INECUAȚII CU MODULE

Rezolvați:

1) $|3-x|+|x-5| \leq 2x+1$

2) $|x-5|-|x+4| \geq 3+2x$

3) $|x+5|+|3-x| > 4-2x$

4) $|x-1|-|1-x|+|x+3| \leq 4-5x$

5) $|2-x|-|x-5|-|x+4| = x+5$

6) $|4-x|-|x+1|+|x-2| < 2x+3$

7) $|4+x|+|x-3|-|x-5| < x-7$

8) $|4-x|-|x+5| > |x+7|$

9) $|5x-10|-|x+4| \leq |2-x|+1$

10) $|x-1|-|x+3| \geq |x-2|+x-3$

11) $|3-x|-|x+5| \geq 2x+1$

12) $|x-3|-|4-x| \leq 2x+6$

13) $|x+1| > 3$

14) $|5-x|-|4-x| \leq 7-x$

Prof. Ovidiu Bădescu

Rezolvați:

$$|3-x|+|x-5| \leq 2x+1$$

$E_1)$ $3-x=0 \Rightarrow x=3$

$x-5=0 \Rightarrow x=5$

$E_2)$

x		3		5	
$3-x$	$+++$	0	$---$		$---$
$x-5$	$---$		$---$	0	$+++$

$E_3)$ Cob I: $x \in (-\infty; 3] \Rightarrow (3-x)-(x-5) \leq 2x+1$

$3-x-x+5 \leq 2x+1 \Rightarrow -4x \leq -7 \mid :(-4) \Rightarrow$

$\Rightarrow x \geq \frac{7}{4} \Rightarrow x \in \left[\frac{7}{4}; \infty\right)$

$S_I = \left[\frac{7}{4}; \infty\right) \cap (-\infty; 3] = \left[\frac{7}{4}; 3\right] \Rightarrow S_I = \left[\frac{7}{4}; 3\right]$

Cob II: $x \in (3; 5] \Rightarrow -(3-x)-(x-5) \leq 2x+1$

$\Rightarrow -3+x-x+5 \leq 2x+1 \Rightarrow -2x \leq -1 \mid :(-2) \Rightarrow x \geq \frac{1}{2}$

$\Rightarrow x \in \left[\frac{1}{2}; \infty\right)$

$S_{II} = \left[\frac{1}{2}; \infty\right) \cap (3; 5] = (3; 5] \Rightarrow S_{II} = (3; 5]$

Cob III: $x \in (5; \infty) \Rightarrow -(3-x)+(x-5) \leq 2x+1 \Rightarrow$

$-3+x+x-5 \leq 2x+1 \Leftrightarrow -8 \leq 1$ adev, $\forall x \in \mathbb{R}$

$S_{III} = \mathbb{R} \cap (5; \infty) = (5; \infty) \Rightarrow S_{III} = (5; \infty)$

$E_4)$ $S_f = S_I \cup S_{II} \cup S_{III} \Rightarrow$

$S_f = \left[\frac{7}{4}; 3\right] \cup (3; 5] \cup (5; \infty) \Rightarrow \boxed{S_f = \left[\frac{7}{4}; \infty\right)}$

Prof. Ovidiu Badescu

Rezolvați:

$$|x-1|-|x+3| \geq |x-2|+x-3$$

E_1) $x-1=0 \Rightarrow x=1$

$x+3=0 \Rightarrow x=-3$

$x-2=0 \Rightarrow x=2$

E_2)

x	$-\infty$		-3		1		2		$+\infty$
$x-1$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	
$x+3$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$	
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	

