $\frac{(n+1)(n+2)\ldots(n+n)}{1\cdot 3\cdot\ldots\cdot(2n-1)}=2^n, \forall n\geq 1$

18) $1-\frac{1}{2}+\frac{1}{3}-\ldots-\frac{1}{2n}=\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}, n\geq 1$

19) $(1+2)(1+2^2)\cdot\ldots\cdot(1+2^{2^n})=2^{2^{n+1}}-1, n\geq 1$

Prof. Vinicu Bădescu

Se cere:

$$\frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\dots+\frac{1}{(3n+2)(3n+5)}=\frac{n}{5(3n+5)},\ \forall n\geq 1$$

E_1) notăm $P(n):\ \frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\dots+\frac{1}{(3n+2)(3n+5)}=\frac{n}{5(3n+5)}$

E_2) $P(1):\ \frac{1}{5\cdot 8}=\frac{1}{5(3+5)}$ adev.

E_3) II P_n. $P(k)$ adev. $\overset{(?)}{\Rightarrow} P(k+1)$ adev.

$P(k):\ \frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\dots+\frac{1}{(3k+2)(3k+5)}=\frac{k}{5(3k+5)}$

$P(k+1):\ \frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\dots+\frac{1}{(3k+5)(3k+8)}=\frac{k+1}{5(3k+8)}$

E_4) Știm că $P(k)$ adev. $\Rightarrow$

$$\frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\dots+\frac{1}{(3k+2)(3k+5)}=\frac{k}{5(3k+5)}\ \Big|+\frac{1}{(3k+5)(3k+8)}$$

$$\Rightarrow \frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\dots+\frac{1}{(3k+5)(3k+8)}=\frac{k}{5(3k+5)}+\frac{1}{(3k+5)(3k+8)}$$

E_5) E suficient să arătăm că

$$\frac{k}{5(3k+5)}+\frac{1}{(3k+5)(3k+8)}=\frac{k+1}{5(3k+8)}\ \Big|\cdot 5(3k+5)(3k+8)$$

$\Leftrightarrow k(3k+8)+5=(k+1)(3k+5)$

$\Leftrightarrow 3k^2+8k+5=3k^2+5k+3k+5$ adev. $\Rightarrow$

$P(k+1)$ adev. $\Rightarrow P(n)$ adev., $\forall n\geq 1$

Prof. Ovidiu Pădescu

Se cere:

$$1+5+5^2+\dots+5^n=\frac{5^{n+1}-1}{4},\ \forall n\geq 1$$

E_1) notăm $P(n)$: $1+5+5^2+\dots+5^n=\frac{5^{n+1}-1}{4}$

E_2) I $P(1)$: $1+5=\frac{5^2-1}{4}\Leftrightarrow 6=\frac{24}{4}$ adev.

E_3) II Ip. $P(k)$ adev. $\overset{(?)}{\Rightarrow} P(k+1)$ adev.

$P(k)$: $1+5+5^2+\dots+5^k=\frac{5^{k+1}-1}{4}$

$P(k+1)$: $1+5+5^2+\dots+5^{k+1}=\frac{5^{k+2}-1}{4}$

E_4) Știm că $P(k)$ adev.

$$\Rightarrow 1+5+5^2+\dots+5^k=\frac{5^{k+1}-1}{4}\ \Big|+5^{k+1}$$

$$\Rightarrow 1+5+5^2+\dots+5^{k+1}=\frac{5^{k+1}-1}{4}+5^{k+1}$$

E_5) E suficient să arătăm că

$$\frac{5^{k+1}-1}{4}+5^{k+1}=\frac{5^{k+2}-1}{4}\ \Big|\cdot 4$$

$$5^{k+1}-1+4\cdot 5^{k+1}=5^{k+2}-1$$

$$5^{k+1}(1+4)=5^{k+2}\ \text{adev.}$$

$\Rightarrow P(k+1)$ adev. $\Rightarrow P(n)$ adev., $\forall n\geq 1$

Prof. Ovidiu Badescu

Se cere: $\frac{3}{4}\cdot\frac{8}{9}\cdot\ldots\cdot\frac{n(n+2)}{(n+1)^2}=\frac{n+2}{2n+2}$, $\forall n\in\mathbb{N}^*$ 3

E_1) notăm $P(n)$

E_2) I $P(1)$: $\frac{3}{4}=\frac{3}{4}$ adev.

