CALCULAREA SUMELOR ȘI PRODUSELOR DE TERMEN GENERAL, $\Sigma 1, \Sigma k, \Sigma k^2, \Sigma k^3$

Calculați următoarele sume și produse, fără a le demonstra prin inducție

1) $S = 1\cdot 2 + 2\cdot 3 + \dots + n(n+1)$

2) $S = 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2$

3) $S = 1\cdot 3 + 2\cdot 4 + \dots + n(n+2)$

4) $S = 1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + \dots + n(n+1)(n+2)$

5) $S = 1\cdot 5 + 5\cdot 9 + \dots + (4n-3)(4n+1)$

6) $P = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\dots\left(1-\frac{1}{n^2}\right)$

7) $P = \frac{2^3-1}{2^3+1}\cdot\frac{3^3-1}{3^3+1}\cdot\dots\cdot\frac{n^3-1}{n^3+1}$

8) $S = 1\cdot 1! + 2\cdot 2! + \dots + n\cdot n!$

9) $S = \frac{1}{2!} + \frac{2}{3!} + \dots + \frac{n}{(n+1)!}$

10) $P = \frac{3}{4}\cdot\frac{8}{9}\cdot\dots\cdot\frac{n(n+2)}{(n+1)^2}$

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Calculați:

$$S = 1\cdot 2 + 2\cdot 3 + \dots + n(n+1)$$

$E_1)$ $S = \sum_{k=1}^{n} k(k+1) = \sum_{k=1}^{n} (k^2 + k)$

$E_2)$ $S = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$

$E_3)$ Dar, $\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$$\sum_{k=1}^{n} k = 1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

$E_4)$ $S = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} =$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right] =$$

$$= \frac{n(n+1)(2n+4)}{2\cdot 3} = \frac{n(n+1)(2n+4)}{6}$$

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Calculați

$$S = 1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + \dots + n(n+1)(n+2)$$

$E_1)$ $S = \sum_{k=1}^{n} k(k+1)(k+2)$

$E_2)$ $k(k+1)(k+2) = k(k^2+3k+2) = k^3+3k^2+2k$

$E_3)$ $S = \sum_{k=1}^{n} (k^3+3k^2+2k) = \sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$

$E_4)$ Dar, $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$,

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$E_5)$ $S = \left[\frac{n(n+1)}{2}\right]^2 + 3\cdot\frac{n(n+1)(2n+1)}{6_2} + 2\cdot\frac{n(n+1)}{2}$

$$S = \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$

$E_6)$ $S = n(n+1)\left[\frac{n(n+1)}{4} + \frac{2n+1}{2} + 1\right]$

$$S = n(n+1)\cdot\frac{n^2+n+4n+2+4}{4}$$

$$S = \frac{n(n+1)\cdot(n^2+5n+6)}{4} = \frac{n(n+1)(n+2)(n+3)}{4}$$

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Calculati:

$$P = \frac{2^3-1}{2^3+1} \cdot \frac{3^3-1}{3^3+1} \cdot \ldots \cdot \frac{n^3-1}{n^3+1}$$

$E_1)$ $P = \prod_{k=2}^{n} \frac{k^3-1}{k^3+1} = \prod_{k=2}^{n} \frac{(k-1)(k^2+k+1)}{(k+1)(k^2-k+1)} =$

$$= \prod_{k=2}^{n} \frac{k-1}{k+1} \cdot \prod_{k=2}^{n} \frac{k^2+k+1}{k^2-k+1}$$

$E_2)$ $\prod_{k=2}^{n} \frac{k-1}{k+1} = \frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5} \cdot \ldots \cdot \frac{n-1}{n+1} =$

$$= 1 \cdot 2 \cdot \frac{1}{n(n+1)} = \frac{2}{n(n+1)}$$

$E_3)$ $\prod_{k=2}^{n} \frac{k^2+k+1}{k^2-k+1} = \frac{7}{3} \cdot \frac{13}{7} \cdot \frac{21}{13} \cdot \ldots \cdot \frac{n^2+n+1}{n^2-n+1} =$

$$= \frac{n^2+n+1}{3}$$

$E_4)$ $P = \frac{2}{n(n+1)} \cdot \frac{n^2+n+1}{3} = \frac{2}{3} \cdot \frac{n^2+n+1}{n^2+n}$

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Calculați

$$S = 1 \cdot 1! + 2 \cdot 2! + \dots + n \cdot n!$$

$E_1)$ $S = \sum_{k=1}^{n} k \cdot k!$

$E_2)$ $k \cdot k! = (k+1-1)\, k! =$

$= (k+1)! - k!$

$\Rightarrow$ $k \cdot k! = (k+1)! - k!$

$E_3)$ $k=1 \Rightarrow 1 \cdot 1! = 2! - 1!$

$k=2 \Rightarrow 2 \cdot 2! = 3! - 2!$

$\dots$

$k=n \Rightarrow n \cdot n! = (n+1)! - n!$

$S = (n+1)! - 1!$

$S = (n+1)! - 1$

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Calculați:

$$S = \frac{1}{2!} + \frac{2}{3!} + \dots + \frac{n}{(n+1)!}$$

$E_1)\ S = \sum_{k=1}^{n} \frac{k}{(k+1)!}$

$E_2)\ \frac{k}{(k+1)!} = \frac{k+1-1}{(k+1)!} = \frac{k+1}{(k+1)!} - \frac{1}{(k+1)!} \Rightarrow$

$$\Rightarrow \frac{k}{(k+1)!} = \frac{1}{k!} - \frac{1}{(k+1)!}$$

$E_3)\ k=1 \Rightarrow \frac{1}{1!} - \frac{1}{2!}$

$k=2 \Rightarrow \frac{1}{2!} - \frac{1}{3!}$

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$k=n \Rightarrow \frac{1}{n!} - \frac{1}{(n+1)!}$

$$S = 1 - \frac{1}{(n+1)!}$$

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