EN / 2025 / MODEL / Nr. 2

$$E(x)=\left(\frac{x+4}{x-3}-\frac{x-3}{x+4}+\frac{49}{x^2+x-12}\right):\frac{7}{x-3},\ x\neq -4,\ x\neq 3$$

a) arătăm că $x^2+x-12=(x-3)(x+4)$

met. I: $T_d=(x-3)(x+4)=x^2+4x-3x-12=x^2+x-12=T_s$

$\Rightarrow$ egalitate

met. II: $T_s=x^2+x-12=x^2+4x-3x-12=$

$=x(x+4)-3(x+4)=(x+4)(x-3)=(x-3)(x+4)$

$\Rightarrow$ egalitate

b) arătăm $N=\sqrt{E(2)+E(4)+\dots+E(16)}$ este natural

$$E(x)=\left(\frac{x+4}{x-3}-\frac{x-3}{x+4}+\frac{49}{(x-3)(x+4)}\right)\cdot\frac{x-3}{7}$$

$$E(x)=\frac{(x+4)^2-(x-3)^2+49}{(x-3)(x+4)}\cdot\frac{x-3}{7}$$

$$E(x)=\frac{(x^2+8x+16)-(x^2-6x+9)+49}{x+4}\cdot\frac{1}{7}$$

$$E(x)=\frac{x^2+8x+16-x^2+6x-9+49}{7(x+4)}=\frac{14x+56}{7(x+4)}=$$

$$=\frac{14(x+4)}{7(x+4)}=2,\ \forall x\in\mathbb{R}\setminus\{-4,3\}\Rightarrow E(2)=2,\ E(4)=2,\ \dots\ E(16)=2$$

Obs. $E(2)+E(4)+\dots+E(16)=\underbrace{2+2+\dots+2}_{8\text{ ori}}=16$

$\Rightarrow N=\sqrt{16}\in\mathbb{N}$

[EN/2024/MODEL/III.2]

$E(x)=\left(\frac{x}{9+3x}-\frac{2}{x+3}+\frac{3}{x^2+3x}\right):\left(\frac{x}{3}+\frac{3}{x}-2\right)$ unde

$x\neq -3, x\neq 0, x\neq 3$

a) arata ca $\frac{x}{9+3x}-\frac{2}{x+3}+\frac{3}{x^2+3x}=\frac{(x-3)^2}{3x(x+3)}$

b) $n=?, n\in\mathbb{N}$ a.i. $5\cdot E(n)\in\mathbb{N}$

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a) $\frac{x}{9+3x}-\frac{2}{x+3}+\frac{3}{x^2+3x}=\frac{x}{3(3+x)}-\frac{2}{x+3}+\frac{3}{x(x+3)}=$

$=\frac{x^2-6x+9}{3x(x+3)}=\frac{(x-3)^2}{3x(x+3)}$

b) $E(x)\overset{a)}{=}\frac{(x-3)^2}{3x(x+3)}:\left(\frac{x}{3}+\frac{3}{x}-2\right)=$

$=\frac{(x-3)^2}{3x(x+3)}:\frac{x^2+9-6x}{3x}=\frac{(x-3)^2}{3x(x+3)}\cdot\frac{3x}{(x-3)^2}=\frac{1}{x+3}$

Dar, $5\cdot E(n)\in\mathbb{N}\Rightarrow \frac{5}{n+3}\in\mathbb{N}$, $n\in\mathbb{N}$ $\Rightarrow n+3\in D_5$

$\Rightarrow n+3\in\{\pm 1;\pm 5\}$

$n+3=1\Rightarrow n=-2\notin\mathbb{N}$

$n+3=-1\Rightarrow n=-4\notin\mathbb{N}$

$n+3=5\Rightarrow n=2\in\mathbb{N}$

$n+3=-5\Rightarrow n=-8\notin\mathbb{N}$

$S_f: n=2$

EN / 2024 / SIMULARE / III 2

$E(x)=(2x+3)^2+(x-2)(x+2)-3(1-x)+2$

a) arată că $E(0)=4$

b) arată că $N=E(n)+6$ este divizibil cu 10, pentru orice număr natural $n$

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a) $E(0)=(0+3)^2+(0-2)(0+2)-3(1-0)+2$