E_3) II P_n. $P(k)$ adev. $\overset{!?}{\Rightarrow}$ $P(k+1)$ adev.

$P(k)$: $\frac{3}{4}\cdot\frac{8}{9}\cdot\ldots\cdot\frac{k(k+2)}{(k+1)^2}=\frac{k+2}{2k+2}$

$P(k+1)$: $\frac{3}{4}\cdot\frac{8}{9}\cdot\ldots\cdot\frac{(k+1)(k+3)}{(k+2)^2}=\frac{k+3}{2k+4}$

E_4) Știm că $P(k)$ adev. $\Rightarrow$

$$\frac{3}{4}\cdot\frac{8}{9}\cdot\ldots\cdot\frac{k(k+2)}{(k+1)^2}=\frac{k+2}{2k+2}\Big|\cdot\frac{(k+1)(k+3)}{(k+2)^2}$$

$$\Rightarrow\frac{3}{4}\cdot\frac{8}{9}\cdot\ldots\cdot\frac{(k+1)(k+3)}{(k+2)^2}=\frac{k+2}{2k+2}\cdot\frac{(k+1)(k+3)}{(k+2)^2}$$

E_5) Ε mg. în ord. că

$$\frac{\cancel{k+2}}{2\cancel{(k+1)}}\cdot\frac{\cancel{(k+1)}(k+3)}{\cancel{(k+2)^2}}=\frac{k+3}{2(k+2)}$$

$$\Rightarrow\frac{k+3}{2(k+2)}=\frac{k+3}{2(k+2)}\text{ adev.}\Rightarrow$$

$P(k+1)$ adev. $\Rightarrow P(n)$ adev., $\forall n\in\mathbb{N}^*$

Prof. Ovidiu Bădescu

Se cere:

$$1+4+7+\dots+(3n-2)=\frac{n(3n-1)}{2},$$

$$\forall n\geq 1$$

E_1) notăm $P(n)$: $1+4+7+\dots+(3n-2)=\frac{n(3n-1)}{2}$

E_2) $P(1)$: $1=\frac{1\cdot 2}{2}\Leftrightarrow 1=1$ adev.

E_2) Pp. $P(k)$ adev. $\overset{(?)}{\Rightarrow} P(k+1)$ adev.

$P(k)$: $1+4+7+\dots+(3k-2)=\frac{k(3k-1)}{2}$

$P(k+1)$: $1+4+7+\dots+(3(k+1)-2)=\frac{(k+1)(3(k+1)-1)}{2}$

$P(k+1)$: $1+4+\dots+(3k+1)=\frac{(k+1)(3k+2)}{2}$

E_3) Știm că $P(k)$ adev.

$$\Rightarrow 1+4+7+\dots+(3k-2)=\frac{k(3k-1)}{2}\Big|+(3k+1)$$

$$\Rightarrow 1+4+\dots+(3k+1)=\frac{k(3k-1)}{2}+(3k+1)$$

E_4) Este suficient să arătăm că

$$\frac{k(3k-1)}{2}+(3k+1)=\frac{(k+1)(3k+2)}{2}\Big|\cdot 2$$

$$\Leftrightarrow k(3k-1)+2(3k+1)=(k+1)(3k+2)$$

$$\Leftrightarrow 3k^2-k+6k+2=3k^2+2k+3k+2$$

$$\Leftrightarrow 0\cdot 2\dots-1\dots-2k^2+5k \text{ adev.} \Rightarrow P(k+1) \text{ adev.} \Rightarrow P(n) \text{ adev. } \forall n\geq 1$$

Prof. Ovidiu Bădescu

Se cere:

$$\frac{1}{3}+\frac{2}{3^2}+\dots+\frac{n}{3^n}=\frac{3}{4}-\frac{2n+3}{4\cdot 3^n},\ \forall n\geq 1$$

E_1) notăm $P(n)$: $\frac{1}{3}+\frac{2}{3^2}+\dots+\frac{n}{3^n}=\frac{3}{4}-\frac{2n+3}{4\cdot 3^n}$

E_2) $P(1)$: $\frac{1}{3}=\frac{3}{4}-\frac{5}{12}$ adev.

E_3) Fie $P(k)$ adev. $\overset{(?)}{\Rightarrow}$ $P(k+1)$ adev.

