EN / 2025 / MODEL / III 5

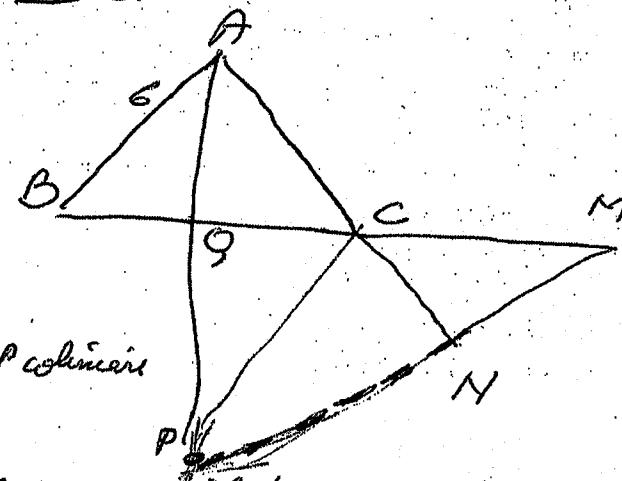
ΔABC echilateral, $AB = 6$

B, C, M coliniare, C mijl BM

N proiecția lui M pe AC

BC mediat. AP

a) arată $NC = 3$ cm b) M, N, P coliniare



E1) $MN \perp CN \Rightarrow m(M\hat{N}C) = 90°$, ΔABC echilat $\Rightarrow m(A\hat{C}B) = 60°$

$\Rightarrow m(M\hat{C}N) = 60° \Rightarrow m(C\hat{M}N) = 30° \Rightarrow CN = \frac{1}{2} CM$

Dar, C mijl $BM \Rightarrow BC \equiv CM = 6$ cm $\Rightarrow NC = 3$ cm

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b) $C \in$ mediat. $AP \Rightarrow CA \equiv CP$ (sau $\Delta CAQ \equiv \Delta CPQ$)

$\Rightarrow CP = 6$ cm și $m(C\hat{P}Q) = 60° \Rightarrow m(A\hat{C}P) = 120° \Rightarrow$

$m(N\hat{C}P) = 60° \Rightarrow N\hat{C}P \equiv N\hat{C}M$ și $CP \equiv CM = 6$, cu bd comună

$\Rightarrow \Delta CMN \equiv \Delta CPN \Rightarrow C\hat{N}M \equiv C\hat{N}P$ și cum $MN \perp AC$

$\Rightarrow m(C\hat{N}P) = 90° \Rightarrow m(M\hat{N}P) = 180° \Rightarrow$ M, N, P coliniare

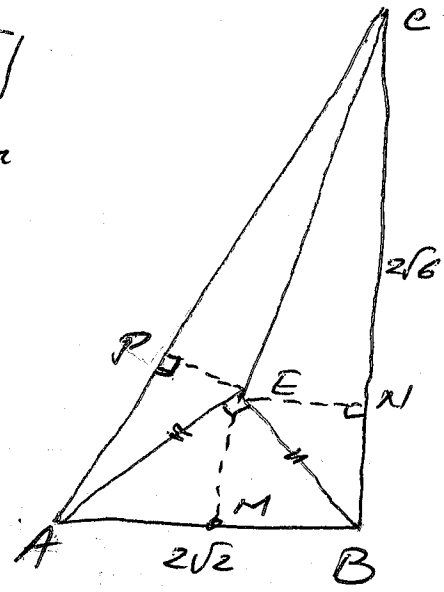
EN / 2024 / MODEL / III 5

$m(A\hat{B}C) = 90°$, $AB = 2\sqrt{2}$ cm, $BC = 2\sqrt{6}$ cm

ΔAEB dreptunghic isoscel cu $AE = EB$

a) arata ca perimetrul ΔABC este $2\sqrt{2}(3+\sqrt{3})$ cm

b) distanta de la E la AC



a) $P_{ABC} = AB + BC + CA =$

$= 2\sqrt{2} + 2\sqrt{6} + CA$

ΔABC dreptunghic $\Rightarrow AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = (2\sqrt{2})^2 + (2\sqrt{6})^2$

$\Rightarrow AC^2 = 8 + 24 = 32 \Rightarrow AC = 4\sqrt{2}$

$P_{\Delta ABC} = 2\sqrt{2} + 2\sqrt{6} + 4\sqrt{2} = 6\sqrt{2} + 2\sqrt{6} = 2\sqrt{2}(3+\sqrt{3})$ cm

b) Fie $EP \perp AC$ si stim a gasi EP, folosim arii

$\mathcal{A}_{AEC} = \frac{AC \cdot EP}{2} = \frac{4\sqrt{2} \cdot EP}{2} = 2\sqrt{2} \cdot EP$ cm^2