E_3) $x \in (-\infty; -3] \Rightarrow -(x-1)+(x+3) \geq -(x-2)+x-3$

$-x+1+x+3 \geq -x+2+x-3$

$4 \geq -1$ adev., $\forall x \in \mathbb{R} \Rightarrow$

$$S_I = \mathbb{R} \cap (-\infty; -3] \Rightarrow S_I = (-\infty; -3]$$

$x \in (-3; 1] \Rightarrow -(x-1)-(x+3) \geq -(x-2)+x-3$

$-x+1-x-3 \geq -x+2+x-3$

$-2x \geq 1 \mid \cdot(-2) \Rightarrow x \leq -\frac{1}{2} \Rightarrow x \in (-\infty; -\frac{1}{2}]$

$$S_{II} = (-\infty; -\tfrac{1}{2}] \cap (-3; 1] = (-3; -\tfrac{1}{2}]$$

$x \in (1; 2] \Rightarrow x-1-(x+3) \geq -(x-2)+x-3 \Rightarrow -4 \geq -1$ fals

$\Rightarrow x \in \emptyset \Rightarrow S_{III} = \emptyset \cap (1; 2] = \emptyset$

$x \in (2; \infty) \Rightarrow x-1-x-3 \geq x-2+x-3 \Rightarrow x \leq \frac{1}{2} \Rightarrow x \in (-\infty; \frac{1}{2}]$

$$S_{IV} = (-\infty; \tfrac{1}{2}] \cap (2; \infty) = \emptyset$$

E_4) $S_{\not\in} = S_I \cup S_{II} \cup S_{III} \cup S_{IV} = (-\infty; -3] \cup (-3; -\frac{1}{2}] \cup \emptyset \cup \emptyset = (-\infty; -\frac{1}{2}]$

Prof. Ovidiu Bădescu

Rezolvați:

$$|3-x|-|x+5| \geq 2x+1$$

E1) $3-x=0 \Rightarrow x=3$

$x+5=0 \Rightarrow x=-5$

E2)

x	$-\infty$	-5		3	$+\infty$
$3-x$	$+$	$+$	$+$	0	$-$
$x+5$	$-$	0	$+$	$+$	$+$

E3) $\overset{\text{Cz. I}}{x \in (-\infty; -5]} \Rightarrow (3-x)+(x+5) \geq 2x+1$

$8 \geq 2x+1 \Rightarrow -2x \geq -7 \mid :(-2) \Rightarrow x \leq \frac{7}{2}$

$\Rightarrow x \in (-\infty; \frac{7}{2}] \cap (-\infty; -5] = (-\infty; -5] = S_I$

Cz. II $x \in (-5; 3] \Rightarrow (3-x)-(x+5) \geq 2x+1$

$-2x-2 \geq 2x+1 \Rightarrow -4x \geq 3 \mid :(-4)$

$x \leq -\frac{3}{4} \Rightarrow x \in (-\infty; -\frac{3}{4}] \cap (-5; 3] = (-5; -\frac{3}{4}]$

Cz. III: $x \in (3; \infty) \Rightarrow -(3-x)-(x+5) \geq 2x+1$ S_{II}

$-3+x-x-5 \geq 2x+1 \Leftrightarrow -2x \geq 9 \mid :(-2)$

$\Leftrightarrow x \leq -\frac{9}{2} \Rightarrow x \in (-\infty; -\frac{9}{2}] \cap (3; \infty) = \emptyset$

$\Rightarrow S_{III} = \emptyset$

E4) $S_f = S_I \cup S_{II} \cup S_{III} = (-\infty; -5] \cup (-5; -\frac{3}{4}] \cup \emptyset =$

$= (-\infty; -\frac{3}{4}]$

Prof. Ovidiu Bădescu

Rezolvați: $|x-3|-|4-x| \leq 2x+6$

E_1) $x-3=0 \Rightarrow x=3$

$4-x=0 \Rightarrow x=4$

E_2)

x		3		4	
$x-3$	$-\ -\ -$	0	$+$	$+$	$+$
$4-x$	$+$	$+$	$+$	0	$-\ -\ -$

E_3) Caz I: $x \in (-\infty, 3] \Rightarrow -(x-3)-(4-x) \leq 2x+6$

$-x+3-4+x \leq 2x+6 \Rightarrow -2x \leq 7 \mid :(-2)$

$\Rightarrow x \geq -\frac{7}{2} \Rightarrow x \in \left[-\frac{7}{2}; \infty\right)$