$E(0)=9-4-3+2 \Rightarrow E(0)=11-7=4$

b) $E(n)=(2n+3)^2+(n-2)(n+2)-3(1-n)+2$

$E(n)=(2n)^2+2\cdot 2n\cdot 3+3^2+n^2-2^2-3+3n+2$

$E(n)=4n^2+12n+9+n^2-4-3+3n+2$

$E(n)=5n^2+15n+4$

$N=E(n)+6=(5n^2+15n+4)+6=5n^2+15n+10$

$\Rightarrow N=5(n^2+3n+2) \Rightarrow N \vdots 5$

Cum $n^2+3n+2=n(n+3)+2$

Dacă $n$ par $\Rightarrow n(n+3)$ par $\Rightarrow n(n+3)+2$ par

$\Rightarrow (n^2+3n+2) \vdots 2 \Rightarrow N \vdots 2$

Dacă $n$ impar $\Rightarrow n+3$ par $\Rightarrow n(n+3)$ par $\Rightarrow$

$n(n+3)+2$ par $\Rightarrow (n^2+3n+2) \vdots 2 \Rightarrow N \vdots 2$

Așadar, $N \vdots 5$ și $N \vdots 2$ iar $(5,2)=1$

$\Rightarrow N \vdots 10$

EN / 2024 / EXAMEN / III 2

$$E(x)=\left(\frac{1}{x^2-3x+2}+\frac{1}{x-1}\right)\cdot(x^2-4)$$

a) arată $x^2-3x+2=(x-1)(x-2)$

b) găsește $n\in\mathbb{Z}$, $n\neq 1$, $n\neq 2$ astfel încât $N=\frac{5}{E(n)}\in\mathbb{N}$

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a) $(x-1)(x-2)=x^2-2x-x+2=x^2-3x+2$

$\Rightarrow x^2-3x+2=(x-1)(x-2)$

b) $E(x)=\left(\frac{1}{(x-1)(x-2)}+\frac{x-2}{x-1}\right)\cdot(x^2-4)$

$$E(x)=\frac{1+x-2}{(x-1)(x-2)}\cdot(x-2)(x+2)$$

$$E(x)=\frac{x-1}{(x-1)}\cdot(x+2)=x+2$$

$$N=\frac{5}{E(n)}=\frac{5}{n+2},\ n\in\mathbb{Z}\Rightarrow n+2\in D_5\Rightarrow n+2\in\{\pm1;\pm5\}$$

cu $n\neq1$, $n\neq2$ și $N\in\mathbb{N}$ $\Rightarrow n+2\in\{1;5\}$

---

$n+2=1\Rightarrow n=1-2=-1\in\mathbb{Z}$

$n+2=5\Rightarrow n=5-2=3\in\mathbb{Z}$

$S_f:\ n\in\{-1;3\}$

EN / 2024 / REZERVĂ / III 2

$$E(x)=\left(\frac{2}{x-2}+\frac{x}{x+2}\right):\frac{x^2+4}{x^2+4x+4},\ x\neq -2,\ x\neq 2$$

a) arată că $\frac{2}{x-2}+\frac{x}{x+2}=\frac{x^2+4}{(x-2)(x+2)}$

b) $n=?$, $n\in\mathbb{N}$, $n\neq 2$ a.î. $N=E(n)-1$ este natural

a) $\frac{2}{x-2}+\frac{x}{x+2}=\frac{2(x+2)+x(x-2)}{(x-2)(x+2)}=\frac{2x+4+x^2-2x}{(x-2)(x+2)}=$

$$=\frac{x^2+4}{(x-2)(x+2)}$$

b) $E(x)\overset{a)}{=}\frac{x^2+4}{(x-2)(x+2)}:\frac{x^2+4}{(x+2)^2}=$

$$=\frac{x^2+4}{(x-2)(x+2)}\cdot\frac{(x+2)^2}{x^2+4}=\frac{x+2}{x-2}$$

$$N=E(n)-1=\frac{n+2}{n-2}-\frac{n-2}{1}=\frac{(n+2)-(n-2)}{n-2}=\frac{4}{n-2}$$

$N\in\mathbb{N}\Rightarrow n-2\in\{1;2;4\}$ și $n\in\mathbb{N}$, $n\neq 2$

$n-2=1\Rightarrow n=3$

$n-2=2\Rightarrow n=4$ $\Rightarrow n\in\{3;4;6\}$

$n-2=4\Rightarrow n=6$

EN / 2023 / MODEL / III 2

$E(x)=\left(\frac{2}{x-2}+\frac{x}{x+2}\right):\frac{x^2+4}{x^2-x-2}$, $x\in\mathbb{R}\setminus\{-2,-1,2\}$

a) arată că $E(x)=\frac{x+1}{x+2}$, $\forall x\in\mathbb{R}\setminus\{-2,-1,2\}$

b) găsește $a\in\mathbb{Z}$ a.î. $E(a)\in\mathbb{Z}$

---

a) $E(x)=\left(\frac{2}{x-2}+\frac{x}{x+2}\right):\frac{x^2+4}{x^2-x-2}=$

$=\frac{2(x+2)+x(x-2)}{(x-2)(x+2)}\cdot\frac{x^2-x-2}{x^2+4}=\frac{2x+4+x^2-2x}{(x-2)(x+2)}\cdot\frac{x^2-x-2}{x^2+4}=$