$\mathcal{A}_{AEC} = \mathcal{A}_{ABC} - \mathcal{A}_{ABE} - \mathcal{A}_{BEC}$

$\mathcal{A}_{ABC} = \frac{AB \cdot BC}{2} = \frac{2\sqrt{2} \cdot 2\sqrt{6}}{2} = 2\sqrt{12} = 2 \cdot 2\sqrt{3} = \boxed{4\sqrt{3}\text{ cm}^2}$

$\mathcal{A}_{BEC} = ?$, Fie $EN \perp BC$ si cum $AB \perp BC \Rightarrow EN \parallel AB$

Fie $EM \perp AB \Rightarrow EMBN$ dreptunghi

Dar, EM inaltime in Δ isoscel EAB cu $AE = BE \Rightarrow$

$\Rightarrow M$ mijl lui $AB \Rightarrow MB = \sqrt{2} \Rightarrow EN = \sqrt{2}$

$\mathcal{A}_{BCE} = \frac{AB \cdot EN}{2} = \frac{2\sqrt{6} \cdot \sqrt{2}}{2} = \sqrt{12} = \boxed{2\sqrt{3}\text{ cm}^2}$

$\mathcal{A}_{AEB} = \frac{AB \cdot EM}{2}$ si EM mediana $\Rightarrow EM = \frac{BC}{2} = \sqrt{2}$

$\Rightarrow \mathcal{A}_{AEB} = \frac{2\sqrt{2} \cdot \sqrt{2}}{2} = \boxed{2\text{ cm}^2} \Rightarrow \mathcal{A}_{AEC} = 4\sqrt{3} - 2\sqrt{3} - 2 = \boxed{2(\sqrt{3}-1)}$

$\mathcal{A}_{AEC} = 2(\sqrt{3}-1)$ cm^2 si $\mathcal{A}_{AEC} = 2\sqrt{2} \cdot EP$ cm^2 $\Rightarrow 2\sqrt{2} \cdot EP = 2(\sqrt{3}-1)$

$\Rightarrow EP = \frac{2(\sqrt{3}-1)}{2\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3}-1)}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{2}\text{ cm}}$

EN/2024/SIMULARE/III 5

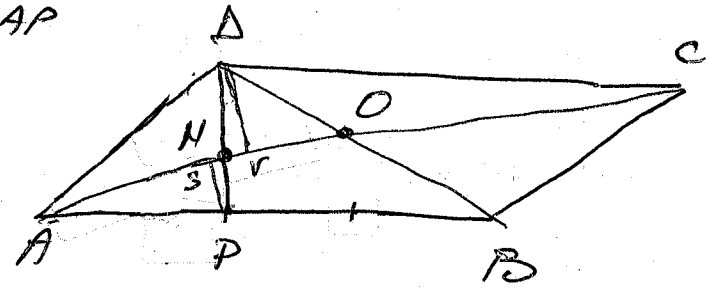
$ABCD$ paralelogram, $PB=2AP$

$AB=15$ cm

$PB=2AP$

a) arate $AP=5$ cm

b) $\frac{\mathcal{A}_{ANP}}{\mathcal{A}_{\Delta NO}}=?$



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a) $AP=x \Rightarrow PB=2x \Rightarrow x+2x=15 \Rightarrow 3x=15 \Rightarrow x=5$

$\Rightarrow AP=5$ cm

b) Fie $PS \perp AN$, $\Delta V \perp AN \Rightarrow \Delta SNP \sim \Delta VN\Delta$ (de ce)

$S\hat{N}P \equiv V\hat{N}\Delta$ (opuse), $PS\hat{N} \equiv \Delta V\hat{N} (=90^\circ)$

$\Rightarrow \frac{SN}{VN}=\frac{NP}{N\Delta}=\frac{PS}{\Delta V}$

Dar, $\Delta ANP \sim \Delta CN\Delta$ ($\hat{A} \equiv \hat{C}$ - alt interne, $A\hat{N}P \equiv C\hat{N}\Delta$ opuse) $\Rightarrow$

$\Rightarrow \frac{AN}{CN}=\frac{NP}{N\Delta}=\frac{PA}{\Delta C}$ (2)