$P(k)$: $\frac{1}{3}+\frac{2}{3^2}+\dots+\frac{k}{3^k}=\frac{3}{4}-\frac{2k+3}{4\cdot 3^k}$

$P(k+1)$: $\frac{1}{3}+\frac{2}{3^2}+\dots+\frac{k+1}{3^{k+1}}=\frac{3}{4}-\frac{2k+5}{4\cdot 3^{k+1}}$

E_4) Știm că $P(k)$ adev. $\Rightarrow$

$$\frac{1}{3}+\frac{2}{3^2}+\dots+\frac{k}{3^k}=\frac{3}{4}-\frac{2k+3}{4\cdot 3^k}\Big|+\frac{k+1}{3^{k+1}}$$

$$\Rightarrow \frac{1}{3}+\frac{2}{3^2}+\dots+\frac{k+1}{3^{k+1}}=\frac{3}{4}-\frac{2k+3}{4\cdot 3^k}+\frac{k+1}{3^{k+1}}$$

E_5) E suficient să arătăm că

$$\frac{3}{4}-\frac{2k+3}{4\cdot 3^k}+\frac{k+1}{3^{k+1}}=\frac{3}{4}-\frac{2k+5}{4\cdot 3^{k+1}}\Big|\cdot(4\cdot 3^{k+1})$$

$$-(2k+3)\cdot 3+4(k+1)=-(2k+5)$$

$$-6k-9+4k+4=-2k-5 \text{ adev.}$$

$\Rightarrow P(k+1)$ adev. $\Rightarrow P(n)$ adev., $\forall n\geq 1$

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Se cere:

$$\left(1+\frac{1}{3}\right)\left(1+\frac{1}{8}\right)\cdots\left(1+\frac{1}{n(n+2)}\right)=\frac{2n+2}{n+2},\ \forall n\geq 1$$

E_1) notăm $P(n)$: $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{8}\right)\cdots\left(1+\frac{1}{n(n+2)}\right)=\frac{2n+2}{n+2}$

E_2) I $P(1)$: $\left(1+\frac{1}{3}\right)=\frac{4}{3} \Leftrightarrow \frac{4}{3}=\frac{4}{3}$ adev.

E_3) II Ip. $P(k)$ adev. $\overset{(?)}{\Rightarrow}$ $P(k+1)$ adev.

$P(k)$: $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{8}\right)\cdots\left(1+\frac{1}{k(k+2)}\right)=\frac{2k+2}{k+2}$

$P(k+1)$: $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{8}\right)\cdots\left(1+\frac{1}{(k+1)(k+3)}\right)=\frac{2k+4}{k+3}$

E_4) Știm că $P(k)$ adev. $\Rightarrow$

$$\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{8}\right)\cdot\cdots\left(1+\frac{1}{k(k+2)}\right)=\frac{2k+2}{k+2}\ \Big|\cdot\left(1+\frac{1}{(k+1)(k+3)}\right)$$

pentru $3=1\cdot(1+2)$; $8=2\cdot(2+2)$, ... după $k(k+2)$ urmează $(k+1)(k+3)$

$$\Rightarrow \left(1+\frac{1}{3}\right)\left(1+\frac{1}{8}\right)\cdots\left(1+\frac{1}{(k+1)(k+3)}\right)=\frac{2k+2}{k+2}\cdot\left(1+\frac{1}{(k+1)(k+3)}\right)$$

E_5) Suficient să arătăm că

$$\frac{2k+2}{k+2}\left(1+\frac{1}{(k+1)(k+3)}\right)=\frac{2k+4}{k+3}\ \Big|\cdot(k+1)(k+2)(k+3)$$

$$\Leftrightarrow (2k+2)\left((k+1)(k+3)+1\right)=(2k+4)(k+1)(k+2)$$

$$\Leftrightarrow 2(k+1)\left(k^2+3k+k+4\right)=2(k+2)(k+1)(k+2)$$

$$k^2+4k+4=k^2+4k+4 \text{ adev.} \Rightarrow$$

$P(k)$ adev. $\Rightarrow P(n)$ adev., $\forall n\geq 1$

Dacă $n! = 1 \cdot 2 \cdot \ldots \cdot n$, de exp. $3! = 1 \cdot 2 \cdot 3 = 6$ și $(n+1)! = n! \cdot n$
($n!$ citește n factorial), arătați că:

$$\frac{1}{3} \cdot \frac{2}{5} \cdot \ldots \cdot \frac{n}{2n+1} = \frac{2^n \cdot (n!)^2}{(2n+1)!}, \forall n \in \mathbb{N}^*$$

E1) not. $P(n): \frac{1}{3} \cdot \frac{2}{5} \cdot \ldots \cdot \frac{n}{2n+1} = \frac{2^n \cdot (n!)^2}{(2n+1)!}$

E2) I $P(1): \frac{1}{3} = \frac{2^1 \cdot (1!)^2}{3!} \Leftrightarrow \frac{1}{3} = \frac{2}{6}$ adev.