$S_I = \left[-\frac{7}{2}; \infty\right) \cap (-\infty; 3] = \left[-\frac{7}{2}; 3\right]$

Caz II: $x \in (3; 4] \Rightarrow (x-3)-(4-x) \leq 2x+6$

$x-3-4+x \leq 2x+6 \Rightarrow -7 \leq 6$ adev, $\forall x \in \mathbb{R}$

$S_{II} = \mathbb{R} \cap (3; 4] = (3; 4]$

Caz III: $x \in (4; \infty) \Rightarrow (x-3)+(4-x) \leq 2x+6$

$x-3+4-x \leq 2x+6 \Rightarrow -2x \leq 5 \mid :(-2)$

$x \geq -\frac{5}{2} \Rightarrow x \in \left[-\frac{5}{2}; \infty\right)$

$S_{III} = \left[-\frac{5}{2}; \infty\right) \cap (4; \infty) = (4; \infty)$

E_4) $S_f = S_I \cup S_{II} \cup S_{III} = \left[-\frac{7}{2}; 3\right] \cup (3; 4] \cup (4; \infty)$

$\Rightarrow S_f = \left[-\frac{7}{2}; \infty\right)$

Profesor Badescu

Rezolvați: $|x+1| > 3$

E_1) $x+1 = 0 \Rightarrow x = -1$

E_2)

x		-1	
$x+1$	$- - - -$	0	$+ + +$

E_3) Cazul I: $x \in (-\infty; -1] \Rightarrow -x-1 > 3 \Rightarrow -x > 4 \mid \cdot(-1)$

$x < -4 \Rightarrow x \in (-\infty; -4)$

$S_I = (-\infty; -4) \cap (-\infty; -1] = (-\infty; -4)$

E_4) Cazul II: $x \in (-1; \infty) \Rightarrow x+1 > 3 \Rightarrow x > 2$

$\Rightarrow x \in (2; \infty)$

$S_{II} = (2; \infty) \cap (-1; \infty) = (2; \infty)$

E_5) $S_f = S_I \cup S_{II} = (-\infty; -4) \cup (2; \infty)$

Prof. Ovidiu Badescu

Rezolvați:

$|5-x|-|4-x| \leq 7-x$

$E_1)$ $5-x=0 \Rightarrow x=5$

$4-x=0 \Rightarrow x=4$

$E_2)$

x		4		5	
$5-x$	$+$	$+$	$+$	0	$-$
$4-x$	$+$	0	$-$	$-$	$-$

$E_3)$ Cazul I: $x \in (-\infty; 4] \Rightarrow (5-x)-(4-x) \leq 7-x$

$5-x-4+x \leq 7-x \Rightarrow x \leq 6 \Rightarrow x \in (-\infty; 6]$

$S_I = (-\infty; 6] \cap (-\infty; 4] = (-\infty; 4]$

Cazul II: $x \in (4; 5] \Rightarrow (5-x)+(4-x) \leq 7-x$

$\Rightarrow 9-2x \leq 7-x \Rightarrow -x \leq -2 \mid \cdot(-1) \Rightarrow x \geq 2$

$\Rightarrow x \in [2; \infty)$, $S_{II} = [2; \infty) \cap (4; 5] = (4; 5]$

Cazul III: $x \in (5; \infty) \Rightarrow -(5-x)+(4-x) \leq 7-x$

$-5+x+4-x \leq 7-x \Rightarrow x \leq 8 \Rightarrow x \in (-\infty; 8]$

$S_{III} = (-\infty; 8] \cap (5; \infty) = (5; 8]$

$E_4)$ $S_f = S_I \cup S_{II} \cup S_{III} = (-\infty; 4] \cup (4; 5] \cup (5; 8]$

$= (-\infty; 8]$

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