Cum $PA=5$ cm, $\Delta C=AB=15$ cm $\Rightarrow \frac{PA}{\Delta C}=\frac{5}{15}=\frac{1}{3}$

$\Rightarrow \frac{NP}{N\Delta}=\frac{1}{3}$ și din (1) $\Rightarrow \frac{SN}{VN}=\frac{PS}{\Delta V}=\frac{1}{3}$

$$\frac{\mathcal{A}_{ANP}}{\mathcal{A}_{\Delta NO}}=\frac{\frac{PS\cdot AN}{2}}{\frac{\Delta V\cdot NO}{2}}=\frac{PS}{\Delta V}\cdot\frac{AN}{NO}=\frac{1}{3}\cdot\frac{AN}{NO}$$

Din (2) $\Rightarrow \frac{AN}{NC}=\frac{1}{3} \Rightarrow 3AN=NC$, $AN=x \Rightarrow NC=3x$

$\Rightarrow AC=4x \Rightarrow AO=2x \mid \Rightarrow NO=x \Rightarrow \frac{AN}{NO}=\frac{x}{x}=1$

Dar, $AN=x$

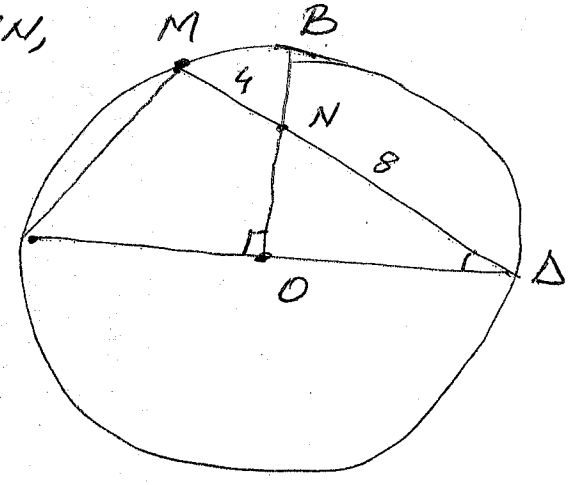
$\Rightarrow \frac{\mathcal{A}_{ANP}}{\mathcal{A}_{\Delta NO}}=\frac{1}{3}\cdot 1=\frac{1}{3}$

EN | 2024 | EXAMEN | III 5

CD diametru, $BO \perp CD$, $DN = 2 \cdot MN$, $MN = 4$ cm

a) arată $m(\hat{CMD}) = 90°$

b) aria $\triangle DON$



a) CD diametru, $M \in$ cercului

$\Rightarrow m(\hat{CMD}) = \frac{m(\overset{\frown}{CD})}{2} = \frac{180°}{2} = 90°$

b) $A_{DON} = ?$ $MN = 4$ cm, $DN = 2MN \Rightarrow DN = 8$ cm $\Rightarrow MD = 12$ cm

$\triangle DON \sim \triangle DMC$ ($\measuredangle D$ comun și $\hat{O} = \hat{M} = 90°$)

$\Rightarrow \frac{DO}{DM} = \frac{ON}{MC} = \frac{ND}{CD}$, notăm $DO = R \Rightarrow DC = 2R$

$\Rightarrow \frac{R}{12} = \frac{ON}{MC} = \frac{8}{2R} \Rightarrow \frac{R}{12} = \frac{8}{2R} \Rightarrow 2R^2 = 96$

$\Rightarrow R^2 = 48 \Rightarrow R = \sqrt{48} = 4\sqrt{3} \Rightarrow \boxed{R = 4\sqrt{3} \text{ cm}}$

Înlocuim în T. lui Pitagora în $\triangle MCD \Rightarrow CD^2 = MC^2 + MD^2 \Rightarrow$

$(8\sqrt{3})^2 = MC^2 + 12^2 \Rightarrow 192 = MC^2 + 144$

$\Rightarrow MC^2 = 192 - 144 = 48 \Rightarrow MC = \sqrt{48} = 4\sqrt{3} \Rightarrow \boxed{MC = 4\sqrt{3}}$

Înlocuim în $\frac{R}{12} = \frac{ON}{MC} \Rightarrow \frac{4\sqrt{3}}{12} = \frac{ON}{4\sqrt{3}} \Rightarrow ON = \frac{4\sqrt{3} \cdot 4\sqrt{3}}{12}$

$\Rightarrow ON = \frac{\overset{4}{\cancel{16} \cdot 3}}{\underset{3}{\cancel{12}}} = \frac{4 \cdot 3}{3} = 4$