E3) Pp. $P(k)$ adev. $\overset{(?)}{\Rightarrow}$ $P(k+1)$ adev.

$$P(k): \frac{1}{3} \cdot \frac{2}{5} \cdot \ldots \cdot \frac{k}{2k+1} = \frac{2^k \cdot (k!)^2}{(2k+1)!}$$

$$P(k+1): \frac{1}{3} \cdot \frac{2}{5} \cdot \ldots \cdot \frac{(k+1)}{2k+3} = \frac{2^{k+1} \cdot [(k+1)!]^2}{(2k+3)!}$$

E4) Știm că $P(k)$ adev. $\Rightarrow$

$$\frac{1}{3} \cdot \frac{2}{5} \cdot \ldots \cdot \frac{k}{2k+1} = \frac{2^k \cdot (k!)^2}{(2k+1)!} \Big| \cdot \frac{k+1}{2k+3}$$

$$\Rightarrow \frac{1}{3} \cdot \frac{2}{5} \cdot \ldots \cdot \frac{k+1}{2k+3} = \frac{2^k \cdot (k!)^2}{(2k+1)!} \cdot \frac{k+1}{2k+3}$$

E5) E suf. să arăt. că

$$\frac{2^k \cdot (k!)^2}{(2k+1)!} \cdot \frac{k+1}{2k+3} = \frac{2^{k+1} \cdot [(k+1)!]^2}{(2k+3)!} \Big| \cdot (2k+3)!$$

$$2^k (k!)^2 (2k+2)(k+1) = 2^{k+1} \cdot [(k+1)!]^2$$

$$2^k \cdot k! \cdot k! \cdot 2 \cdot (k+1)(k+1) = 2^{k+1} (k+1)! (k+1)!$$

$$(k+1)! (k+1)! = (k+1)! (k+1)! \text{ adev.}$$

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Arătați că:

$$1^2+2^2+3^2+\dots+n^2=\frac{n(n+1)(2n+1)}{6},\ \forall n\in\mathbb{N}^*$$

E1) $\underline{I}$ $P(1): 1^2=\frac{1\cdot(1+1)(2\cdot1+1)}{6} \Leftrightarrow 1=1$ adev.

E2) P_n. $P(k)$ adev. $\overset{(?)}{\Longrightarrow}$ $P(k+1)$ adev

$$P(k): 1^2+2^2+\dots+k^2=\frac{k(k+1)(2k+1)}{6}$$

$$P(k+1): 1^2+2^2+\dots+(k+1)^2=\frac{(k+1)(k+2)(2k+3)}{6}$$

E3) $1^2+2^2+\dots+(k+1)^2=\underbrace{1^2+2^2+\dots+k^2}_{P(k)}+(k+1)^2$

$$=\frac{k(k+1)(2k+1)}{6}+(k+1)^2=\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$=\frac{(k+1)\left[k(2k+1)+6(k+1)\right]}{6}=$$

$$=\frac{(k+1)(2k^2+k+6k+6)}{6}=\frac{(k+1)\left[2k^2+4k+3k+6\right]}{6}=$$

$$=\frac{(k+1)\left[2k(k+2)+3(k+2)\right]}{6}=\frac{(k+1)(k+2)(2k+3)}{6}$$

$\Rightarrow P(k+1)$ adev. $\Rightarrow P(n)$ adev., $\forall n\geq 1$

Calculați: $S = 8+88+888+\ldots+\underbrace{888\ldots8}_{n \text{ ori}}$ și demonstrați prin inducție

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$E_1)\ S = 1\cdot 8 + 8\cdot 11 + 8\cdot 111 + \ldots + 8\cdot \underbrace{11\ldots1}_{n} =$