$=\frac{x^2+4}{(x-2)(x+2)}\cdot\frac{x^2-x-2}{x^2+4}=\frac{x^2-x-2}{(x-2)(x+2)}$

Dar, $x^2-x-2=x^2-2x+x-2=x(x-2)+(x-2)=(x-2)(x+1)$

$\Rightarrow E(x)=\frac{(x-2)(x+1)}{(x-2)(x+2)}=\frac{x+1}{x+2}$

b) $E(a)=\frac{a+1}{a+2}\in\mathbb{Z}$ cu $a\in\mathbb{Z}$

Dar $\frac{a+1}{a+2}=\frac{a+2-1}{a+2}=\frac{a+2}{a+2}-\frac{1}{a+2}=1-\frac{1}{a+2}\in\mathbb{Z}$

$\Rightarrow a+2\in D_1\Rightarrow a+2\in\{\pm1\}$ $\begin{cases}a+2=1\Rightarrow a=-1\\ a+2=-1\Rightarrow a=-3\end{cases}$

Dar, $a\notin\{-2,-1,2\}\Rightarrow \boxed{a=-3}$ CONVINE

met II: $\frac{a+1}{a+2}\in\mathbb{Z}\Rightarrow a+2\mid a+1$

Dar, $a+2\mid a+2\Rightarrow a+2\mid(a+1)-(a+2)$

$\Rightarrow a+2\mid -1$

$\Rightarrow a+2\in D_{-1}\Rightarrow a+2\in\{\pm1\}\Rightarrow$ $\begin{cases}a+2=1\Rightarrow a=-1\\ a+2=-1\Rightarrow a=-3\end{cases}$

Și $a\notin\{-2,-1,2\}\Rightarrow a=-3$ CONVINE

EN / 2023 / SIMILARE / III 2

$$E(x)=\left(\frac{x^2-9}{x^2-16}-1\right):\left(\frac{1}{x+4}+\frac{1}{x-4}-\frac{3}{x^2-16}\right),$$

$x$ real, $x\neq -4$, $x\neq 4$, $x\neq \frac{3}{2}$

a) arată că $E(x)=\frac{7}{2x-3}$

b) $n=?$, $n\in\mathbb{N}$ astfel încât $E(n)\in\mathbb{N}$

a) deoarece $x^2-9=(x-3)(x+3)$ și $x^2-16=(x-4)(x+4)$

$$\Rightarrow E(x)=\left(\frac{x^2-9}{x^2-16}-\frac{x^2-16}{1}\right):\left(\frac{\overset{x-4}{1}}{x+4}+\frac{\overset{x+4}{1}}{x-4}-\frac{3}{x^2-16}\right)$$

$$E(x)=\frac{x^2-9-(x^2-16)}{x^2-16}:\frac{x-4+x+4-3}{x^2-16}$$

$$E(x)=\frac{\cancel{x^2}-9-\cancel{x^2}+16}{\cancel{x^2-16}}\cdot\frac{\cancel{x^2-16}}{2x-3}$$

$$E(x)=\frac{7}{1}\cdot\frac{1}{2x-3}=\frac{7}{2x-3}$$

b) $n\in\mathbb{N}$, $E(n)=\frac{7}{2n-3}\in\mathbb{N}\Rightarrow 2n-3\in D_7$

$\Rightarrow 2n-3\in\{1;7\}$

$2n-3=1\Rightarrow 2n=4\Rightarrow n=2\in\mathbb{N}$

$2n-3=7\Rightarrow 2n=10\Rightarrow n=5\in\mathbb{N}$

$\Rightarrow n\in\{2;5\}$

# EN/2023/EXAMEN/III 2

Fie $E(x)=\left(\frac{1}{(x+1)(x+2)}+\frac{1}{x+2}\right):\frac{x+3}{5(x+1)}$

cu $x\in\mathbb{R}$, $x\neq-3$, $x\neq-2$, $x\neq-1$

a) arată că $\frac{1}{(x+1)(x+2)}+\frac{1}{x+2}=\frac{1}{x+1}$

b) găsește mulțimea soluțiilor lui $E(x)=\frac{x-3}{8}$ unde $x\neq-3$, $x\neq-2$, $x\neq-1$

---

a) $\frac{1}{(x+1)(x+2)}+\frac{x+1}{x+2}=\frac{1+(x+1)}{(x+1)(x+2)}=\frac{x+2}{(x+1)(x+2)}=\frac{1}{x+1}$

b) Folosind a) $\Rightarrow E(x)=\frac{1}{x+1}\cdot\frac{5(x+1)}{x+3}=\frac{5}{x+3}$

ec devine $\frac{5}{x+3}=\frac{x-3}{8}\Leftrightarrow 40=(x-3)(x+3)\Leftrightarrow$

$40=x^2-9\Leftrightarrow x^2=49\Rightarrow x=\pm7$ și oricare din ele verifică $x\neq-3$, $x\neq-2$, $x\neq-1\Rightarrow x\in\{-7;7\}$