$A_{DON} = \frac{DO \cdot ON}{2} = \frac{4\sqrt{3} \cdot 4}{2} = 8\sqrt{3} \text{ cm}^2$

EN / 2024 / REZERVĂ / III 5

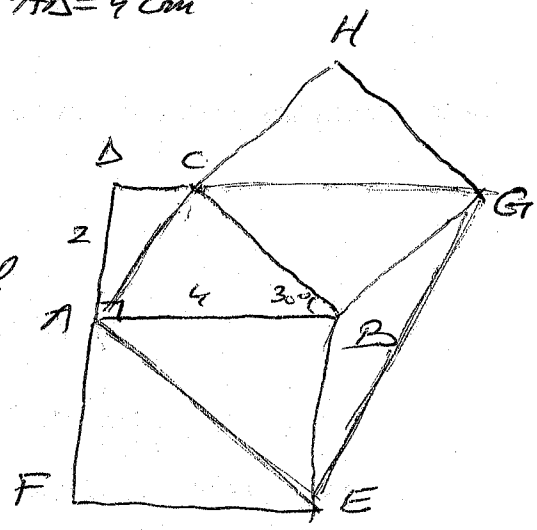
$ABCD$ trapez, $AB \parallel CD$, $AB = 2 \cdot AD = 4$ cm

$m(BAD) = 90°$, $m(ABC) = 30°$

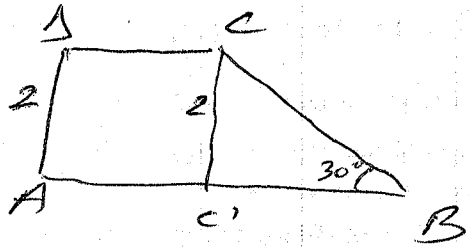
$ABEF$ și $BCHG$ pătrate

a) Să se arate $BC = 4$ cm

b) Arată că $AEGC$ trapez isoscel



a) $\left.\begin{matrix} AB = 4\text{cm} \\ AB = 2AD \end{matrix}\right\} \Rightarrow AD = 2$ cm



Fie $CC' \perp AB$, $m(C') = 90°$, $m(B) = 30°$

$\Rightarrow CC' = \frac{BC}{2}$

Dar, $AD = CC' \Rightarrow CC' = 2$ cm $\left.\right\}$

$\Rightarrow \frac{BC}{2} = 2$ cm $\Rightarrow BC = 4$ cm

b) Cum $BC = 4$cm, $AB = 4$cm $\Rightarrow \Delta ABC$ isoscel, fie $m(\hat{CAB}) = x$

$\Rightarrow$ teorema unghi $\Delta ABC \Rightarrow 2x + 30° = 180° \Rightarrow x = \frac{150°}{2} = 75°$

$\Rightarrow m(\hat{CAE}) = 75° + 45° = 120° \Rightarrow \boxed{m(\hat{CAE}) = 120°}$ (1)

Cum $ABEF$ pătrat $\Rightarrow BE = AB \Rightarrow BE = 4$ cm $\left.\right| \Rightarrow BE = BG$

$CBGH$ pătrat $\Rightarrow BC = BG \Rightarrow BG = 4$cm $\left.\right| \Rightarrow \Delta GBE$ isoscel

Scriem $\measuredangle$ în jurul lui $B \Rightarrow 30° + 90° + m(\hat{EBG}) + 90° = 360°$

$\Rightarrow \boxed{m(\hat{EBG}) = 150°}$

ΔGBE isoscel, fie $m(\hat{BEG}) = y \Rightarrow 2y + 150° = 180°$

$\Rightarrow 2y = 30° \Rightarrow y = 15°$

$\Rightarrow m(\hat{AEG}) = 45° + 15° \Rightarrow \boxed{m(\hat{AEG}) = 60°}$ (2)

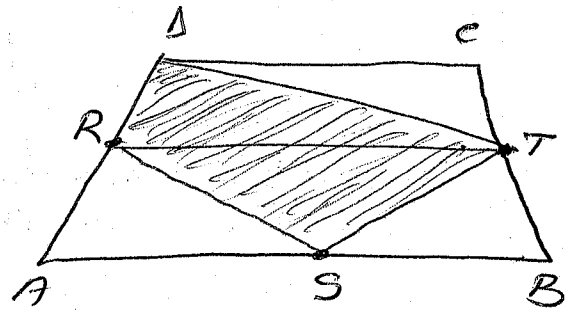
Din (1) + (2) $\Rightarrow m(\hat{CAE}) + m(\hat{AEG}) = 120° + 60° = 180° \Rightarrow AC \parallel EG$