$= 8\cdot 1 + 8(10+1) + 8(10^2+10+1) + \ldots + 8(10^{n-1}+10^{n-2}+\ldots+1) =$

$= 8\cdot\frac{10^1-1}{10-1} + 8\cdot\frac{10^2-1}{10-1} + 8\cdot\frac{10^3-1}{10-1} + \ldots + 8\cdot\frac{10^n-1}{10-1}$

$= \frac{8}{9}\left[(10+10^2+10^3+\ldots+10^n) - n\right] =$

$= \frac{8}{9}\left[10(1+10+10^2+\ldots+10^{n-1}) - n\right] = \frac{8}{9}\left[10\cdot\frac{10^n-1}{10-1} - n\right]$

$= \frac{8}{9}\cdot\frac{10^{n+1}-10-9n}{9} = \frac{8}{81}(10^{n+1}-10-9n)$

$E_2)$ Arătăm că $8+88+888+\ldots+\underbrace{88\ldots8}_{n \text{ ori}} = \frac{8}{81}(10^{n+1}-10-9n)$

I $P(1)$: $8 = \frac{8}{81}(10^2-10-9) \Leftrightarrow 8 = \frac{8}{81}\cdot 81$ adev.

II Fie $P(k)$ adev. $\overset{(1)}{\Rightarrow} P(k+1)$

$P(k)$: $8+88+888+\ldots+\underbrace{88\ldots8}_{k \text{ ori}} = \frac{8}{81}(10^{k+1}-10-9k)$

$P(k+1)$: $\underbrace{8+88+\ldots+\underbrace{88\ldots8}_{k \text{ ori}}}_{P(k)} + \underbrace{88\ldots8}_{k+1} = \frac{8}{81}(10^{k+2}-10-9k-9)$

$\Rightarrow \frac{8}{81}(10^{k+1}-10-9k) + 8\cdot\underbrace{11\ldots1}_{k+1} = \frac{8}{81}(10^{k+1}-10-9k) + 8\cdot(\underbrace{10^k+10^{k-1}+\ldots+1}_{k+1})$

$= \frac{8}{81}(10^{k+1}-10-9k) + 8\cdot\frac{10^{k+1}-1}{10-1} = \frac{8}{81}\left[10^{k+1}-10-9k+9(10^{k+1}-1)\right]$

$= \frac{8}{81}(10^{k+1}-10-9k+9\cdot 10^{k+1}-9) = \frac{8}{81}\left(\frac{10\cdot 10^{k+1}-19-9k}{\boxed{10^n+10^{n-1}+\ldots+1 = \frac{10^{n+1}-1}{10-1}}}\right.$

Arătați că, $\forall n \geq 1$ avem

$$1 \cdot 4 + 2 \cdot 7 + \ldots + (n+1)(3n+4) = (n+1)(n+2)^2,$$

E_1) I $P(1)$: $1 \cdot 4 + 2 \cdot 7 = 2 \cdot (1+2)^2$

$\Leftrightarrow 4 + 14 = 18$ adev.

E_2) II Pp. $P(k)$ adev. $\overset{(?)}{\Longrightarrow} P(k+1)$ adev, unde

$P(k)$: $1 \cdot 4 + 2 \cdot 7 + \ldots + (k+1)(3k+4) = (k+1)(k+2)^2$

$P(k+1)$: $1 \cdot 4 + 2 \cdot 7 + \ldots + (k+2)(3k+7) = (k+2)(k+3)^2$

E_3) Știm că $P(k)$ adev $\Rightarrow$

$1 \cdot 4 + 2 \cdot 7 + \ldots + (k+1)(3k+4) = (k+1)(k+2)^2 \,\big|\, + (k+2)(3k+7)$

$\Rightarrow 1 \cdot 4 + 2 \cdot 7 + \ldots + (k+2)(3k+7) = (k+1)(k+2)^2 + (k+2)(3k+7)$

Este suficient să arăt că:

$(k+1)(k+2)^2 + (k+2)(3k+7) = (k+2)(k+3)^2 \,\big|\, :(k+2)$

$(k+1)(k+2) + (3k+7) = (k+3)^2$

$k^2 + 2k + k + 2 + 3k + 7 = k^2 + 6k + 9$

$k^2 + 6k + 9 = k^2 + 6k + 9$ adev.

$\Rightarrow P(k+1)$ adev $\Rightarrow P(n)$ adev., $\forall n \geq 1$

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