INDUCTIA CU PASUL „2"

1) Fie $x_{n+1}=3x_n+x_{n-1}$ cu $x_0=3$, $x_1=1$, se cere $x_n \vdots 3$, $\forall n \geq 0$

2) Fie $x_{n+2}=x_{n+1}+7x_n$, $x_0=0$, $x_1=10$, se cere $x_n \vdots 5$, $\forall n \geq 0$

3) $x_n=\frac{5}{2}x_{n-1}-x_{n-2}$, $x_0=2$, $x_1=\frac{5}{2}$, se cere $x_n=2^n+2^{-n}$, $\forall n \geq 0$

4) $x_{n+1}=3x_n-2x_{n-1}$, $x_1=3$, $x_2=5$, se cere $x_n=2^n+1$, $n \geq 1$

5) $x_n=x_{n-1}+x_{n-2}$, $x_1=1$, $x_2=1$, se cere $x_n=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right]$

6) $x_n=5x_{n-1}-6x_n$, $x_0=6$, $x_1=17$, se cere $x_n=2^n+5\cdot 3^n$, $n \geq 0$

7) $x_{n+2}=x_{n+1}+6x_n$, $x_0=3$, $x_1=4$, se cere $x_n=(-2)^n+2\cdot 3^n$, $n \geq 0$

Prof. Ovidiu Bădescu

Fie $x_{n+2} = x_{n+1} + 7x_n$, $x_0 = 0$, $x_1 = 10$

Se cere $x_n \vdots 5$, $\forall n \in \mathbb{N}$

E_1) Observăm că un termen al șirului depinde de 2 termeni anteriori $\Rightarrow$ folosim inducția cu pas 2

E_2) I: $P(0): x_0 \vdots 5 \Leftrightarrow 0 \vdots 5$ adev.

$P(1): x_1 \vdots 5 \Leftrightarrow 10 \vdots 5$ adev.

II Ip. $P(k)$, $P(k+1)$ adev. $\overset{(?)}{\Longrightarrow} P(k+2)$ adev.

$P(k): x_k \vdots 5$

$P(k+1): x_{k+1} \vdots 5$

$P(k+2): x_{k+2} \vdots 5$

$x_{k+2} \overset{\text{ip}}{=\!=} x_{k+1} + 7x_k$ $\Bigg\} \Rightarrow$

Dar, $x_{k+1} \vdots 5$ și $x_k \vdots 5$

$\Rightarrow x_{k+2} \vdots 5 \Rightarrow P(k+2)$ adev. $\Rightarrow$

$\Rightarrow P(n)$ adev., $\forall n \in \mathbb{N}$

Prof. Ovidiu Badescu

Dacă $x_0=2$, $x_1=\frac{5}{2}$, $x_n=\frac{5}{2}x_{n-1}-x_{n-2}$,

să se arate $x_n=2^n+2^{-n}$, $\forall n\geq 0$

E_1) inducție cu pas 2, $P(n)$: $x_n=2^n+2^{-n}$

E_2) $P(0)$: $x_0=2^0+2^{-0} \Leftrightarrow 2=1+1$ adev.

$P(1)$: $x_1=2^1+2^{-1} \Leftrightarrow \frac{5}{2}=2+\frac{1}{2}$ adev.

E_3) Ip. $P(k)$, $P(k+1)$ adev. $\overset{(?)}{\Longrightarrow} P(k+2)$ adev.

$P(k)$: $x_k=2^k+2^{-k}$

$P(k+1)$: $x_{k+1}=2^{k+1}+2^{-k-1}$

$P(k+2)$: $x_{k+2}=2^{k+2}+2^{-k-2}$

E_4) din $x_n=\frac{5}{2}x_{n-1}-x_{n-2} \Rightarrow$

$\Rightarrow x_{k+2}=\frac{5}{2}x_{k+1}-x_k \xrightarrow{P(k),P(k+1)}$

$\Rightarrow x_{k+2}=\frac{5}{2}\left(2^{k+1}+2^{-k-1}\right)-\left(2^k+2^{-k}\right)$

$x_{k+2}=\frac{5}{2}\cdot 2^{k+1}+\frac{5}{2}\cdot 2^{-k-1}-2^k-2^{-k}$

$x_{k+2}=2^k\left(\frac{5}{2}\cdot 2-1\right)+2^{-k}\left(\frac{5}{2}\cdot\frac{1}{2}-1\right)$

$x_{k+2}=2^k\cdot 4+2^{-k}\cdot\left(\frac{5}{4}-1\right) \Rightarrow x_{k+2}=2^{k+2}+\frac{2^{-k}}{2^2}$

$\Rightarrow x_{k+2}=2^{k+2}+2^{-k-2} \Rightarrow P(k+2)$ adev.

Prof. Ovidiu Bădescu