Arătăm că $AE \nparallel GG$

$m(\hat{CGE}) + m(\hat{GEA}) = (45° + 15°) + (15° + 45°) = 120° \neq 120°$

$\Rightarrow \boxed{AE \nparallel GG} \Rightarrow AEGC$ trapez

[EN/2023/MODEL/III 5]



$ABC\Delta$ trapez, $AB=6$ cm, $C\Delta=4$ cm
și R, S, T mijloace
a) arată că $RT=5$ cm
b) arată că aria lui ΔRST este jumătate din aria lui $ABC\Delta$

---

a) R, T mijloace $\Rightarrow RT$ l mijl $\Rightarrow RT=\frac{AB+C\Delta}{2}=\frac{6+4}{2}=5$ cm

b) $\mathcal{A}_{\Delta RST}=\mathcal{A}_{\Delta RT}+\mathcal{A}_{RST}$

Fie $\Delta\Delta_1\perp RS$, not $\Delta\Delta_1=h_1$ și $TT_1\perp AB\Rightarrow TT_1=h_1$ deoarece RS linie mijlocie $\Rightarrow \Delta\Delta_1\equiv TT_1$

$\mathcal{A}_{\Delta RT}=\frac{RT\cdot h_1}{2}=\frac{5h_1}{2}$; $\mathcal{A}_{RST}=\frac{RT\cdot h_1}{2}=\frac{5h_1}{2}$

$\Rightarrow \mathcal{A}_{\Delta RST}=\frac{5h_1}{2}+\frac{5h_1}{2}=\frac{10h_1}{2}=5h_1$

$\mathcal{A}_{ABC\Delta}=\frac{(AB+C\Delta)\cdot 2h_1}{2}=(6+4)\cdot h_1=10h_1$

$\Rightarrow \mathcal{A}_{\Delta RST}=\frac{1}{2}\mathcal{A}_{ABC\Delta}$

EN/2025/SIMULARE/III 5

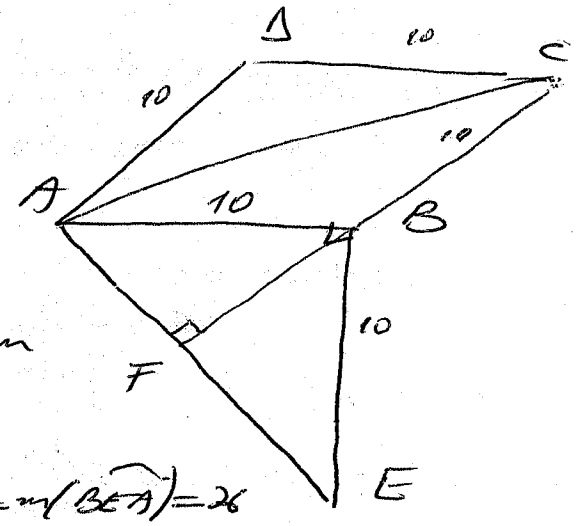
$ABCD$ romb, $m(B\hat{A}D) = 45^\circ$

ΔABE dreptunghic isoscel cu

$AB = BE = 10$ cm

a) arătați că $DA \perp AE$

b) arătați $tg(C\hat{A}E) = (\sqrt{2}+1)$ cm



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a) ΔABE dreptunghic isoscel $\Rightarrow m(B\hat{A}D) = m(B\hat{E}A) = x$

suma unghiurilor $\Delta ABE \Rightarrow 2x + 90^\circ = 180^\circ \Rightarrow 2x = 90^\circ$

$\Rightarrow x = 45^\circ \Rightarrow m(B\hat{A}E) = 45^\circ$

$m(D\hat{A}E) = m(D\hat{A}B) + m(B\hat{A}E) = 90^\circ + 90^\circ = 180^\circ$

b) se cere $tg(C\hat{A}E)$, încadrăm $\sphericalangle CAE$ într-un triunghi dreptunghic

Cum $DA \perp AE$ și $CB \parallel DA \Rightarrow CB \perp AE$, fie $CB \cap AE = \{F\}$

$\Rightarrow BF$ înălțime în Δ isoscel $\Rightarrow F$ mijl lui AE

În Δ Pitag $\Rightarrow AB^2 + BE^2 = AE^2 \Rightarrow 100 + 100 = AE^2 \Rightarrow$

