9.1.18. ECUAȚII STANDARD CU PARTE ÎNTREAGĂ

Rezolvați ecuațiile:

1) $\left[\frac{2x-7}{2}\right] = \frac{x-2}{3}$

2) $\left[\frac{x+1}{3}\right] = \frac{x-1}{2}$

3) $\left[\frac{x+1}{2}\right] = \frac{2x+1}{3}$

4) $\left[\frac{3x-1}{4}\right] = \frac{x-1}{2}$

5) $\left[\frac{x}{2}\right] = x-2$

6) $\left[\frac{4x+1}{3}\right] = \frac{x-5}{7}$

7) $\left[\frac{-2x+1}{5}\right] = \frac{2+x}{3}$

8) $\left[\frac{4+x}{3}\right] = \frac{3x+6}{7}$

9) $\left[\frac{2x+1}{5}\right] = \frac{3x+1}{7}$

10) $\left[\frac{4-x}{5}\right] = \frac{2x+7}{3}$

Prof. Ovidiu Bădescu

Rezolvați: $\left[\frac{x+1}{2}\right]=\frac{2x+1}{3}$

E_1) not $\left[\frac{x+1}{2}\right]=k\in\mathbb{Z}\Rightarrow\frac{2x+1}{3}=k$

$\Rightarrow 2x+1=3k\Rightarrow 2x=3k-1\Rightarrow x=\frac{3k-1}{2}$

E_2) Folosim $[t]\leq t<[t]+1$

$\Rightarrow\left[\frac{x+1}{2}\right]\leq\frac{x+1}{2}<\left[\frac{x+1}{2}\right]+1$

E_3) $k\leq\frac{\frac{3k-1}{2}+1}{2}<k+1\ \Big|\cdot 2$

$2k\leq\frac{3k-1}{2}+1<2k+2\ \Big|\cdot 2$

$4k\leq 3k+1<4k+4\Rightarrow\begin{cases}4k\leq 3k+1\\ 3k+1<4k+4\end{cases}\Leftrightarrow$

$\Leftrightarrow\left\{\begin{array}{l}k\leq 1\\ k>-3\end{array}\right.\Rightarrow k\in(-3;1]$ Dar, $k\in\mathbb{Z}\Big\}\Rightarrow$

$\Rightarrow k\in\{-2;-1;0;1\}$

E_4) Dar, $x=\frac{3k-1}{2}\Rightarrow x\in\{-\frac{7}{2};-2;-\frac{1}{2};1\}$

Prof. Ovidiu Bădescu

Rezolvați: $\left[\frac{4x+1}{3}\right] = \frac{x-5}{7}$

E_1) $\left[\frac{4x+1}{3}\right] \leq \frac{x-5}{7}$

not $\left[\frac{4x+1}{3}\right] = k \in \mathbb{Z} \Rightarrow \frac{x-5}{7} = k \Rightarrow$

$\Rightarrow x = 7k+5$

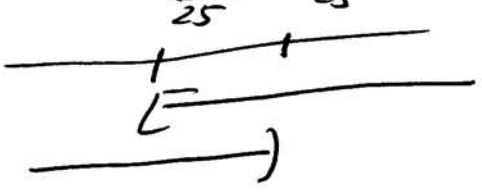
E_2) folosim $\left[\frac{4x+1}{3}\right] \leq \frac{4x+1}{3} < \left[\frac{4x+1}{3}\right] + 1$

$k \leq \frac{4 \cdot (7k+5)+1}{3} < k+1 \mid \cdot 3$

$3k \leq 28k + 21 < 3k + 3$

E_3) $\begin{cases} 3k \leq 28k+21 \Rightarrow -25k \leq 21 \mid :(-25) \\ 28k+21 < 3k+3 \Rightarrow 25k < -18 \mid :25 \end{cases}$

$\Rightarrow \begin{cases} k \geq -\frac{21}{25} \\ k < -\frac{18}{25} \end{cases} \Rightarrow$



$k \in \left[-\frac{21}{25}; -\frac{18}{25}\right) \cap \mathbb{Z} \Rightarrow k = \emptyset$

$\Rightarrow x \in \emptyset$

Prof. Ciucu Badescu

Rezolvați: $\left[\frac{4-x}{5}\right] = \frac{2x+7}{3}$

E_1) not $\left[\frac{4-x}{5}\right] = k \in \mathbb{Z} \Rightarrow \frac{2x+7}{3} = k \Rightarrow x = \frac{3k-7}{2}$

E_2) folosim $\left[\frac{4-x}{5}\right] \leq \frac{4-x}{5} < \left[\frac{4-x}{5}\right] + 1$

$\Rightarrow k \leq \frac{4 - \frac{3k-7}{2}}{5} < k+1 \;\Big|\cdot 5$

$5k \leq 4 - \frac{3k-7}{2} < 5k+5 \;\Big|\cdot 2$

$10k \leq 8 - 3k + 7 < 10k + 10$

E_3) $\begin{cases} 10k \leq 15 - 3k \\ 15 - 3k < 10k + 10 \end{cases} \Leftrightarrow \begin{cases} 13k \leq 15 \\ 13k > 5 \end{cases} \Rightarrow$

$\Rightarrow k \in \left(\frac{5}{13}; \frac{15}{13}\right]$

E_4) Dar, $k \in \mathbb{Z} \Rightarrow k \in \left(\frac{5}{13}; \frac{15}{13}\right] \cap \mathbb{Z}$

$\Rightarrow k = 1$

E_5) $x = \frac{3k-7}{2} = \frac{3-7}{2} = -2 \Rightarrow \boxed{x = -2}$

Prof. Ciprian Bădescu