$AE^2 = 200 \Rightarrow AE = 10\sqrt{2} \Rightarrow \boxed{AF = 5\sqrt{2} \text{ cm}}$

În Δ Pitag $\Delta ABF \Rightarrow AF^2 + BF^2 = AB^2 \Rightarrow 50 + BF^2 = 100$

$\Rightarrow BF^2 = 50 \Rightarrow \boxed{BF = 5\sqrt{2}}$

(se putea direct, $BF = \frac{AE}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$)

(se putea $BF = \frac{c_1 \cdot c_2}{h}$) $\Rightarrow CF = 10 + 5\sqrt{2} = 5(2+\sqrt{2})$

$tg(C\hat{A}F) = \frac{\text{cat opus}}{\text{cat adj}} = \frac{CF}{AF} = \frac{5(2+\sqrt{2})}{5\sqrt{2}} = \frac{\sqrt{2}}{2+\sqrt{2}} =$

$= \frac{2\sqrt{2}+2}{2} = \frac{2(\sqrt{2}+1)}{2} = (\sqrt{2}+1)$ cm

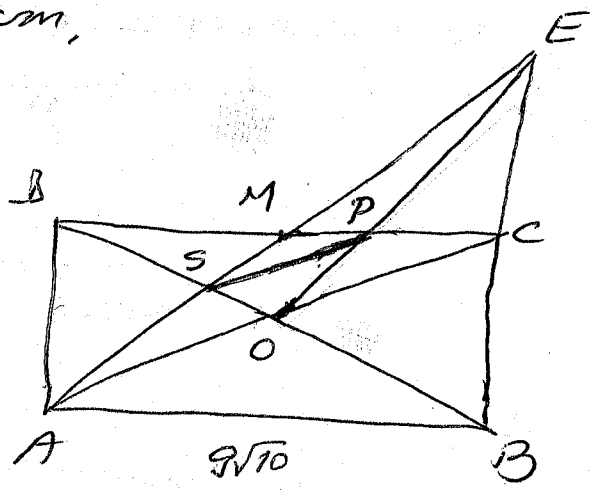
Cum $C\hat{A}F = C\hat{A}E \Rightarrow tg(C\hat{A}E) = (\sqrt{2}+1)$ cm

[EN/2023/EXAMEN/III 4]

$ABCD$ dreptunghi, $AB = 9\sqrt{10}$ cm, $AC = 30$ cm, M mijl lui CD

a) arată că aria lui $ABCD$ este $270\ cm^2$

b) arată că $SP = 10$ cm



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a) ΔABC, $m(\hat{B}) = 90°$ $\xrightarrow{T.\ Pitagora}$

$AC^2 = AB^2 + BC^2 \Rightarrow 30^2 = (9\sqrt{10})^2 + BC^2 \Rightarrow 900 = 810 + BC^2$

$\Rightarrow BC^2 = 900 - 810 \Rightarrow BC^2 = 90 \Rightarrow BC = 3\sqrt{10}$

$A_{ABCD} = AB \cdot BC = 9\sqrt{10} \cdot 3\sqrt{10} = 27 \cdot 10 = 270\ cm^2$

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b) O mijl lui $AC \Rightarrow EO$ mediană în ΔACE (1)

M mijl lui CD, $MC = \frac{CD}{2} \Rightarrow MC = \frac{AB}{2}$ și $MC \parallel AB \Rightarrow$

MC linie mijlocie în $\Delta ABE \Rightarrow M$ mijl $AE \Rightarrow CM$ mediană

în $\Delta ACE \xrightarrow{(1)} P$ centru greutate $\Delta ACE \Rightarrow$

$\Rightarrow \frac{MP}{MC} = \frac{1}{3}$ (2)

Analog, în ΔACD, O mijl $AC \Rightarrow DO$ mediană
M mijl $CD \Rightarrow AM$ mediană $\Big\} \Rightarrow$

$\Rightarrow S$ centru greutate $\Delta ACD \Rightarrow \frac{MS}{MA} = \frac{1}{3}$ (3)

Din (2) + (3) $\Rightarrow \frac{MP}{MC} = \frac{MS}{MA} \Rightarrow \Delta MPC \sim \Delta MCA$

$\Rightarrow \frac{MP}{MC} = \frac{MS}{MA} = \frac{SP}{AC} = \frac{1}{3}$

Cum $AC = 30$ cm $\Big\} \Rightarrow \frac{SP}{30} = \frac{1}{3} \Rightarrow SP = \frac{30}{3}$

$\Rightarrow SP = 10$ cm